Reduction of Guard Interval by Impulse Compression for DMT Modulation on Twisted Pair Cables

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Abstract- **In multicarrier modulation schemes like Orthogonal Frequency Division Multiplex (OFDM) or Discrete Multi Tone (DMT) a guard interval represented by a cyclic prefix (CP) is inserted between successive multicarrier symbols in order to reduce intersymbol interference (ISI). The length of the guard interval is determined by the length of the channel impulse response (CIR). For a fixed guard interval, the longer the CIR, the higher the ISI is. In this paper, we have designed a time domain equalizer (TEQ) to compress the impulse response of several twisted pair lines with and without bridged taps. The reduction of ISI is shown as well as the resulting signal to noise ratio (SNR) in the presence of white Gaussian noise, near and far end crosstalk (NEXT, FEXT) for various equalizer designs.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) or Discrete Multi Tone (DMT) modulation techniques are currently applied in many applications like digital audio broadcasting (DAB), digital terrestrial video broadcasting (DVB-T) and asymmetrical digital subscriber line (ADSL) [1], [2]. Both modulation schemes, OFDM and DMT, use a guard interval, represented by a cyclic prefix (CP) of length G samples, to reduce intersymbol interferences (ISI) between multicarrier symbols. Therefore, the guard interval has to be at least as long as the channel impulse response (CIR), which is of length K samples. So the following condition

$$G \geq K - 1 \tag{1}$$

has to be met. As will be shown later, the guard interval reduces the effective symbol rate by the factor

$$\eta = \frac{1}{1 + G/N} \tag{2}$$

where N is the size of the Discrete Fourier Transform (DFT), because within the guard interval no further information is transmitted. Obviously, by increasing N for a given G, the effective symbol rate increased. However, the computational complexity, the peak-to-average factor of the transmitter signal, the memory requirements and the system delay increase simultaneously, which is not desirable. To increase the effective symbol rate also for long channel impulse responses the idea is to feed this impulse response into a time domain equalizer (TEQ) which will be designed such that the overall impulse response is compressed. Consequently, a short guard interval G can be used to maximize the efficiency.

In section II, the discrete time model of an OFDM/DMT system is described. In section III, the computation of the optimum filter coefficients of the TEQ is presented. As a design criterion for compressing the impulse response, we maximize the ratio

$$\gamma = \frac{E_{in}}{E_{out}} \tag{3}$$

between the energy E_{in} inside a specified time interval and the energy E_{out} outside this interval. In contrast to the design technique of [3], we do not require the rather stringent requirement of a positive definite matrix in our quadratic cost function. The reduction of intersymbol interference by the TEQ in a DMT system as a function of the number of TEQ coefficients and their quantization is evaluated for various practical twisted pair subscriber loops in section IV. Also, the performance in the presence of additive white Gaussian noise (AWGN) as well as near end and far end crosstalk (NEXT and FEXT) is examined in quite some detail. Finally, section V concludes the paper.

II. SYSTEM MODEL

Fig. 1 shows a DMT multicarrier system. In contrast to OFDM, which is a passband modulation scheme, DMT represents the equivalent baseband modulation. In this paper a discrete representation of an OFDM/DMT multicarrier system with the following parameters is considered:

- The length of the cyclic prefix is G samples.
- The size of the DFT is N samples with $N > G$, N even.
- The time base of the samples at the output of the transmitter is $T_A = 1/f_A$.
- Impulse response $h(n)$ of the time invariant twisted pair channel is of FIR type with length K samples.
- The TEQ is a FIR-filter with impulse response $a(n)$ of length L_F samples.
- Throughout the paper, we denote the number of samples of an FIR response or the number of coefficients of an FIR filter as the length of the response or the filter.

IEEE Global Telecommunications Conference, Conference Record pp. 1632-1636, San Francisco (USA), Nov. 2000

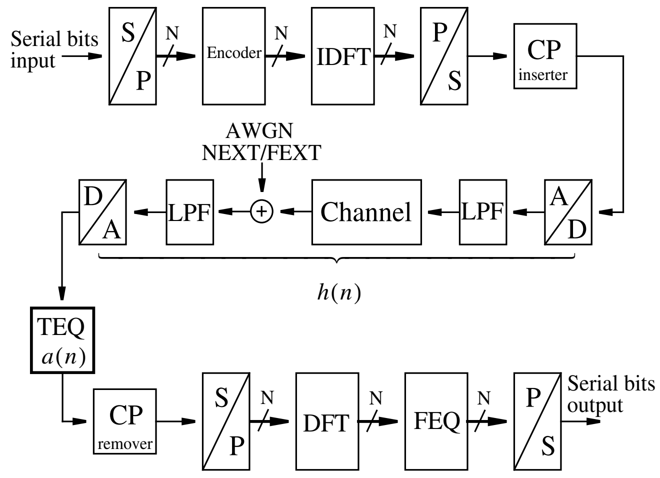

Fig. 1: Block diagram of a DMT transmission system

Applying a DFT of size N, there is a maximum number of $N/2$ tones in a DMT-system and N tones in an OFDM system. The duration of a single OFDM/DMT symbol at the output of the CP inserter is $T_S = (N+G)\cdot T_A$.
The ratio between the duration of an OFDM/DMT symbol without cyclic prefix and a symbol with cyclic prefix obviously is $\eta = N/(N+G)$ which is given in (2). The purpose of the TEQ in Fig. 1 is to compress the overall impulse response $h_{eff}(n) = a(n) * h(n)$ to keep G low and thus the symbol rate reduction factor η close to 1. $h_{eff}(n)$ is called the effective impulse response.

III. TIME DOMAIN EQUALIZER

We consider a DMT system. Thus, the channel impulse response $h(n)$ in Fig. 1 is real. As a consequence, real valued TEQ coefficients $a(n)$ are sufficient. Certainly, the length of $h_{eff}(n)$ is greater than the length K of $h(n)$, but the intention is to design the TEQ such, that $h_{eff}(n)$ has samples of significant size only within a window of $G+1<K$ consecutive samples. The remaining $|h_{eff}(n)|$ should be very small. We call an impulse $h_{eff}(n)$ with this property as compressed. The window begins with $h_{eff}(\xi)$ and ends with $h_{eff}(\xi+G)$. $h_{eff}(n)$ can be calculated in vector notation:

$$\vec{h}_{eff}^T = \underbrace{\begin{bmatrix} a(0) & \dots & a(L_F-1) \end{bmatrix}}_{\vec{a}^T} \cdot$$

$$\cdot \underbrace{\begin{bmatrix} h(0) & h(1) & \dots & h(K-1) & 0 & \dots & 0 \\ 0 & h(0) & h(1) & \dots & h(K-1) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & h(0) & h(1) & \dots & h(K-1) \end{bmatrix}}_{\|H\|}$$

$$= \vec{a}^T \cdot \|H\|$$

$$= [h_{eff}(0)\dots h_{eff}(\xi-1) \quad h_{eff}(\xi)\dots h_{eff}(\xi+G) \quad h_{eff}(\xi+G+1)\dots h_{eff}(K+L_F-2)] \tag{4}$$

We now separate $h_{eff}(n)$ into a part inside and another part outside the window. Thereby, the matrix $\|H\|$ is splitted into two parts $\|H\|_{in}$ and $\|H\|_{out}$.

$$\vec{h}_{eff,in}^T = \vec{a}^T \cdot \|H\|_{in} = [h_{eff}(\xi)\dots h_{eff}(\xi+G)]$$

$$\vec{h}_{eff,out}^T = \vec{a}^T \cdot \|H\|_{out}$$

$$= [h_{eff}(0)\dots h_{eff}(\xi-1) h_{eff}(\xi+G+1)\dots h_{eff}(K+L_F-2)] \tag{5}$$

Compression of $h_{eff}(n)$ can be achieved e. g. by minimization the energy $E_{out}(\xi)$ of $h_{eff}(n)$ outside the window, as it was done by [3]. However, this leads to quadratic optimization problem, where the corresponding matrix $\|H\|_{in} \cdot \|H\|_{in}^T$ has to be positive definite. For positive semidefinite matrices, the design of the TEQ is more complicated. To overcome this problem, we take in contrast to [3] the impulse energy ratio in (3) as a cost function to be optimized by maximizing $E_{in}(\xi)$. $E_{in}(\xi)$ is the energy of $h_{eff}(n)$ inside the window.

$$E_{in}(\xi) = \vec{a}^T \cdot \underbrace{\|H\|_{in} \cdot \|H\|_{in}^T}_{\|\psi\|_{in}} \cdot \vec{a} = \vec{a}^T \cdot \|\psi\|_{in} \cdot \vec{a} \ \geq 0 \tag{6}$$

$$E_{out}(\xi) = \vec{a}^T \cdot \underbrace{\|H\|_{out} \cdot \|H\|_{out}^T}_{\|\psi\|_{out}} \cdot \vec{a} = \vec{a}^T \cdot \|\psi\|_{out} \cdot \vec{a} \ > 0 \tag{7}$$

$\|\psi\|_{in}$ is a symmetric and positive semidefinite matrix. Of course, there is only a need for impulse compression, if at least one sample of $h_{eff,out}(n)$ is unequal to zero. Therefore $E_{out}(\xi) > 0$ holds and the matrix $\|\psi\|_{out}$ is a positive definite matrix. An eigenvalue decomposition can be performed as follows

$$\|\psi\|_{out} = \|\Theta\| \cdot \|\Lambda\| \cdot \|\Theta\|^T = (\|\Theta\| \cdot \sqrt{\|\Lambda\|}) \cdot (\|\Theta\| \cdot \sqrt{\|\Lambda\|})^T$$

$$= \|L\| \cdot \|L\|^T \tag{8}$$

where $\|\Lambda\|$ is a diagonal matrix containing the eigenvalues of $\|\psi\|_{out}$ and $\|\Theta\|$ a matrix containing the orthogonal eigenvectors of $\|\psi\|_{out}$. A Cholesky decomposition of $\|\psi\|_{out}$ is also adequate.
By inserting (8) into (7), we obtain

$$E_{out}(\xi) = \underbrace{\vec{a}^T \cdot \|L\|}_{\vec{v}^T} \cdot \underbrace{\|L\|^T \cdot \vec{a}}_{\vec{v}} = \vec{v}^T \cdot \vec{v} \tag{9}$$

$$\text{with} \quad \vec{v} = \|L\|^T \cdot \vec{a} \quad \text{and} \quad \vec{a} = (\|L\|^T)^{-1} \cdot \vec{v} \tag{10}$$

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Setting (10) into (6), we get

$$E_{in}(\xi) = \vec{a}^T \cdot \|\psi\|_{in} \cdot \vec{a} = ((\|L\|^T)^{-1} \cdot \vec{v})^T \cdot \|\psi\|_{in} \cdot ((\|L\|^T)^{-1} \cdot \vec{v})$$

$$= \vec{v}^T \cdot \underbrace{\|L\|^{-1} \cdot \|\psi\|_{in} \cdot (\|L\|^T)^{-1}}_{\|W\|} \cdot \vec{v} \qquad (11)$$

Now, γ in (3) becomes

$$\gamma(\xi) = \frac{E_{in}(\xi)}{E_{out}(\xi)} = \frac{\vec{v}^T \cdot \|W\| \cdot \vec{v}}{\vec{v}^T \cdot \vec{v}} \qquad (12)$$

In mathematics this term is known as Rayleigh-Quotient. It is independent of the absolute value of $\vec{v}$. For a given window position ξ, the Rayleigh-Quotient can range between the smallest and the largest eigenvalue, $\mu_{min}(\xi)$ and $\mu_{max}(\xi)$ of $\|W\|$ [4]. Therefore we obtain from (12):

$$\gamma_{max}(\xi) = (E_{in}(\xi)/E_{out}(\xi))_{max} = \mu_{max}(\xi) \qquad (13)$$

So, the only thing that is left to do for a given window position ξ, is to chose $\vec{v}$ in such a way, that the maximum eigenvalue of $\|W\|$ results. $\gamma_{max}(\xi)$ is obtained, if $\vec{v}$ is chosen as the eigenvector corresponding to $\mu_{max}(\xi)$. The coefficient vector of the TEQ can then be calculated by using (10). To obtain the largest value of $\mu_{max}(\xi)$ depending on ξ, the window position ξ is varied. The position with the largest $\mu_{max}(\xi)$ is indicated by $\xi = \xi_{max}$.

The solution described above that uses the maximum eigenvalue results in the same TEQ coefficients as the minimum eigenvalue solution given by [3]. This is plausible, because the cost functions are reciprocal.

IV. SIMULATION RESULTS

We have simulated the complete DMT communication system for various twisted pair subscriber lines on a computer [5]. Fig. 2 shows the different loops, which are used (transmitter at the left end and receiver at the right end). The wire diameter is 0,4 mm (AWG 26 cable type) and the sampling rate is $f_A = 1/T_A = 2{,}208\ MHz$ as used by ADSL [2]. The length of $h(n)$ amounts to $K = 1024$ samples.

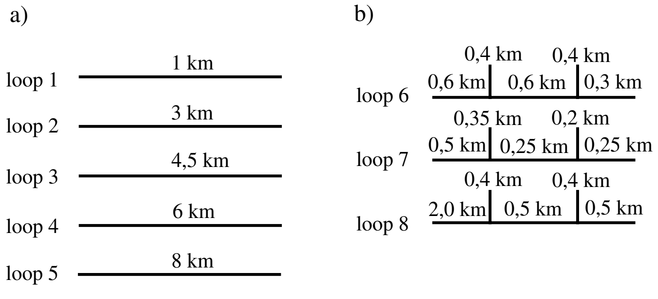

Fig. 2: Twisted pair subscriber loops (a) without and (b) with bridged taps. Wire diameter is 0,4 mm.

In Fig. 3, the channel impulse response $h(n)$ for loop 3 as well as the corresponding compressed impulse response $h_{eff}(n)$ are shown. The compressing effect of the TEQ of length $L_F = 8$ is clearly visible. The impulse energy ratio

$$\gamma_{max}(\xi_{max})/dB = 10 \cdot \log(E_{in}(\xi_{max})/E_{out}(\xi_{max})) \qquad (14)$$

improves from about 11 dB without TEQ to 53 dB with time domain equalization.

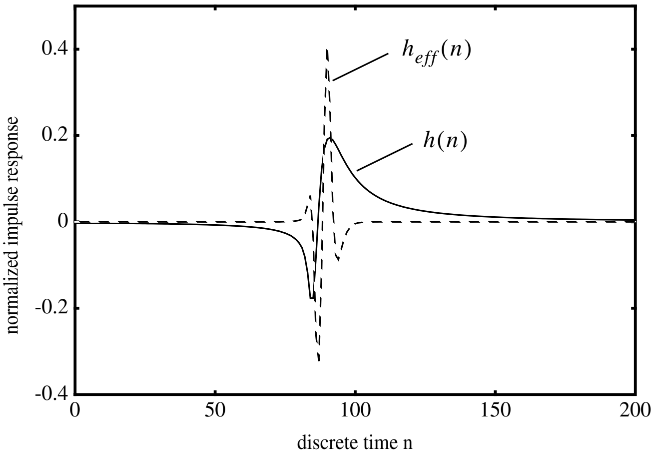

Fig. 3: Normalized impulse response $h(n)$ of loop 3 and compressed impulse response $h_{eff}(n)$ at the output of the TEQ with $L_F = 8$ coefficients. ADSL guard interval is $G = 32$.

In Fig. 4, the impulse energy ratio $\gamma_{max}(\xi_{max})$ is plotted versus the number L_F of equalizer coefficients for different loops without bridged taps. As can be seen, $L_F = 7$ to 10 is almost sufficient. For a given L_F, the longer the line, the lower $\gamma_{max}(\xi_{max})$ is.

Loops 6 to 8 in Fig. 2 contain bridged taps. These are branching lines to connect prospective new customers. Bridged lines mostly are not terminated correctly and thus cause reflections. Thus, the impulse response normally shows long tails and is broadened [6].

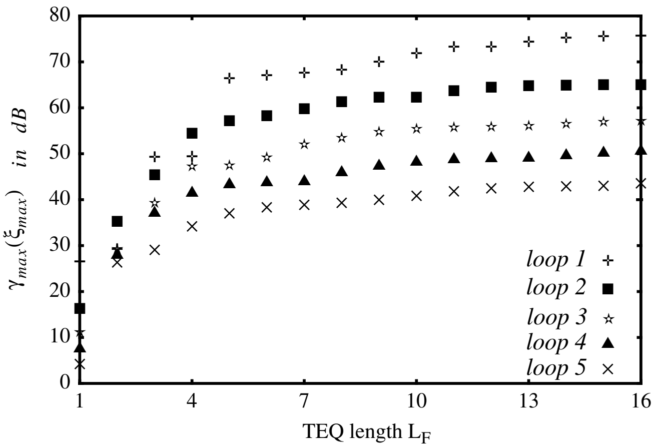

Fig. 4: Impulse energy ratio $\gamma_{max}(\xi_{max})$ versus equalizer length L_F for subscriber loops without bridged taps.

In Fig. 5, $\gamma_{max}(\xi_{max})$ is shown as a function of the number L_F of equalizer coefficients for loops 6 to 8 with bridged taps. To achieve a significant compressing effect, a higher number of coefficients $L_F > 15$ is required.

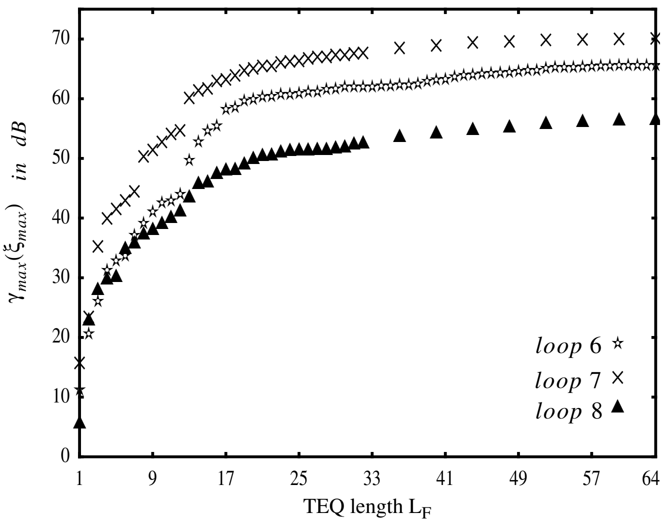

Fig. 5: Impulse energy ratio $\gamma_{max}(\xi_{max})$ versus equalizer length L_F for subscriber loops with bridged taps.

The minimum number L_F of TEQ coefficients required to achieve a certain $\gamma_{max}(\xi_{max})$ as a function of the window length $G+1$ is shown in Fig. 6 for loop 3 without bridged taps and in Fig. 7 for loop 6 with bridged taps. Already a small number L_F leads to a significant improvement of $\gamma_{max}(\xi_{max})$. If the window length $G+1$ is chosen very small, the required L_F rises enormously.

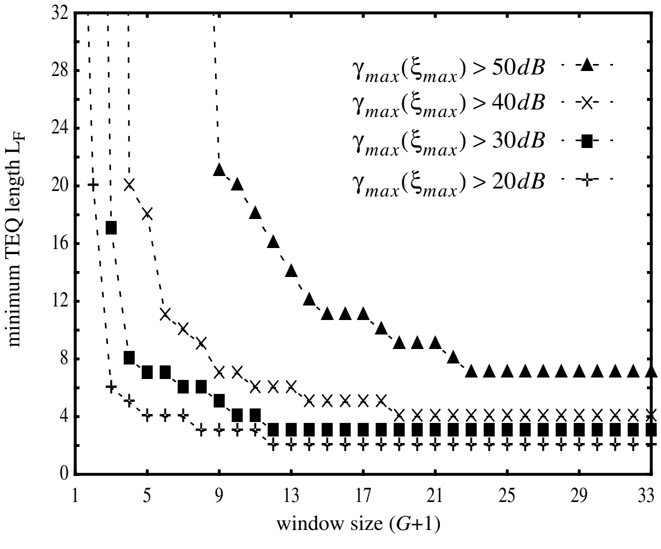

Fig. 6: Minimum TEQ length L_F versus window size $G+1$ for loop 3 without bridged taps.

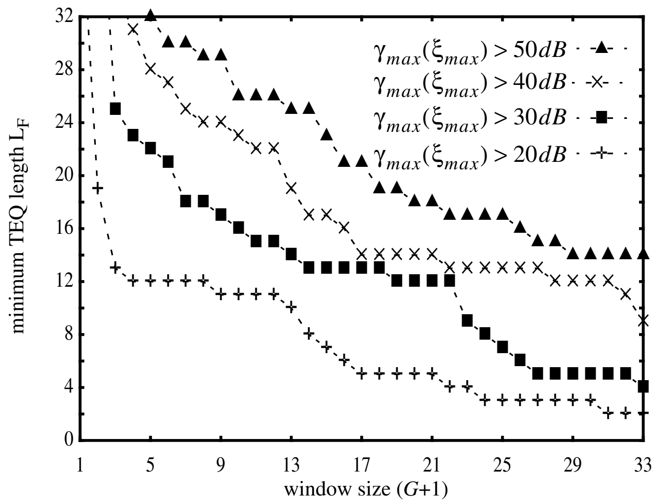

Fig. 7: Minimum TEQ length L_F versus window size $G+1$ for loop 6 with bridged taps.

For implementation of TEQ, the filter coefficients have to be quantized. In Table 1 $\gamma_{max}(\xi_{max})$ for different coefficient quantization levels, $G = 32$ and TEQ lengths $L_F = 8$ (loop 3) and $L_F = 16$ (loop 8) is listed. With 16 *bits/coefficient*, the results are almost ideal.

Tab. 1: $\gamma_{max}(\xi_{max})$ for TEQ with quantized coefficients. Example for loop 3 and loop 8, both with $G = 32$

	$\gamma_{max}(\xi_{max})$ *in dB*	
coefficient quantization	**loop 3** $L_F = 8$	**loop 8** $L_F = 16$
without TEQ	11,1	5,5
TEQ with 2 bits/coefficient	38,2	26,5
TEQ with 4 bits/coefficient	51,2	34,2
TEQ with 8 bits/coefficient	51,3	44,7
TEQ with 16 bits/coefficient	53,4	47,2
TEQ with no quantization	53,4	47,2

So far, we have seen, that TEQ can provide an output impulse $h_{eff}(n)$ which has minimum energy outside a given guard interval and thus is able to reduce intersymbol interference of adjacent DMT symbols. However, the noise spectrum given by the channel is changed at the output of the TEQ. This issue is discussed next.

Fig. 8 shows the signal-to-noise ratio (SNR) for the subchannels of an ADSL system with $N = 512$, if additive white Gaussian noise (AWGN) is present at the input of the TEQ. Transmission loop 2 is used. The power density spectrum (pds) of the AWGN is assumed to be -120 dBm/Hz and the pds of the transmitting signal is -40 dBm/Hz [2].

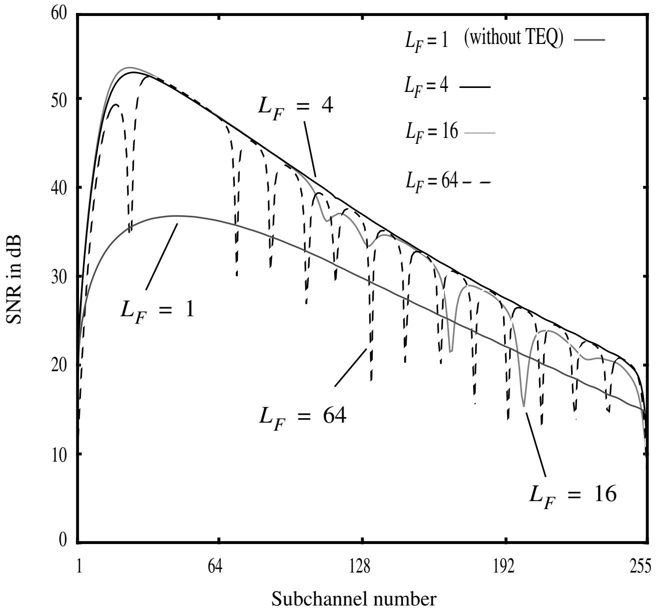

Fig. 8: SNR versus subchannel number of an ADSL system with loop 2 corrupted by AWGN and with TEQ of various lengths L_F.

Apart from the edges and without TEQ, the SNR function is continuously declining with increasing subchannel number (carrier frequency) due to the increasing attenuation of the line. If a TEQ is present, the SNR is increasing in most subchannels caused by the reduction of interferences. But simultaneously, notches in the SNR function occur. The number and depth of such notches increases with TEQ length. Altogether, the geometric mean value of the SNR, averaged over all subchannels, gets lower with increasing TEQ length for $L_F > 1$.
In Table 2, the geometric mean of the SNR for different lengths of the TEQ is listed. Results from system simulations with AWGN as well as with NEXT/FEXT disturbances caused by similar signals in the adjacent pairs within the same cable are shown.

Tab. 2: Geometric mean of the SNR as a function of TEQ length L_F for loop 2

	SNR in dB	
TEQ length L_F	**AWGN**	**FEXT / NEXT**
without TEQ ($L_F = 1$)	28,0	27,0 / 7,7
TEQ with $L_F = 4$	36,3	31,4 / 6,5
TEQ with $L_F = 16$	35,2	27,8 / 6,4
TEQ with $L_F = 64$	34,1	26,7 / 5,2

The NEXT/FEXT is modeled according to [7] and its sources are assumed to be white and gaussian with equal power. In both cases, the average SNR decreases with increasing TEQ length. Furthermore, the achieved SNR with AWGN is much higher than for NEXT/FEXT distortion. The impact of L_F on symbol error probability in the presence of AWGN and NEXT/FEXT is for further study.

V. CONCLUSION

We have investigated the compression of the channel impulse response by introducing an FIR time domain equalizer (TEQ) into an OFDM/DMT system. As a result, the length of the guard interval can be reduced and thus the symbol rate is increased. The TEQ was designed by maximizing $\gamma(\xi) = E_{in}(\xi)/E_{out}(\xi)$, where $E_{in}(\xi)$ is the energy inside and $E_{out}(\xi)$ is the energy outside a given window of the output impulse of the TEQ. Several optimum TEQ have been designed for a DMT system on twisted pairs. A TEQ with about 10 coefficients can increase γ by 30 dB and even much more, depending on the subscriber loop without bridged taps. For loops with bridged taps, an equalizer with more than about 15 coefficients is required. For a digital implementation of the TEQ, quantization of coefficients should be done at least with 8 bits/coefficient. We have investigated the performance of the TEQ in the presence of additive white Gaussian noise as well as near and far end crosstalk. The SNR averaged over all DMT carriers is reduced with increased length of the TEQ and $L_F > 1$. For a TEQ with 64 coefficients, the SNR penalty is about 2 dB for AWGN and about 5 dB for FEXT compared to a scheme with a 4 tap TEQ.

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