A rate one–half code for approaching the Shannon limit by 0.1dB

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Abstract

A serially concatenated code is presented which exhibits a turbo cliff at 0.28dB. The concatenation consists of an outer rate one–half repetition code and an inner rate one recursive convolutional code. The iterative decoding scheme was designed using the extrinsic information transfer chart (EXIT chart).

Keywords

Iterative decoding, serially concatenated codes

Introduction: The discovery of parallel concatenated codes (PCC) [1] has motivated the search for other code concatenations and corresponding iterative decoders which can operate close to the theoretical capacity limit. For the binary input/continuous output additive white Gaussian noise channel, the Shannon capacity limit [2] is at $E_b/N_0 = 0.19$dB (code rate one–half). In this Letter we present a serially concatenated code (SCC) [3] which achieves a bit error rate (BER) of less than 10^{-5} at $E_b/N_0 = 0.28$dB. The code was designed using the EXIT chart [4], [5]. For large interleavers, the EXIT chart can predict the iterative decoding convergence solely based on mutual information transfer characteristics of individual component codes.

Coding scheme: The encoder is shown in Fig. 1. An outer rate one–half repetition code is connected through a random interleaver with an inner rate one recursive convolutional code of memory 3 (feedback polynomial $G_r = 017$, feedforward polynomial $G = 07$). A switch allows to substitute inner coded bits by inner systematic bits ("doping"), with a systematic to coded bit–ratio of n_s/n_c. The term "doping" is used rather than "puncturing" in order to express that the coded bits are *substituted* by their respective systematic counterparts without changing the inner code rate; the number of systematic bits is typically much smaller than the number of coded bits, i. e. $n_s/n_c << 1$. For the results in this Letter a doping ratio of $n_s/n_c = 1 : 100$ was used.

Iterative Decoder: The inputs to the inner soft in/soft out decoder (BCJR algorithm [6]) are channel observations on the inner coded bits and *a priori* log–likelihood ratios A_1 (L–values [7]) on the inner information (i. e. systematic) bits. The inner decoder outputs extrinsic and channel information E_1 which is forwarded through a deinterleaver to become the *a priori* input A_2 for the outer soft in/soft out repetition decoder. The repetition decoder computes extrinsic information E_2 which is re–interleaved and fed back as *a priori* knowledge A_1 to the inner decoder to reduce the BER in further iterative decoding steps. Note that the *a posteriori* probability decoding rule for the repetition decoder turns out to be a swapping operation: For two outer *a priori* L–values $A_{2,0}$, $A_{2,1}$ stemming from the same outer information bit, the *a posteriori* L–values are easily calculated to $D_{2,0} = D_{2,1} = A_{2,0} + A_{2,1}$ and thus the corresponding extrinsic L–values at the decoder output are $E_{2,0} = D_{2,0} - A_{2,0} = A_{2,1}$ and $E_{2,1} = D_{2,1} - A_{2,1} = A_{2,0}$, which is a simple swapping operation performed on the outer coded bits.

Extrinsic information transfer chart: Rather than rehearsing the EXIT chart technique in detail, we will focus on the particularities of the code concatenation of interest.

Since the outer repetition decoder is a plain swapper, its extrinsic transfer characteristic [4] is represented by a diagonal line $I_{E_2} = I_{A_2}$ in the EXIT chart (Fig. 2). We computed the extrinsic information transfer characteristics of all inner rate one recursive convolutional codes up to memory 6 at $E_b/N_0 = 0.5$dB. In this code search we were looking for transfer characteristics which do not intersect with the characteristic of the outer repetition code. The preferred inner transfer characteristic should resemble the shape of a straight line from $I_{E_1}(I_{A_1} = 0) > 0$ to $(I_{E_1}, I_{A_1}) = (1, 1)$ to allow for steady convergence. We noticed that the transfer characteristics of the most promising candidates start at the origin $I_{E_1}(I_{A_1} = 0) \approx 0$, which, unfortunately, makes these codes unsuitable for iterative decoding; the iteration would not even get started. We observed that it is possible to "open up" those inner transfer characteristics (i. e. achieve $I_{E_1}(I_{A_1} = 0) > 0$) by systematic doping, while, however, sacrificing some extrinsic output strength at higher *a priori* input I_{A_1}. Fig. 2 depicts an inner recursive convolutional code whose extrinsic transfer characteristic has the desired properties. By systematic doping we made it usable for iterative decoding. The trajectory at 0.4dB visualises the exchange of extrinsic information between inner and outer decoder. The trajectory is a simulation result of the *iterative decoder*, whereas the extrinsic transfer characteristics are based on simulations of the *individual component codes*. The good agreement of trajectory and transfer characteristics verifies the convergence prediction capabilities of the EXIT chart.

Bit error rate chart: For the BER curves of Fig. 3 we simulated 10^7 information bits. The proposed SCC with inner memory 3 code $(G_r, G) = (017, 07)$ outperforms the classic rate one–half PCC of [1] with memory 4 constituent codes $(G_r, G) = (037, 021)$, although many iterations are needed. However, more iterations would not improve the PCC performance, as indicated by the *pinch–off limit*. It denotes the E_b/N_0–value at which both transfer characteristics are just about to intersect; below the pinch–off limit, no convergence of iterative decoding towards low BER is possible.

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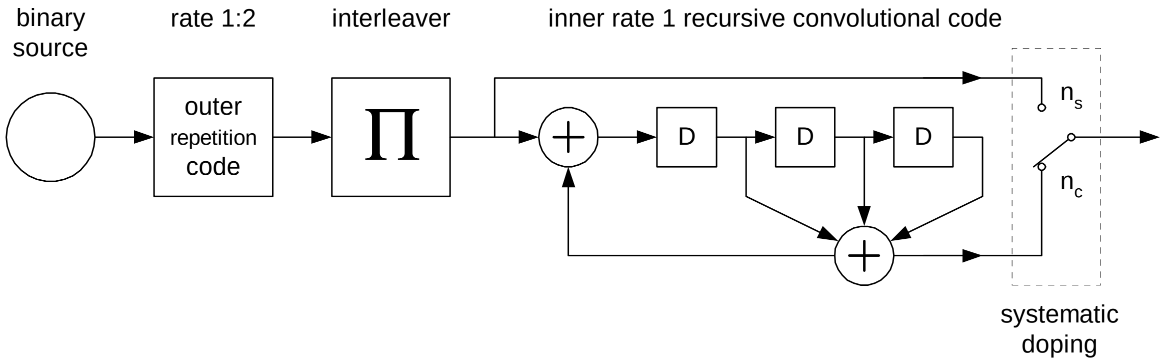

Fig. 1. Serially concatenated code with outer rate 1:2 repetition code and inner rate 1 recursive convolutional code.

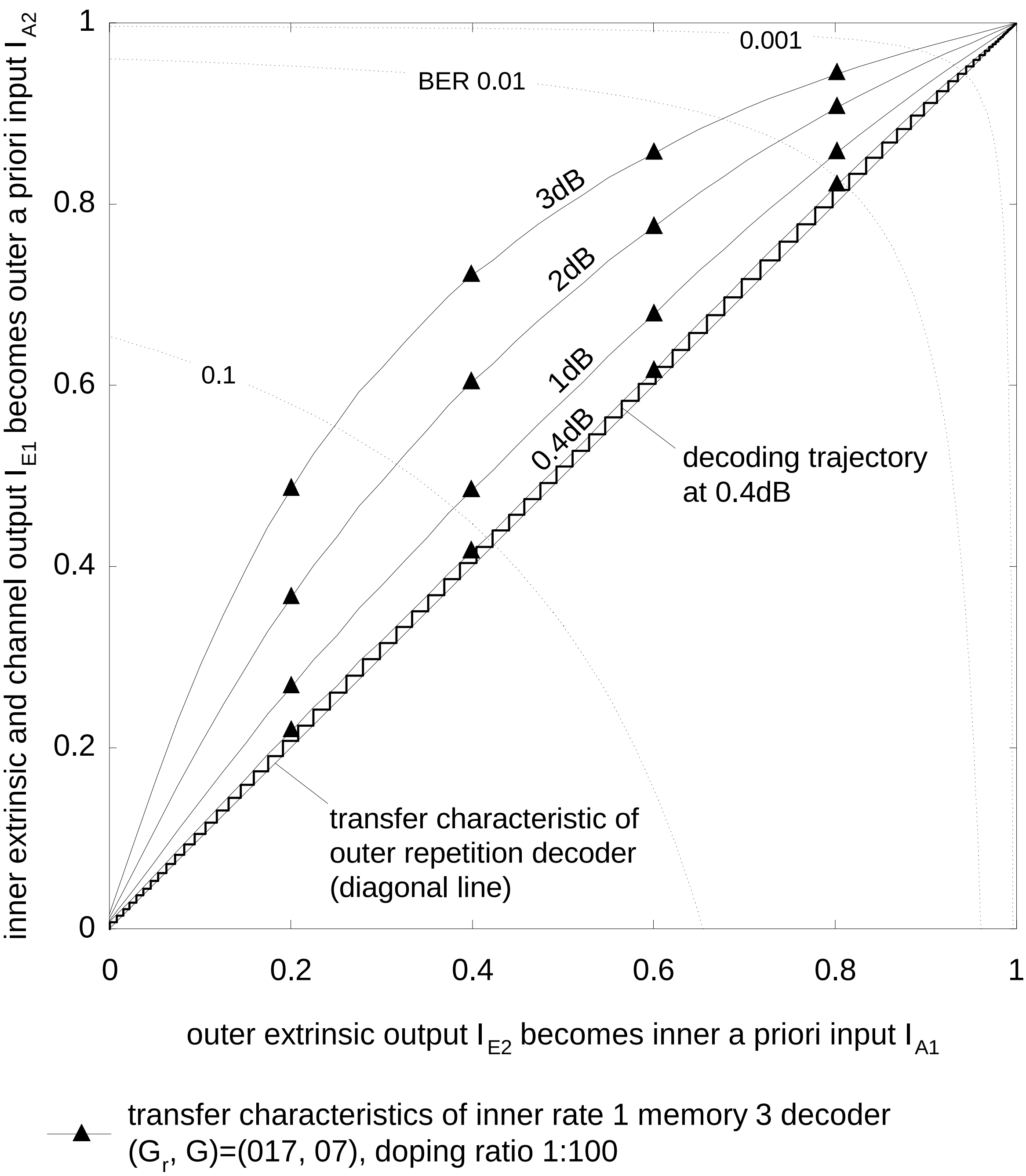

Fig. 2. Extrinsic information transfer chart with inner decoder transfer characteristics for a set of E_b/N_0–values; the BER is given as contour lines.

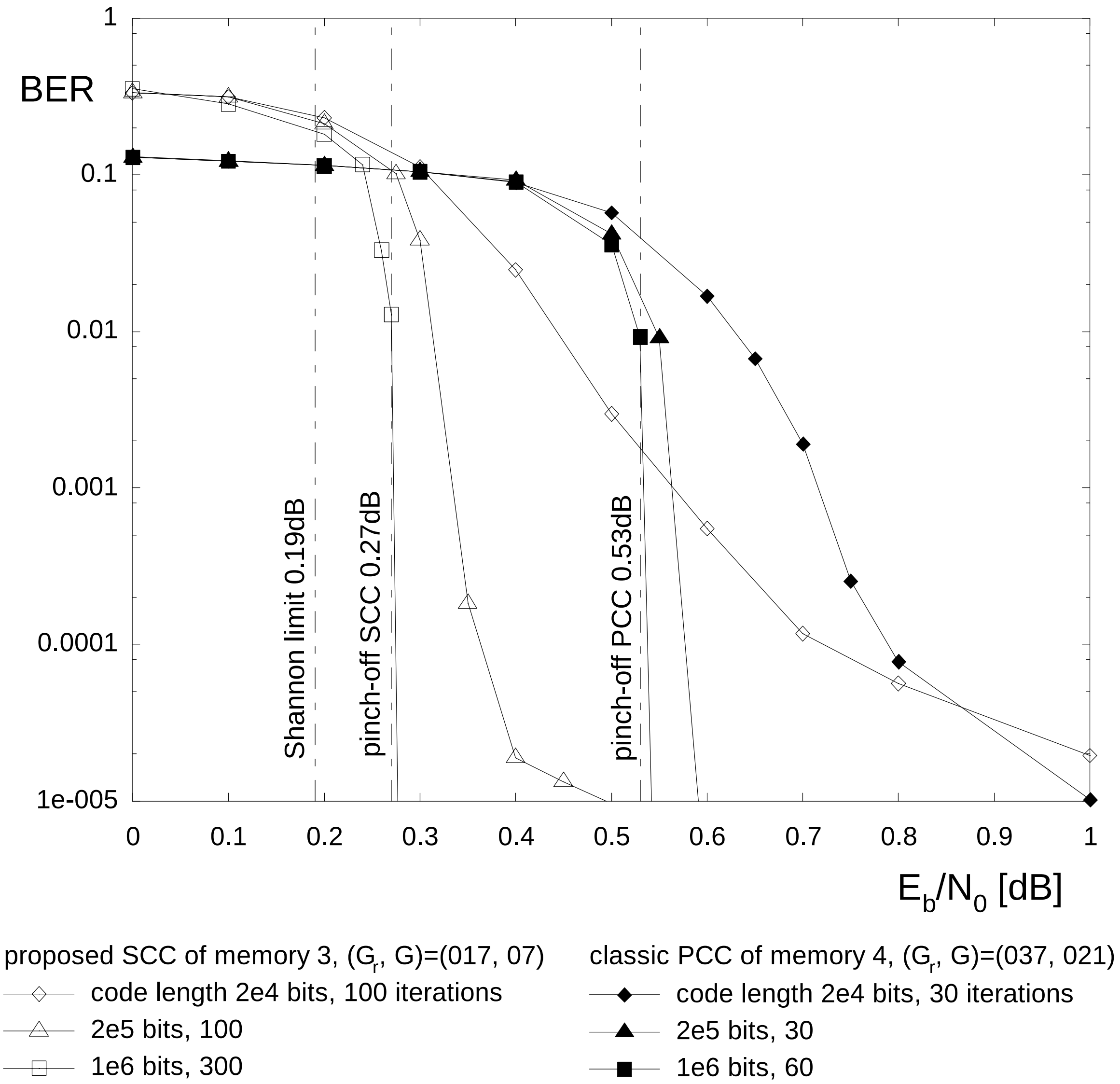

Fig. 3. Bit error rate chart of proposed SCC in comparison with classic PCC; code rate one–half.