Extended Lapped Transforms for Digital Multicarrier Modulation

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Abstract **– In today's multicarrier modulation systems such as orthogonal frequency division multiplexing (OFDM) or discrete multitone (DMT), modulation is accomplished digitally by the use of the inverse discrete Fourier transform (IDFT). Instead of the IDFT, any other orthogonal transform can be used. In this paper a multicarrier scheme based on extended lapped transforms (ELT) is investigated. With the ELT, a better spectral containment of the subchannels is achieved, which also leads to a superior power spectral density of the multicarrier signal. Further, the ELT multicarrier modulation is shown to be more robust against frequency selective interferers present on the transmission channel.**

I. Introduction

In multicarrier modulation, a transmission channel is partitioned into a multitude of subchannels, each with its own associated carrier. Each subchannel carries only a portion of the data to be transmitted by the entire multicarrier system. In today's systems such as orthogonal frequency division multiplexing (OFDM) and discrete multitone modulation (DMT) the modulation is accomplished digitally, using the inverse discrete Fourier transform (IDFT). For demodulation the discrete Fourier transform (DFT) is used [1]. The DMT is a special case of OFDM, which produces a real-valued baseband signal.

Instead of the IDFT/DFT any other orthogonal transform can be used for digital multicarrier modulation [2]. This paper presents a multicarrier system based on extended lapped transforms (ELT) and shows advantages in spectral containment over a DMT system. The ELT is compared with the DMT, because both produce a baseband signal.

The DFT and the ELT can also be described with an equivalent FIR filterbank [2]. The impulse responses of the filters are the basis functions of the direct or inverse transform, respectively. The inverse ELT (IELT) is used for modulation, the ELT for demodulation. The ELT based filterbank is expected to show better filter characteristics than the IDFT, because the filter impulse responses have more coefficients in the ELT case, which will be explained in section II. Section III provides a general model of a filterbank for multicarrier modulation that can be used with either transform, and some differences between the ELT and DFT are described. In section IV, properties such as power spectral density (PSD), time domain spread and frequency domain spread of the filterbank impulse and frequency responses for both filterbanks will be presented. The robustness in the presence of a frequency selective interferer on the transmission channel will be investigated.

II. Overview of Extended Lapped Transforms

With a conventional block transform such as the IDFT/DFT, for M input symbols M output symbols are computed. In contrast, the IELT maps M input symbols (Fig. 1(a)) onto $2KM$ output symbols (Fig. 1(b)). The variable K is called overlapping factor and can be any integer greater than or equal to 1. Consecutive output blocks overlap by $2KM - M$ symbols. In order to get the resulting symbol sequence, the symbols in the overlapping areas of the output blocks are added (overlap-add-algorithm). Thus, for every M input symbols, M output symbols are ready for transmission at the modulator output (Fig. 1(c)) with a delay of $D = 2K - 1$. The transform is called "extended" in order to distinguish it from the lapped orthogonal transform (LOT) [3].

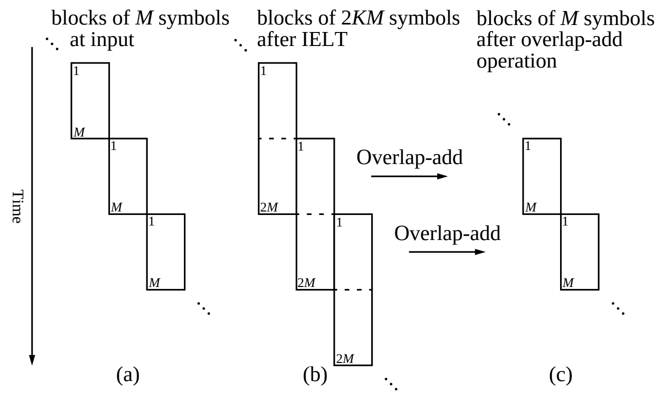

Fig. 1. IELT and overlap-add operation for overlapping factor $K = 1$

The direct ELT, which is used at the demodulator, maps input blocks of $2KM$ symbols onto blocks of M output symbols. It is clear, that consecutive $2KM$ input symbols overlap by $2KM - M$ symbols, because of the overlap-add-operation at the modulator.

If the IELT and the ELT are orthogonal transforms, the M output symbols from the demodulator are identical to the M input symbols to the modulator, provided that the transmission channel is ideal. This property is also called perfect reconstruction (PR)[4]. If the transform basis functions are longer than M, orthogonality not only means orthogonality of the transform basis functions themselves, but also orthogonality of shifts by integer multiples of M of the transform basis functions [2]. As the basis functions are $2KM > M$ long for the IELT or ELT, respectively, they need to satisfy this general-

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ized orthogonality. If the IELT is expressed by the $2KM \times M$ real-valued transform matrix

$$\boldsymbol{P} = \left[\boldsymbol{P}_0^T \; \boldsymbol{P}_1^T \; \dots \; \boldsymbol{P}_{2K-1}^T\right]^T \tag{1}$$

with the $M \times M$ submatrices $\boldsymbol{P}_i$, perfect reconstruction is achieved, if [3]

$$\sum_{m=0}^{2K-1-l} \boldsymbol{P}_m \boldsymbol{P}_{m+l}^T = \delta(l) \cdot \boldsymbol{I}$$
$$l = 0, 1, \dots, 2K-1 \quad , \tag{2}$$

where $(\cdot)^T$ denotes the transpose operator, $\delta(l)$ means the Kronecker symbol, and $\boldsymbol{I}$ is the identity matrix. The transform basis functions in the columns of $\boldsymbol{P}$ are the impulse responses of the synthesis filterbank (modulator). Thus, the impulse responses have $2KM$ real coefficients. The transform matrix for the direct ELT is the $M \times 2KM$ real-valued matrix $\boldsymbol{P}^T$, which is just the transpose of the transform matrix for the IELT. The rows of $\boldsymbol{P}^T$ are the impulse responses of the analysis filterbank (demodulator). Again, the impulse responses have $2KM$ coefficients.

The basis functions of the IELT can be generated by modulation of a real-valued lowpass prototype function $h(n)$ with a cosine function. The transform coefficients are expressed as

$$p_{n,k} = h(n)\sqrt{\frac{2}{M}}\cos\left[\left(n + \frac{M+1}{2}\right)\left(k + \frac{1}{2}\right)\frac{\pi}{M}\right]$$
$$n = 0, 1, \dots, 2KM-1 \qquad k = 0, 1, \dots, M-1 \,, \tag{3}$$

where $p_{n,k}$ denotes the element in the n-th row and the k-th column of the transform matrix $\boldsymbol{P}$. An ELT based on (3) allows for perfect reconstruction, if the lowpass prototype $h(n)$ has even symmetry

$$h(2KM-1-n) = h(n)$$
$$n = 0, 1, \dots, 2KM-1 \tag{4}$$

and satisfies

$$\sum_{i=0}^{2K-2s-1} h(n+iM)h(n+iM+2sM) = \delta(s)$$
$$n = 0, 1, \dots, M/2-1 \qquad s = 0, 1, \dots, K-1 \,. \tag{5}$$

Suitable lowpass prototypes $h(n)$ can be numerically computed, using the algorithms described in [3].

The factor $(k + 1/2)$ in the argument of the cosine in (3) has the effect, that the modulation frequencies[1] are the centers of the intervals $[k(\pi/M), (k+1)(\pi/M)]$. That means, that the frequency interval $[0, \pi]$ is divided into M subchannels. The channel spacing is illustrated in Fig. 2.

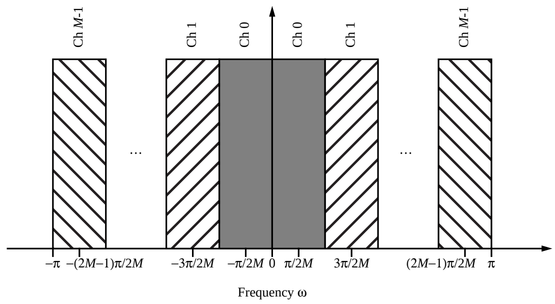

Fig. 2. Channel spacing for IELT with cosine modulated lowpass prototype $h(n)$

Note, that the IELT channel spacing is different from the IDFT channel spacing, where the center frequencies of the complex subchannels are $\omega_{k,IDFT} = 2k\pi/M$ and the subchannels are wider.

III. System model

In this section, system models for an ELT multicarrier system and for a reference DMT system are presented. Both systems will be able to transmit $M/2$ complex QAM symbols while generating a baseband signal. The basic filterbank structure is the same for both cases. The main difference is the loading of symbols onto the carriers.

A. ELT multicarrier system

As the ELT system is composed of M channels with real-valued impulse responses, the complex QAM symbols are split into their real and imaginary parts, which are then transmitted separately in adjacent frequency bands as shown in Fig. 3. The complete ELT system is thus capable of transmitting $M/2$ complex QAM symbol series $X_0(m), \dots, X_{M/2-1}(m)$ while generating a baseband signal at the output of the synthesis filterbank. The impulse responses of the synthesis filters $f_k(n)$ and analysis filters $h_k(n)$ are related to the transform matrix $\boldsymbol{P}$ by

$$f_k(n) = h_k(2KM-1-n) = p_{n,k}$$
$$k = 0, 1, \dots, M-1 \qquad n = 0, 1, \dots, 2KM-1 \,. \tag{6}$$

The system model in Fig. 3 also includes the transmission channel with additive noise. In this paper, ELTs with overlapping factor $K = 1$ and $K = 2$ will be investigated.

B. DMT system

The DMT system also carries $M/2$ complex QAM symbol series. In order to obtain a real-valued baseband signal, one half of the M inputs to the synthesis filterbank is loaded with QAM symbols, while the other half is loaded with the conjugate complex symbols of the first half. The impulse responses

[1] Please note that throughout this paper, (radian) frequencies (e.g. ω, f) are normalized. In order to introduce a physical frequency $\tilde{f}$ in Hz, use the sampling frequency $\tilde{f}_s$ and the relation $\omega = 2\pi\tilde{f}/\tilde{f}_s$

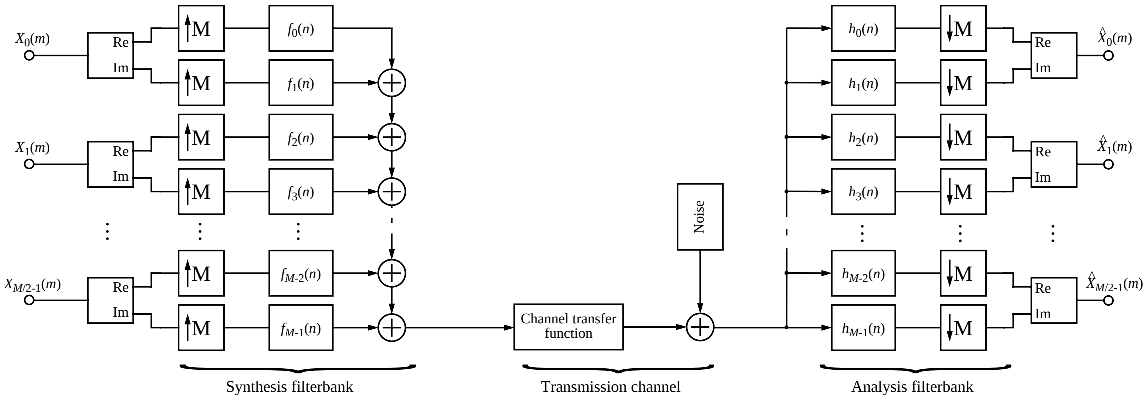

Fig. 3. System model of ELT multicarrier system for transmission of M / 2 complex 16-QAM symbols

of the filterbanks are obtained through complex modulation of the lowpass prototype

$$h_{DMT}(n) = \begin{cases} (1/\sqrt{M}), & 0 \le n \le M-1 \\ 0, & \text{else} \end{cases} \quad . \tag{7}$$

The model for the DMT system is quite similar to the ELT model in Fig. 3. The differences lie only within the filter impulse responses and the loading of the symbols. For a detailed description of a DMT system including the impulse responses of the synthesis and analysis filters, please refer to [5]. For the reference DMT system in this paper, no guard interval will be inserted.

IV. Performance Analysis

In this section, the filters of the ELT system will be analyzed and compared to the reference DMT system. First the magnitude frequency responses of the filters are presented. Then the time domain and frequency domain spread are calculated. Also, the power spectral density (PSD) for ELT and DMT systems will be examined. Finally, the overall system performance in presence of a frequency selective interferer on the channel will be investigated.

A. Magnitude frequency responses

Fig. 4 shows the magnitude frequency responses of 6 contiguous ELT filters with $K = 2$ and 3 contiguous DMT filters, both for $M = 128$. Note, that in both cases, 3 complex QAM symbols could be transmitted. The stopband attenuation is 30 dB for the ELT, while it is only 13 dB for the DMT. The much higher spectral containment of the subchannels of the ELT is clearly observable from Fig. 4.

B. Time and frequency domain spread

We define the time domain spread of a discrete time function $g(n)$ by

$$D_n = \frac{1}{E} \sum_{n=-\infty}^{\infty} (n-\bar{n})^2 |g(n)|^2 . \tag{8}$$

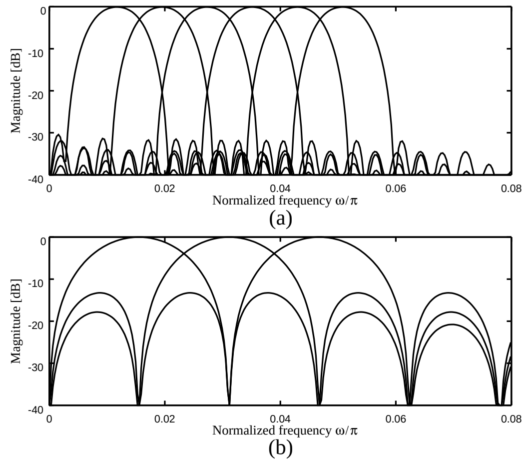

Fig. 4. Contiguous magnitude frequency responses of: (a) ELT with K=2; (b) DMT

The energy E and the time center $\bar{n}$ are given as

$$E = \sum_{n=-\infty}^{\infty} |g(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|G(e^{j\omega})\right|^2 d\omega \tag{9}$$

$$\bar{n} = \frac{1}{E} \sum_{n=-\infty}^{\infty} n |g(n)|^2 , \tag{10}$$

where

$$G(e^{j\omega}) = \sum_{n=-\infty}^{\infty} g(n) e^{-j\omega n} . \tag{11}$$

Similar to (8), we define the frequency domain spread as

$$D_\omega = \frac{1}{2\pi E} \int_{-\pi}^{\pi} (\omega - \bar{\omega})^2 \left|G(e^{j\omega})\right|^2 d\omega \tag{12}$$

where the frequency center is

$$\bar{\omega} = \frac{1}{2\pi E} \int_{-\pi}^{\pi} \omega \left|G(e^{j\omega})\right|^2 d\omega . \tag{13}$$

The lower the time domain spread, the more concentrated is the energy of a function in the time domain, which makes the signal more robust against inter-symbol interference (ISI). The lower the frequency domain spread, the more concentrated is the energy in the frequency domain, which makes the signal more robust against inter-channel interference (ICI). There always has to be a trade-off between both the frequency and time domain spread. So it is desired, that the product of both is small. Table I gives both time and frequency domain spreads and their product for the DMT and the ELT prototypes with overlapping factors $K = 1$ and $K = 2$ for $M = 128$.

TABLE I
TIME AND FREQUENCY DOMAIN SPREAD FOR ELT AND DMT

	DMT	ELT	
		$K = 1$	$K = 2$
Time domain spread D_n	$1.4 \cdot 10^3$	$2.7 \cdot 10^3$	$3.5 \cdot 10^3$
Frequency domain spread D_ω	$2.2 \cdot 10^{-2}$	$1.2 \cdot 10^{-3}$	$2.3 \cdot 10^{-4}$
Product $D_n \cdot D_\omega$	30.8	3.24	0.805

From Table I, it can be concluded that the DMT prototype has the lowest and the ELT with overlapping factor $K = 2$ has the highest time domain spread D_n. This is to be expected, as the DMT filter prototype has only M coefficients while the ELT filter prototypes have $2KM$ coefficients. The benefits from having more coefficients can be seen very clearly when looking at the frequency domain spread D_ω in Table I. The ELT with $K = 2$ has the lowest frequency domain spread D_ω, whereas the DMT has the highest D_ω. Thus, the more coefficients the prototype functions have, the lower the frequency domain spread. A low frequency domain spread of a filter means, that it is narrow band. Thus, the conclusions that can be drawn from the results in Table I are, that ELT filterbanks concentrate the energy of a specific subchannel stronger around the center frequency of that subchannel than the DMT does. The spectral overlap of subchannels is much less significant for ELTs than for the DMT. Also, the spectral concentration of ELTs gets better with increasing overlapping factor K. As a direct result from the higher spectral concentration by the ELT filterbanks, an improved robustness against frequency selective interferers on the transmission channel is to be expected. This will be investigated later in this paper. Finally, the value of $D_n \cdot D_\omega$ for the ELT with $K = 1$ is lower by about factor 10 than for the DMT, and even about factor 40 lower than the DMT value for the ELT with $K = 2$. That means that the trade-off between frequency and time domain spread is superior with the ELTs.

C. Power spectral density

We compare the PSD of the output signals of the two modulation schemes. As the spectral containment of the subchannels is better for ELTs than for the DMT, impacts on the PSD are also to be expected. Fig. 5(a) shows the PSD for a DMT signal and Fig. 5(b) and (c) for ELT signals with overlapping factors $K = 1$ and $K = 2$. In each case, 107 out of $M = 128$ filterbank inputs are loaded with uncorrelated symbols with equal power.

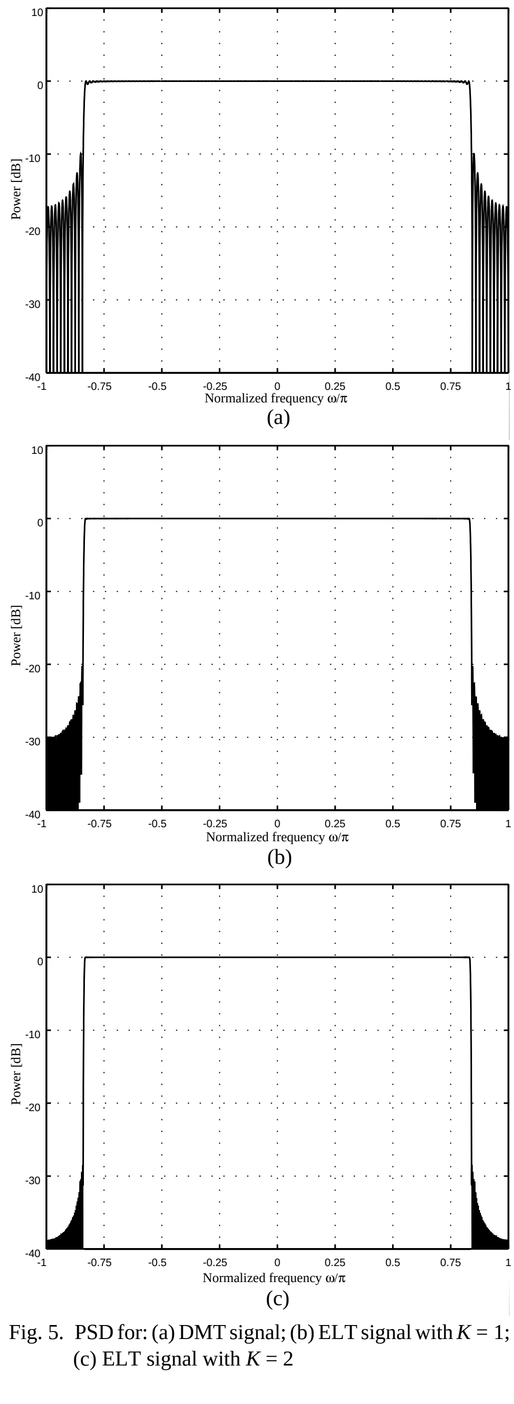

Fig. 5. PSD for: (a) DMT signal; (b) ELT signal with $K = 1$; (c) ELT signal with $K = 2$

Again, it can be observed that ELT signals have a better spectral containment than the DMT signal. The higher the overlapping factor *K*, the better is the spectral concentration in the passband. For the ELT with $K = 1$, the power in the unused frequency range (i.e. unmodulated carriers) is up to 13dB lower than the DMT power there, and even up to 22dB lower for the ELT with $K = 2$. This dramatically reduces the requirements for analog postfiltering or frequency guardbands in frequency division multiplexing (FDM) of several multicarrier signals.

D. Frequency selective interferer

In this section, the symbol error ratios (SER) for ELT and DMT multicarrier transmission with a frequency selective interferer present on the transmission channel are investigated by computer simulation. Such ingress noise at the receiver may be the result of a nearby AM station, for example. For the simulations, the interferer is modeled as a sinusoid and thus exhibiting two spectral lines. One is at $\omega_{i,\,neg} = -2\pi(f_i/M)$ and the other one is at $\omega_{i,\,pos} = 2\pi(f_i/M)$, where f_i is a variable normalized frequency factor with $0 < f_i < M/2$. The power of the interferer is chosen such that the signal-to-noise ratio (SNR) is -2dB. In addition to the interferer, there is additive white Gaussian noise (AWGN) with zero mean on the channel, which yields an SNR of 18dB, if no interferer is present. Fig. 6 shows the SER of each channel for a DMT system and for ELT systems with $K = 1$ and $K = 2$, with $f_i = 10$ in Fig. 6a and $f_i = 10.5$ in Fig. 6b. Both systems carried 16-QAM symbols on the $M/2$ subchannels, where $M = 128$.

It can be easily observed, that for the DMT system, the SER strongly depends on the frequency of the interferer. The ELT systems are more robust against the interferer frequency and the SER curves keep their shapes while they are slightly shifted over the channel index. The total SER, which is the weighted sum of the SER of each channel, does not vary much for the ELT systems, if the interferer frequency varies. For the DMT system there are order-of-magnitude variations, because the interferer can be at one of the frequency response minima common to all but one channel, or it can be at any other frequency, where all channels have only low attenuation (cf. Fig. 4). The minimum achievable SER is in the same order of magnitude for all three systems [6]. As real interferers will have arbitrary frequencies, the ELT systems are better suited than the DMT system in a transmission scenario with frequency selective interferers present on the transmission channel.

V. Conclusions

We have investigated a digital multicarrier modulation system based on extended lapped transforms with respect to the spectral containment of the subchannels, the power spectral density and the robustness against frequency selective interferers. The spectral containment of the subchannels in ELT systems is superior to conventional DMT systems. The higher spectral containment of the subchannels leads to a higher spectral containment of the resulting multicarrier signal, meaning an improved PSD. For frequency division multiplexing of several multicarrier signals, ELT multicarrier signals can be easier used than DMT multicarrier signals. The high spectral containment of the subchannels for ELT systems also leads to an improved robustness in the case of frequency selective interferers present on the transmission channel.

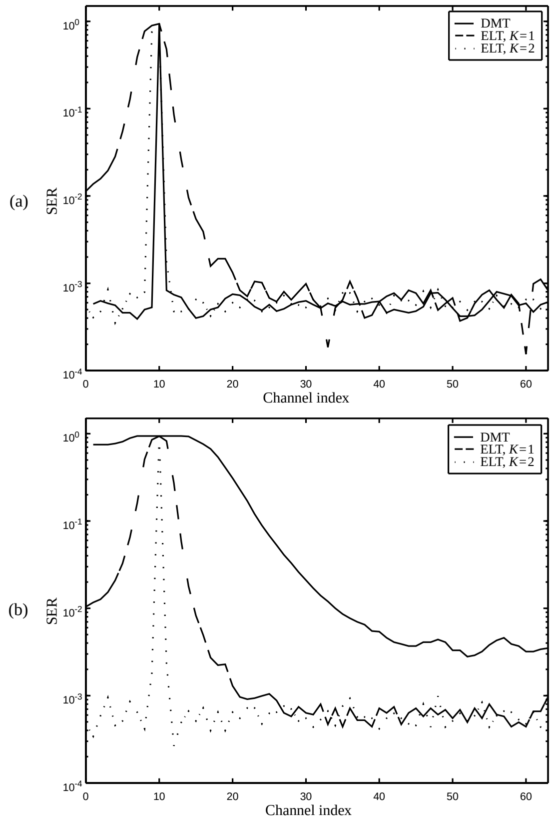

Fig. 6. Symbol error ratio (SER) vs. channel index for 16-QAM symbols, $M = 128$, in the presence of AWGN and sinusoidal interferer with: (a) f_i=10; (b) f_i=10.5

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