An Adaptive Wiener-Filter for Improved Channel Estimation in mobile OFDM-Systems

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Abstract—**In this paper we propose an improved channel estimator for a pilot based, mobile OFDM-system by adaptive Wiener-filtering in the frequency direction. In order to achieve a better estimation, the conventional fixed Wiener-Filter is replaced by an adaptive filter which adapts to the actual channel characteristic. The primary effect is a bandwidth reduction which minimizes the impact of noise onto the channel estimate. We have computed the BER and achieved a gain in E_b/N_0 of up to 2dB, compared to a fixed Wiener-Filter. Various simulations show the good results for different channels at high Doppler frequencies.**

I. Introduction

Over the past decade, OFDM has become a popular transmission scheme. One advantage of OFDM is that it turns a frequency selective channel into a flat fading channel for each sub–carrier. For a coherent demodulation at the receiver side, each of those flat fading channels can be equalized by a one-tap equalizer. The necessary channel estimation can conveniently be performed by inserting a regular pattern of pilot symbols into the data stream at the transmitter side. As the receiver knows the pattern and the transmitted pilot symbols, the time variant channel transfer function $H(f,t)$ can be calculated at the pilot positions. The result is a 2-dimensional sampling of the channel transfer function, both in the frequency and time direction. If the sampling theorem is fulfilled, the receiver can obtain an estimate of the channel transfer function by 2D lowpass filtering. Better estimation results can be achieved by using Wiener-Filters. It is well understood that this 2D estimation process can mathematically be split up into two 1D filter processes [1]. Höher et. al. have shown that two consecutive 1D filters in the time- and frequency-direction deliver almost the same performance as one 2D-filter [2].

In a conventional receiver, Wiener-Filters designed for a worst-case scenario are used. In the 1D case, the Wiener-Filter for estimation in time-direction is designed for the maximum assumed Doppler-shift, and the filter in frequency-direction for the worst-case channel delay spread.

Often, the transmission channel behaves much better than the assumed worst case. In those situations, Wiener-Filters with a smaller bandwidth would yield better estimation results by reducing the noise on the channel estimate. This paper will demonstrate how to create an adaptive Wiener-Filter, which perfectly aligns to the present channel characteristic. The noise on the channel estimate is minimized and BER is decreased significantly.

This paper is structured as follows: The investigated system model is presented in section II. Section III gives a brief summary of pilot based channel estimation and introduces the adaptive Wiener-Filter. Finally, section IV presents the simulated BER for different channels.

II. System Model

The regarded system model is based on the standard for Digital Video Broadcasting – Terrestrial (DVB-T) [3]. At the transmitter, the signal from a binary source is encoded by a rate $\frac{1}{2}$

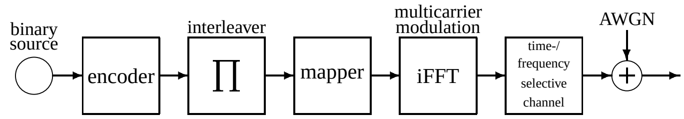

Fig. 1. Transmitter and channel model.

convolutional encoder, interleaved, mapped (QPSK, gray mapping), and modulated onto K orthogonal sub–carriers by an iFFT–block (Fig. 1). Additionally, pilot symbols are inserted according to [3] to allow for coherent detection at the receiver.

The wide–sense stationary uncorrelated scattering (WSSUS) channel model introduced in [4] was used for the mobile channel. The frequency response of the channel can be expressed as

$$H(f,t) = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} e^{j(\varphi_i + 2\pi f_{D_i} t - 2\pi f \tau_i)} \quad , \qquad (1)$$

where φ_i is the phase, f_{D_i} the Doppler frequency and τ_i the delay of the ith path. The variable M denotes the number of propagation paths. The φ_i, f_{D_i} and τ_i are randomly chosen depending on the corresponding joint probability density function $p_{\varphi,f_D,\tau}(\varphi, f_D, \tau)$ of the channel model.

After multicarrier demodulation, the received signal is fed to the channel estimation stage as shown in Fig. 2.

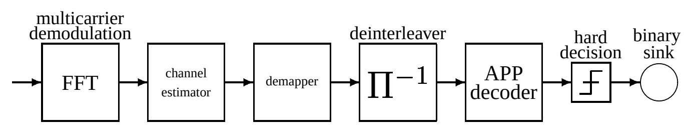

Fig. 2. Receiver.

For our further considerations we assume the channel characteristic to be approximately unchanged for the duration of one OFDM symbol (however, the simulations were performed with a true mobile channel). Under this assumption and provided that the guard interval is longer than the delay spread of the channel, the cyclic prefix avoids inter–carrier interference (ICI) and inter–symbol interference (ISI). Then the received constellation points compute as

$$Y_{k,l} = H_{k,l} \cdot X_{k,l} + N_{k,l} \qquad (2)$$

whereby l is the OFDM symbol index, k is the sub–carrier index, $X_{k,l}$ are the transmitted signal constellation points and $N_{k,l}$ are independent and identically distributed complex Gaussian noise variables with component–wise noise power $\sigma_n^2 = N_0/2$. The $H_{k,l}$ are sample values of the channel frequency response (cfr):

$$H_{k,l} = H(k\,\Delta f, l\,T_s) \qquad (3)$$

The demapper calculating log–likelihood ratios (L–values [5]) on the coded bits follows the estimation stage. After deinterleaving and soft–in/soft–out decoding with an APP algorithm [5], the estimates on the transmitted information bits are available at the output of the decoder. The final blocks are the decider and the binary sink.

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III. Channel Estimation and Adaptive Wiener-Filter

At the channel estimation stage the channel frequency response is only known at pilot positions with

$$\hat{H}_{k_P,l_P} = \frac{Y_{k_P,l_P}}{X_{k_P,l_P}} = H_{k_P,l_P} + \frac{N_{k_P,l_P}}{X_{k_P,l_P}} \quad , \tag{4}$$

whereby k_P and l_P describe a pilot position and X_{k_P,l_P} is the transmitted pilot signal which is known to the receiver. To allow for coherent detection the receiver has to know the channel frequency response over the whole time/frequency–grid. Therefore, the remaining $H_{k,l}$ have to be estimated by means of interpolation based on the known $\hat{H}_{k_P,l_P}$:

$$\hat{H}_{k,l} = \sum_{\{k_P,l_P\}\in\mathcal{P}} w^{k,l}_{k_P,l_P} \cdot \hat{H}_{k_P,l_P} \quad , \tag{5}$$

where $\mathcal{P}$ is a set of locations containing the nearest pilots with respect to the position $\{k,l\}$. The FIR–filter coefficients $w^{k,l}_{k_P,l_P}$ in (5) are based on the Wiener design criterion [6]. The filter coefficients depend on the autocorrelation of the channel frequency response $R_{HH}(k,l) = E\{H_{\tilde{k},\tilde{l}} \cdot H^*_{\tilde{k}-k,\tilde{l}-l}\}$ and the noise power $2 \cdot \sigma_n^2$ [6],[1].

It can be shown that $R_{HH}(k,l)$ can be split up into two independent parts according to [1]

$$R_{HH}(k,l) = R_t(l) \cdot R_f(k) \quad , \tag{6}$$

with $R_t(l)$ and $R_f(k)$ representing the autocorrelation of the channel transfer function in time- resp. frequency direction. This makes it possible to replace the 2D-filter by two 1D-filters, one for estimation in time-direction and one for estimation in frequency-direction. This concept is illustrated in Fig. 3 for the DVB-T-system.

The first 1D filter step estimates the channel transfer function on every third subcarrier by Wiener-filtering in the time direction. Since the time t is related to the Doppler-frequency f_d by a Fourier-transform, the bandwidth of this filter must be at least equal to the maximum Doppler-frequency, but must still be small enough to suppress the image-spectra of the sampled channel transfer function.

In the second 1D filter step, the channel transfer function of one OFDM-symbol is estimated in the frequency direction by a second Wiener-Filter. The frequency f is related to the channel delay τ by a Fourier-transform. Thus the bandwidth of the filter must be at least as large as the maximum delay spread of the channel, but small enough to filter out the image-spectra.

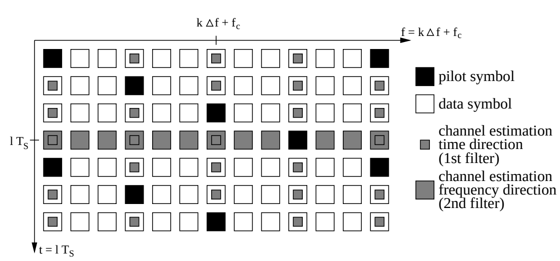

Fig. 3. Pilot pattern for DVB-T, and channel estimation with two 1D filters.

In this paper we will focus on the Wiener-Filter in the frequency direction. The presented concepts apply to the 1D-filter in the time direction and the 2D-case as well. Since only one OFDM timestep at a time is considered, the time t resp. the discrete time index l is dropped for convenience.

For a Wiener-Filter with input signal u_k and desired output signal d_k the following relation for the w_k can be found [7]:

$$\underline{w} = \underline{r}_{uu}^{-1} \underline{r}_{du} \quad , \tag{7}$$

with

$$\underline{r}_{uu} = \begin{pmatrix} R_{uu}(0) & R^*_{uu}(1) & \cdots & R^*_{uu}(K) \\ R_{uu}(1) & R_{uu}(0) & \cdots & R^*_{uu}(K{-}1) \\ \vdots & \vdots & \ddots & \vdots \\ R_{uu}(K) & R_{uu}(K{-}1) & \cdots & R_{uu}(0) \end{pmatrix} \quad , \tag{8}$$

$$\underline{w} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_K \end{pmatrix} \text{ and } \underline{r}_{du} = \begin{pmatrix} R_{du}(0) \\ R_{du}(1) \\ \vdots \\ R_{du}(K) \end{pmatrix} \quad , \tag{9}$$

$$R_{uu}(k) = E\{u_{\tilde{k}} u^*_{\tilde{k}-k}\} \quad , \tag{10}$$

$$R_{du}(k) = E\{d_{\tilde{k}} u^*_{\tilde{k}-k}\} \quad . \tag{11}$$

In case of channel estimation, the u_k are the calculated or estimated $\hat{H}_k$ on every third subcarrier. The filter output contains the remaining samples of the estimated channel transfer function $\hat{H}_k$. Because the desired filter-output d_k is the channel transfer function H_k itself, the $R_{uu}(k)$ and $R_{du}(k)$ contain elements of the autocorrelation $R_f(k)$ of H_k. Additionally, the main diagonal of $\underline{r}_{uu}$ needs to be extended by a factor which is the noise power $2\sigma_m^2 = E\{m_k \cdot m_k^*\}$, with m being the noise at the filter input. A detailed introduction to Wiener-filtering for channel estimation can be found in [6].

As a consequence, R_f and the noise power $2\sigma_m^2$ are the only parameters needed to determine the filter coefficients w_k. For a non adaptive Wiener-Filter, a worst-case estimate of R_f is used in [6] to determine the w_k.

Before we present our solution of an adaptive Wiener-Filter we consider the optimal filter in frequency direction which is given for the case that the autocorrelation function of the channel is known exactly.

A. Optimal Wiener Filter

The impulse response $h(\tau)$ of the channel is the inverse Fourier-Transform of $H(f)$ with respect to τ. Obviously, $h(-\tau)$ is the Fourier-Transform of $H(f)$.

In Fig. 4 the delay response $|F(\tau)|$ in dB of the optimal Wiener-Filter is shown for three different values of $2\sigma_m^2$, where R_f was assumed to be known. The filter was designed with 36 filter taps for the channel model Hilly Terrain (HT) described in [8], which has a power delay profile $P_\tau(\tau)$ as shown in Fig. 5.

The noise power $2\sigma_m^2 = E\{m_k \cdot m_k^*\}$ plays an important part in the design of the filter, where m_k is the noise on the filter input. For big values of $2\sigma_m^2$ it is important to filter as much noise as possible (left diagram in Fig. 4), whereas for small values of $2\sigma_m^2$ it is better to spend the effort on filtering the image spectra (starting at $-74\frac{2}{3}\mu$s and $-149\frac{1}{3}\mu$s resp.). If no noise is present, only the image spectra need to be filtered out (right diagram in Fig. 4).

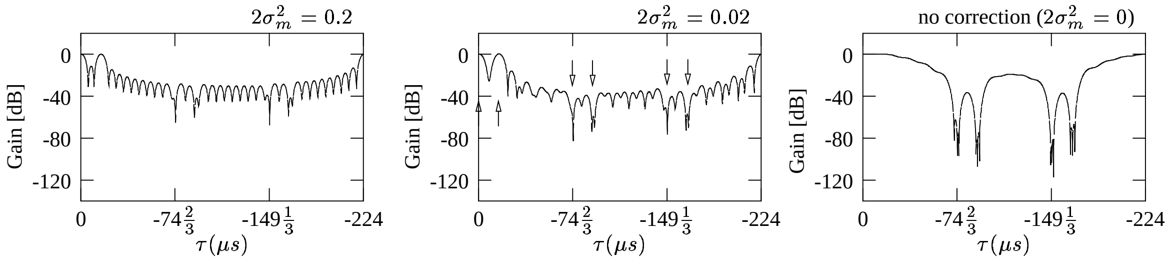

Fig. 4. Delay response $|F(\tau)|$ of the Wiener-Filter for channel model HT and different values of $2\sigma_m^2$

Fig. 4 shows that the Wiener-Filter aligns to the channel's power delay profile as expected. This is highlighted by the arrows in the second diagram of Fig. 4. The filter aligns to the two parts of the HT power delay profile by creating a gain minimum for each of them at the positions of the image spectra (arrow pairs at $-74\frac{2}{3}\mu$s and $-149\frac{1}{3}\mu$s). Alike, the filter gain is $0dB$ at the two positions 0μs and 16μs of the original spectrum, but drops in between to filter out as much noise as possible (left arrow pair).

B. *Adaptive Wiener Filter*

The proposed channel estimator to realize the adaptive filter is shown in Fig. 6. Its goal must be to find a very good estimate of the actual autocorrelation function R_f. The input data u_{k_P,l_P} of the estimator are the samples of the two-dimensional channel transfer function $H_{k,l}$ at the positions of the pilot symbols. In the lower branch a rough channel estimation is performed by two lowpass filters. The resulting output $\tilde{H}_k$ is a first estimate of the channel transfer function for the currently decoded OFDM-symbol at time step l. $\tilde{H}_k$ is superimposed by noise according to

$$\tilde{H}_k = H_k + m_k \quad , \tag{12}$$

where m_k is the noise. The following correlator calculates the short time correlation $\tilde{R}_f^1$ of $\tilde{H}$:

$$\tilde{R}_f^1 = R_f + R_{mm} \quad , \tag{13}$$

where R_{mm} is the autocorrelation of the noise. Note that the H_k and m_k are statistically independent, which is why all crosscorrelation terms vanish.

If the autocorrelation properties of the noise on the channel are known, R_{mm} can be calculated and subtracted. Usually, AWGN can be assumed on each subcarrier. Furthermore it can be assumed that all noise processes on the subcarriers are statistically independent. The noise $\tilde{n}_k$ in the $\tilde{u}_k$ is therefore still AWG, and its autocorrelation function is known to be an impulse. R_{mm} is the autocorrelation of the noise $\tilde{n}_k$ after it was filtered by the second lowpass-filter and needs to be subtracted from $\tilde{R}_f^1$.

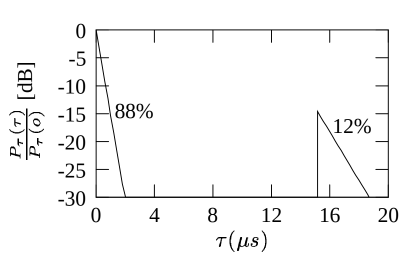

Fig. 5. Channel model HT (Hilly Terrain) as known from [6]

The relation between the autocorrelation of the input and the output of a FIR filter with filter coefficients g_k is known to be (see for example [9]):

$$R_{mm}(k) = g_k * g_{-k} * R_{\tilde{n}\tilde{n}}(k) \quad . \tag{14}$$

Here the g_k are the filter coefficients of the lowpass in the frequency direction. R_{mm} is calculated by the Noise-Correction-block, using the filter coefficients g_k of the lowpass in frequency direction. Additionally, the design of the lowpass in time-direction has to be considered in order to determine the power $2\sigma_m^2$ of the noise on the channel estimate in the time direction (see also [2]).

A good estimate $\tilde{R}_f^2$ of the autocorrelation is already obtained after subtracting R_{mm} from $\tilde{R}_f^1$. However, $\tilde{R}_f^2$ is still noisy, which has a big impact on the creation of the adaptive filter. Therefore, an additional step to further reduce the noise on the autocorrelation function is necessary.

Consider the Fourier-transforms of the cfr $H(f)$ and its autocorrelation function $R_f'(f)$ (WSSUS case):

$$\begin{array}{lcl} H(f) & \bullet\!\!-\!\!\circ & h(\tau) \\ R_f'(f) & \circ\!\!-\!\!\bullet & P_\tau \end{array} \tag{15}$$

$H(f)$ is related to $h(\tau)$ by an inverse Fourier-Transform. On the other hand, $R_f'(f)$ is related to P_τ by a Fourier-Transform. As the Wiener-Filter aligns to the power-delay profile P_τ of $h(\tau)$, a filter created from $\tilde{R}_f$ to reduce the noise on the estimated $\hat{H}_k$ can be used to reduce the noise on $\tilde{R}_f$ itself. To generate this filter, a big enlargement of the main diagonal of $\underline{r}_{uu}$ needs to be chosen, since the goal is to reduce the noise (cmp. eq. (9)). The elements of $\underline{r}_{uu}$ are still noisy, but the big enlargement of the main diagonal reduces the impact of this noise on the filter design, and a filter good for noise reduction can be created.

Finally, the autocorrelation $\tilde{R}_f^3$ in Fig. 6 is normalized and used to create a Wiener-Filter for channel estimation. A problem is introduced for small noise powers by channels with a very short maximum delay τ_{max}, such as channel Rural Area (RA) from [8]. As τ_{max} decreases, the autocorrelation R_f approaches a constant value. For $2\sigma_m^2 = 0$ all elements of the matrix $\underline{r}_{uu}$ are then equal (cmp. eq. (9)). In this case, $\underline{r}_{uu}$ is not invertible, and the filter coefficients cannot be computed from equation (7) anymore. However, $\underline{r}_{uu}$ would still be invertible if $2\sigma_m^2 > 0$. For the filter creation it is therefore necessary to assign a minimum value to $2\sigma_m^2$. Although $2\sigma_m^2 > 0$ would theoretically be sufficient, $2\sigma_m^2$ was chosen such that the absolute value of the difference between the main diagonal elements of $\underline{r}_{uu}$ and the secondary diagonal elements is not smaller than 0.05 for numerical reasons. This is an empirical value determined by simulation.

For the whole process, the receiver must have knowledge of the noise power $2\sigma_n^2$. For the simulation it was assumed that the

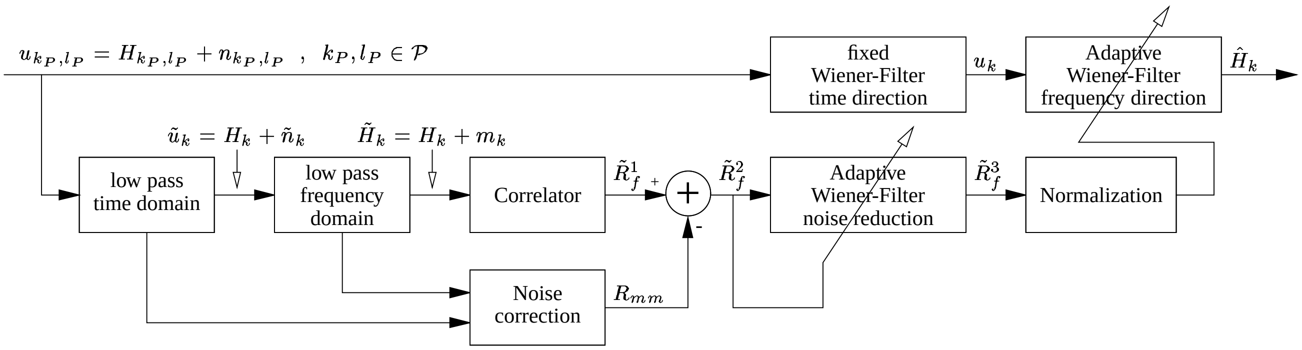

Fig. 6. System model of channel estimator

receiver knows $2\sigma_n^2$, but in practice methods similar to those explained in [10] and [11] can be used to obtain estimates of $2\sigma_n^2$. No additional complexity is introduced hereby, since the value of $2\sigma_n^2$ is needed for the MAP-algorithm of the inner channel decoder anyway. Our simulations showed, that the BER remains almost the same even if the value of $2\sigma_n^2$ was assumed to be wrong by 20%.

IV. Simulations

Simulations were performed for a DVB-T system operating in the 2k-mode with an OFDM-symbol duration of 224μs and a guard interval length of 56μs. Such a system allows for operation with channels having a large power delay spread of 56μs. The fixed Wiener-Filter for a classical channel estimator therefore needs to be designed with a large bandwidth. A channel estimator based on the adaptive Wiener-Filter has a big potential to improve performance, while still maintaining the capability of handling channels with the same long maximum path delay.

The DVB-T standard requires a BER of $2 \cdot 10^{-4}$ after the inner decoder to achieve quasi-error-free reception after the outer Reed-Solomon decoder. Depending on the channel characteristic the gain in $\frac{E_b}{N_0}$ after the inner decoder at a BER of $2 \cdot 10^{-4}$ varies between 0.5dB and approximately 2dB in the simulated cases. Fig. 7 and 8 show the BER-curves for the channel models Rural Area (RA), Typical Urban (TU) and Hilly Terrain (HT) [8]. The modulation scheme was 4-QAM, and the maximum Doppler-frequency $f_{dmax} = 193$Hz. Plotted are the BER-curves when using only fixed Wiener-Filters, compared to the BER-curves for the proposed channel estimator with an adaptive Wiener-Filter with 36 filter taps. The performance gain over the fixed Wiener-Filter is approximately 1.7dB for channel TU, and 1.2dB for channel HT. The biggest improvement of about 2dB was achieved with the channel model Rural Area (RA) [8], which has a maximum delay spread of only 0.7μs.

V. Conclusion

We have presented a new adaptive Wiener-Filter for pilot based channel estimation in a mobile OFDM system. The efficient method adds only a small complexity to conventional estimation schemes. However it can provide an $\frac{E_b}{N_0}$ improvement of up to 2dB, which was shown by computer simulation using various mobile channel models. The scheme was successfully applied to mobile DVB-T.

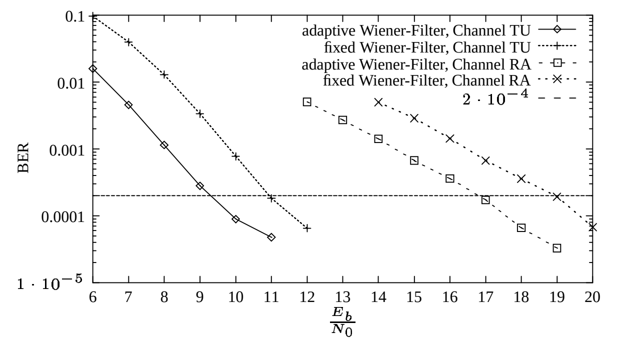

Fig. 7. BER after inner channel decoder for channels TU and RA with $f_{dmax} = 193Hz$

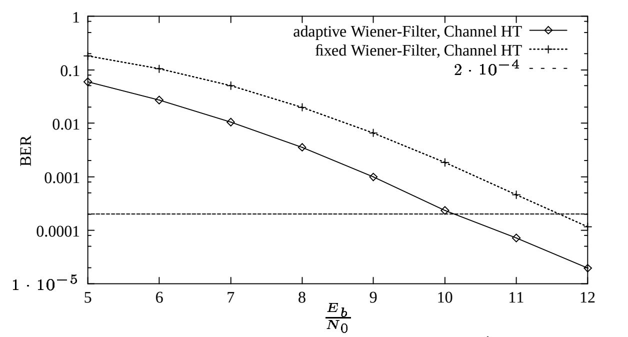

Fig. 8. BER after inner channel decoder for channel HT with $f_{dmax} = 193Hz$

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