A Closed-Form Bound on Correlated MIMO Channel Capacity

Mario Kiessling, Joachim Speidel
Institute of Telecommunications
University of Stuttgart, Germany

Ingo Viering, Markus Reinhardt
Siemens AG, Ulm, Germany

***Abstract*—The exact calculation of the ergodic MIMO channel capacity with channel correlation is mathematically at least very challenging. Having no closed-form analytical expression available for the capacity is making it difficult to derive optimum stochastic waterfilling schemes that are based on long-term channel state information (channel correlation) only. We therefore derive a closed-form tight upper bound on the ergodic capacity of correlated MIMO channels. The bound takes into account both the effects of correlation at the transmitter as well as the receiver. Furthermore, we give a recursive algorithm for its efficient calculation. Simulations demonstrate the tightness of the bound and show that a long-term waterfilling scheme based on the new bound yields almost the same performance as a scheme with full short-term (instantaneous) channel state information.**

Keywords—MIMO; capacity; channel correlation

I. Introduction

It is well known that the capacity potential offered by multiple antennas on both RX and TX side of the wireless link suffers from fading correlation between antenna elements ([4][5]). This is especially true in outdoor scenarios with a scarce scattering environment and 'blind' transmission, i.e. without channel state information (CSI) at the TX. However, it was demonstrated in [7] that with adequate transmit filtering, the capacity of a semi-correlated MIMO channel (TX correlation only) can outnumber uncorrelated capacity even in cases where only long-term (LT) CSI is available at the TX. Those results inspire the development of transceiver architectures that adapt to the prevailing LT fading conditions and thus optimally exploit the various gains of the MIMO link (diversity, beamforming, multiplexing).

LT processing is especially appealing in FDD systems with different fast fading conditions in uplink and downlink, but relatively stable LT channel characteristics that can be obtained via uplink-downlink frequency conversion or feedback. Furthermore, LT processing requires only moderate processing resources in the TX.

While the seminal papers [1][2] studied MIMO capacity of uncorrelated channels by numerical integration (using results on the Eigen values of complex Wishart matrices [8][9]) and bounding techniques, the authors of [4][5][7] considered MIMO capacities with correlated fading again using simplified bounds. However, those bounds were derived for semi-correlated channels only. In [3] the authors propose to use a bound based on the dominating correlation matrix of the link. Unfortunately, for fully correlated channels with strong correlation at both TX and RX the bounds become very loose.

In this paper we derive a closed-form tight upper bound on the ergodic capacity of a correlated MIMO channel. The bound takes into account the effects of TX as well as RX correlation. Based on the analysis presented in this paper, adaptive MIMO TX signal processing algorithms can be derived that exploit LT CSI in order to improve the wireless MIMO link in terms of various quality criteria such as bit error rate and data throughput ([14][15]).

II. Signal And Channel Model

In the following, bold face lower case letters denote column vectors, bold face upper case letters denote matrices, $\mathbf{I}$ is an identity matrix, $\mathbf{A}^H$ denotes the Hermitian of $\mathbf{A}$ and k! is the factorial of k. The expectation of a term will be written as E[.] in the following. Furthermore, let $|\mathbf{A}|_{\hat{\delta}_k}^{\hat{\gamma}_k}$ denote the determinant of a submatrix of $\mathbf{A}$ that results from selecting the row and column subset from the n×n matrix $\mathbf{A}$ indexed by $\hat{\gamma}_k = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ and $\hat{\delta}_k = \{\delta_1, \delta_2, \ldots, \delta_k\}$, respectively. The cardinalities of the subsets $\hat{\gamma}_k$ and $\hat{\delta}_k$ are $\langle \hat{\gamma}_k \rangle = \langle \hat{\delta}_k \rangle = k$.

We consider a flat fading MIMO link modeled by

$$\mathbf{y} = \mathbf{HFs} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is the $M_{TX}\times 1$ TX symbol vector, $\mathbf{F}$ is a $M_{TX}\times M_{TX}$ linear transmit matrix filter, $\mathbf{H}$ is the $M_{RX}\times M_{TX}$ MIMO channel matrix with correlated complex Gaussian distributed path gains (i.e. we are assuming a Rayleigh faded channel), $\mathbf{n}$ is the $M_{RX}\times 1$ colored Gaussian noise vector and $\mathbf{y}$ is the $M_{RX}\times 1$ receive vector. The noise covariance matrix is $\mathbf{R}_{nn}$, the TX signal covariance is generally given by $\mathbf{R}_{ss}$, however we will without loss of generality restrict the derivation to the case $\mathbf{R}_{ss}=\mathbf{I}$ in order to simplify matters.

With the common assumptions on the MIMO propagation model given e.g. in [4][5][7], the correlated channel can mathematically be expressed as the matrix product

$$\mathbf{H} = \mathbf{A}^H\mathbf{GB}, \tag{2}$$

where **G** is a matrix of complex i. i. d. Gaussian variables of unity variance and matrix square roots **A** and **B**, $\mathbf{A}^H\mathbf{A}=\mathbf{R}_{RX}$ and $\mathbf{B}^H\mathbf{B}=\mathbf{R}_{TX}$, where $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ is the receive and transmit correlation matrix, respectively.

III. New Capacity Bound

A. Derivation

The ergodic capacity in bit per channel use of the MIMO link above is given by (cf. [12])

$$C = E\left[\log_2\left[\det\left(\mathbf{I}+\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{nn}^{-1}\mathbf{H}\mathbf{F}\right)\right]\right]. \tag{3}$$

Introducing the singular value decompositions (SVD)

$$\mathbf{A}\mathbf{R}_{nn}^{-1/2} = \mathbf{U}_A\mathbf{\Sigma}_A\mathbf{V}_A^H \qquad \mathbf{B} = \mathbf{U}_B\mathbf{\Sigma}_B\mathbf{V}_B^H \tag{4}$$

and noticing that the statistics of the resulting matrix after left or right multiplication of **G** with an unitary matrix do not change, leads to

$$C = E\left[\log_2\left[\det\left(\mathbf{I}+\mathbf{F}^H\mathbf{V}_B\mathbf{\Sigma}_B\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\mathbf{\Sigma}_B\mathbf{V}_B^H\mathbf{F}\right)\right]\right]. \tag{5}$$

The Eigen vectors in $\mathbf{V}_B$ and their corresponding Eigen values in $\mathbf{\Sigma}_B^2$ will be referred to as LT Eigen modes in the following. Without loss of generality, we can choose $\mathbf{F}=\mathbf{V}_B\mathbf{\Phi}_f$ with an arbitrary matrix $\mathbf{\Phi}_f$ and due to the concavity of the log function we get via Jensen's inequality and (A.4) the upper bound

$$\begin{aligned} C \le \bar{C} &= \log_2 E\left[\det\left(\mathbf{I}+\mathbf{\Phi}_f^H\mathbf{\Sigma}_B\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\mathbf{\Sigma}_B\mathbf{\Phi}_f\right)\right] \\ &= \log_2 E\left[\det\left(\mathbf{I}+\mathbf{\Sigma}_B\mathbf{\Phi}_f\mathbf{\Phi}_f^H\mathbf{\Sigma}_B\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right)\right] \end{aligned}. \tag{6}$$

Note that $\mathbf{\Phi}_f$ can be controlled by the TX to maximize capacity. Furthermore, from (6) it is obvious that again without loss of generality we can assume that the structure of $\mathbf{\Phi}_f$ is $\mathbf{\Phi}_f=\mathbf{V\Lambda}$ with unitary **V** and diagonal **Λ**.

The expectation in (6) can be calculated in closed form via determinant expansion and the theory of complex Wishart matrices (cf. [8][9]). To this end, we are applying formula (A.2) to (6) and get

$$\begin{aligned} \bar{C} &= \log_2 E\left[\sum_{k=0}^{n}\sum_{\hat{\alpha}_k}\left|\mathbf{\Sigma}_B\mathbf{\Phi}_f\mathbf{\Phi}_f^H\mathbf{\Sigma}_B\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}\right] \\ &= \log_2 E\left[\sum_{k=0}^{n}\sum_{\hat{\alpha}_k}\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}\right] \end{aligned} \tag{7}$$

with $n=M_{TX}$ and $\mathbf{U}=\mathbf{\Sigma}_B\mathbf{\Phi}_f\mathbf{\Phi}_f^H\mathbf{\Sigma}_B$ for brevity. The sub determinants of size k×k can then again be expanded via (A.3)

$$\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k} = \sum_{\hat{\beta}_k}\sum_{\hat{\gamma}_k}\sum_{\hat{\sigma}_k}\left|\mathbf{U}\right|_{\hat{\beta}_k}^{\hat{\alpha}_k}\cdot\left|\mathbf{G}^H\right|_{\hat{\gamma}_k}^{\hat{\beta}_k}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\sigma}_k}^{\hat{\gamma}_k}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\sigma}_k}. \tag{8}$$

Note that the expansion will vanish for $k>M_{RX}$, which is the case for $M_{TX}\ge M_{RX}$. We therefore have to assure $k\le m=\min(M_{TX}, M_{RX})$. In a next step, the diagonal structure of $\mathbf{\Sigma}_A$ can be exploited by noticing that

$$\left|\mathbf{\Sigma}_A^2\right|_{\hat{\sigma}_k}^{\hat{\gamma}_k} = 0 \quad \textit{for } \hat{\gamma}_\kappa \ne \hat{\sigma}_\kappa, \tag{9}$$

as the sub matrix has at least one row that contains only zeros. Now we get from (8)

$$\begin{aligned} \left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k} &= \sum_{\hat{\beta}_k}\sum_{\hat{\gamma}_k}\left|\mathbf{U}\right|_{\hat{\beta}_k}^{\hat{\alpha}_k}\cdot\left|\mathbf{G}^H\right|_{\hat{\gamma}_k}^{\hat{\beta}_k}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_k}^{\hat{\gamma}_k}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\gamma}_k} \\ &= \sum_{\hat{\beta}_k}\sum_{\hat{\gamma}_k}\left|\mathbf{U}\right|_{\hat{\beta}_k}^{\hat{\alpha}_k}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_k}^{\hat{\gamma}_k}\cdot\left|\mathbf{G}^H\right|_{\hat{\gamma}_k}^{\hat{\beta}_k}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\gamma}_k} \end{aligned}, \tag{10}$$

Equation (10) shall be further simplified. To this end, we are considering special cases. Note from (6) that for high SNRs (large $\mathbf{\Sigma}_A$) the ergodic capacity is dominated by the principal determinant $\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|$ (the identity matrix can be neglected). We simply get

$$\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right| = \left|\mathbf{U}\right|\left|\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|. \tag{11}$$

This means that the value of the determinant only depends on the Eigen values of the matrix **U**. For the high SNR region we are therefore free to choose the matrix **U** and equivalently $\mathbf{\Phi}_f$ to be diagonal. Equivalently, for k=1 we get from (10)

$$\begin{aligned} E\left[\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_1}^{\hat{\alpha}_1}\right] &= E\left[\sum_{\hat{\beta}_1}\sum_{\hat{\gamma}_1}\left|\mathbf{U}\right|_{\hat{\beta}_1}^{\hat{\alpha}_1}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_1}^{\hat{\gamma}_1}\cdot\left|\mathbf{G}^H\right|_{\hat{\gamma}_1}^{\hat{\beta}_1}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_1}^{\hat{\gamma}_1}\right] \\ &= \sum_{\hat{\beta}_1}\sum_{\hat{\gamma}_1}\left|\mathbf{U}\right|_{\hat{\beta}_1}^{\hat{\alpha}_1}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_1}^{\hat{\gamma}_1}\cdot E\left[\left|\mathbf{G}^H\right|_{\hat{\gamma}_1}^{\hat{\beta}_1}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_1}^{\hat{\gamma}_1}\right] \end{aligned}. \tag{12}$$

These terms dominate the low SNR region. With independent matrix elements in **G**, obviously we get

$$E\left[\left|\mathbf{G}^H\right|_{\hat{\gamma}_1}^{\hat{\beta}_1}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_1}^{\hat{\gamma}_1}\right] = \delta\left(\hat{\alpha}_1,\hat{\beta}_1\right), \tag{13}$$

with the Kronecker symbol δ. Now (12) reads

$$E\Big[\Big| \quad {}^{H}\;{}^{2}_{A}\Big|_{\hat{\alpha}_1}^{\hat{\alpha}_1}\Big] \quad \Big| \; \Big|_{\hat{\alpha}_1}^{\hat{\alpha}_1} \quad \hat{\gamma}_1 \quad \Big| \; {}^{2}_{A}\Big|_{\hat{\gamma}_1}^{\hat{\gamma}_1}$$

of the trace of the matrix **U**. Without loss of generality, **U** and equivalently $\mathbf{\Phi}_f$ may be assumed to be diagonal.

Thus, in order to simplify matters, we will assume a diagonal matrix $\mathbf{\Phi}_f$ in the following, which was shown above to be adequate at least for the high and low SNR regions. Then we get analogously to (9)

$$\left|\mathbf{U}\right|_{\hat{\beta}_k}^{\hat{\alpha}_k} = 0 \quad for\ \hat{\alpha}_k \neq \hat{\beta}_k \tag{15}$$

and from (10) we can derive

$$\left|\mathbf{U}\mathbf{G}^H\mathbf{\Sigma}_A^2\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k} = \sum_{\hat{\gamma}_k}\left|\mathbf{U}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}\cdot\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_k}^{\hat{\gamma}_k}\cdot\left|\mathbf{G}\right|_{\hat{\alpha}_k}^{\hat{\gamma}_k}\cdot\left|\mathbf{G}^H\right|_{\hat{\gamma}_k}^{\hat{\alpha}_k}. \tag{16}$$

Obviously, the product of the last two sub determinants in (16) may be combined to result in the generalized variance of a $\tilde{\mathbf{W}}_k(k,\mathbf{I})$ distributed matrix (see appendix). Combining (7), (16) and using (A.1), we finally can derive the upper bound for diagonal $\mathbf{\Phi}_f$

$$\bar{C} = \log_2\left[\sum_{k=0}^{m}(k!)\cdot\left(\sum_{\hat{\alpha}_k}\left|\mathbf{\Sigma}_B^2\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}\cdot\left|\mathbf{\Phi}_f^2\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}\right)\cdot\left(\sum_{\hat{\gamma}_k}\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_k}^{\hat{\gamma}_k}\right)\right], \tag{17}$$

where the sums are over all possible index subsets $\hat{\alpha}_k$ and $\hat{\gamma}_k$. Note that the SNR is implicitly given in $\mathbf{\Sigma}_A$.

B. Special Cases

When considering a MIMO system without power allocation with uncorrelated fading and white Gaussian noise, i.e. $\mathbf{R}_{nn}=N_0\cdot\mathbf{I}$ and $\mathbf{\Phi}_f=\mathbf{A}=\mathbf{B}=\mathbf{I}$, we get from (17)

$$\begin{aligned}\bar{C}_i &= \log_2\left[\sum_{k=0}^{m}(k!)\cdot\binom{M_{TX}}{k}\cdot\binom{M_{RX}}{k}\cdot\left(\frac{1}{N_0}\right)^k\right] \\ &= \log_2\left[\sum_{k=0}^{m}\binom{M_{TX}}{k}\cdot\frac{M_{RX}!}{(M_{RX}-k)!}\cdot\left(\frac{1}{N_0}\right)^k\right]\end{aligned}. \tag{18}$$

A similar result has already been derived in [6]. Furthermore, the $M_{RX}=M_{TX}=2$, i.e. short (2, 2), antenna case with arbitrary correlation and noise covariance results in the simple formula

$$\bar{C}_2 = \log_2\left[1+tr\left(\mathbf{\Phi}_f^2\mathbf{\Sigma}_B^2\right)tr\left(\mathbf{\Sigma}_A^2\right)+2\cdot\det\left(\mathbf{\Phi}_f^2\mathbf{\Sigma}_A^2\mathbf{\Sigma}_B^2\right)\right]. \tag{19}$$

This formula will be used below to derive a closed form stochastic waterfilling (WF) scheme. Note, that in the (2, 2) case a diagonal $\mathbf{\Phi}_f$ maximizes the new capacity bound over the total SNR range.

C. Recursive Capacity Calculation

Especially for a higher number of transmit and receive antennas the calculation of the partial sums in (17) can be computationally challenging. Therefore, we are proposing a recursive calculation of the capacity. To this end, we are focusing on the coefficients

$$a_k = \left(\sum_{\hat{\gamma}_k}\left|\mathbf{\Sigma}_A^2\right|_{\hat{\gamma}_k}^{\hat{\gamma}_k}\right). \tag{20}$$

These coefficients can be seen to be the coefficients of the polynomial

$$P(z) = \prod_{n=1}^{M_{RX}}\left(1+\sigma_{A,n}^2 z\right) = \sum_{k=0}^{M_{RX}} a_k z^k, \tag{21}$$

where the σ_A are the diagonal elements of $\mathbf{\Sigma}_A$. In [13], a recursive algorithm was given for the calculation of the a_k

$$\begin{aligned} &S_i = \sum_{n=1}^{M_{RX}}\sigma_{A,n}^{2i} \quad, i\geq 1 \\ &E_0 = 1,\quad E_k = \sum_{i=1}^{k}(-1)^{i-1}P_{i-1}^{k-1}S_iE_{k-i} \quad, k\geq 1 \\ &a_k = \frac{E_k}{k!} \quad, k = 0\ldots M_{RX}\end{aligned} \tag{22}$$

with

$$P_i^n = n(n-1)\cdots(n-i+1). \tag{23}$$

The other coefficients in (17) have the same structure as (20) and can be calculated in the same way.

D. Capacity Plots

If not stated otherwise, in the following we assume an angular spread of 10 degrees (Laplacian distributed) on each side of the MIMO link, 20 degrees direction of arrival (departure) and 0.5 wavelength element spacing. We will refer to this as fully correlated channel (FCC).

A plot of the new bound with blind transmission ($\mathbf{\Phi}_f=\mathbf{I}$) is given in Fig. 1 for a (4, 4), antenna system in a correlated scenario. For comparison, a simulated curve of the true ergodic capacity is also depicted. The bound is seen to be very tight. Another bound that can also be derived via Jensen's inequality and the concavity of the log det function [7], is solely based on the transmit correlation matrix (7) $\mathbf{R}_{TX}$

$$\bar{\bar{C}} = \log_2\left[\det\left(\mathbf{I}+\mathbf{F}^H E\left[\mathbf{H}^H\mathbf{R}_{nn}^{-1}\mathbf{H}\right]\mathbf{F}\right)\right]. \tag{24}$$

It can be seen to be noticeably looser.

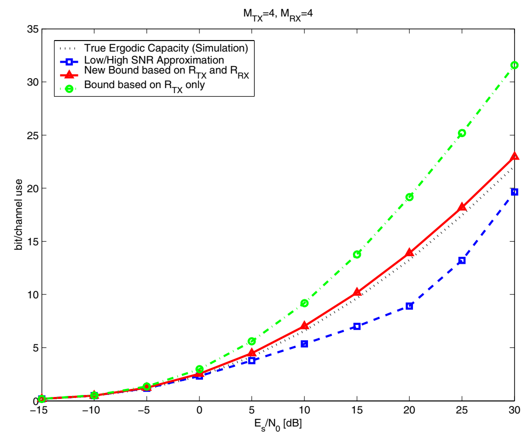

Fig. 1: Bounds based on R_{TX} and both R_{TX}, R_{RX} (FCC)

Also included in Fig. 1 is an approximation for the low and high SNR region based on the new bound. As mentioned earlier, it just consists of a truncation of (17), taking into account only the terms for k={0,1,m}. Note that the approximation converges with the bound for low and high SNR.

The new bound is also tighter for scenarios with TX correlation only. This is demonstrated in Fig. 2 for a (6, 8) MIMO system. We will use the expression semi-correlated channel (SCC) for this type of propagation scenario.

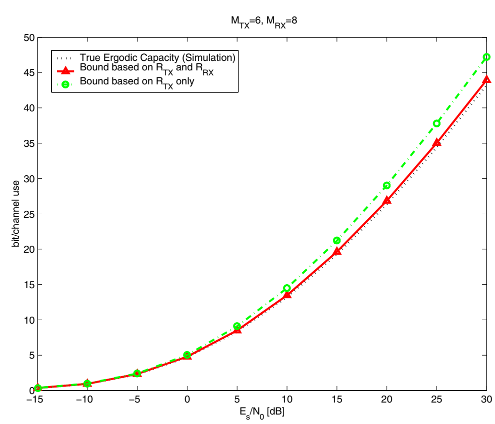

Fig. 2: Bounds based on R_{TX} and both R_{TX}, R_{RX} (SCC)

IV. Long-Term Waterfilling

Direct capacity maximization of (3) via the TX matrix filter **F** would require an explicit expression for the expectation. This expectation is at least very hard to calculate. Therefore, we deploy our new capacity bound to derive the capacity maximizing power allocation scheme. Limiting the total TX power to ρ we get from (19) the following constrained maximization problem for the optimum long-term power allocation policy based on the new bound

$$\max_{\mathbf{\Phi}_f} \bar{C} \quad s.t. \quad tr\left(\mathbf{\Phi}_f^2\right) = \rho . \tag{25}$$

We shall restrict ourselves to the (2, 2) antenna case to allow for a concise analytical solution. Omitting details, the diagonal elements of $\mathbf{\Phi}_f^2$ obtained from a Lagrange optimization process are

$$\begin{aligned} \phi_{f,1}^2 &= \frac{\rho \det\left(\mathbf{\Sigma}_A^2 \mathbf{\Sigma}_B^2\right) + \frac{1}{2}\left(\sigma_{B,1}^2 - \sigma_{B,2}^2\right) tr\left(\mathbf{\Sigma}_A^2\right)}{2\det\left(\mathbf{\Sigma}_A^2 \mathbf{\Sigma}_B^2\right)} \\ \phi_{f,2}^2 &= \frac{\rho \det\left(\mathbf{\Sigma}_A^2 \mathbf{\Sigma}_B^2\right) + \frac{1}{2}\left(\sigma_{B,2}^2 - \sigma_{B,1}^2\right) tr\left(\mathbf{\Sigma}_A^2\right)}{2\det\left(\mathbf{\Sigma}_A^2 \mathbf{\Sigma}_B^2\right)} , \end{aligned} \tag{26}$$

where we have to assure that $\phi_{f,1/2}>0$.

Note that in agreement with results stated in [4], the total transmit power is equally distributed on both subchannels, i.e. $\phi_{f,1}^2 = \phi_{f,2}^2 = \rho/2$, if no TX correlation is present, i.e. $R_{TX}=\mathbf{I}$, no matter what RX correlation is prevailing.

Plots of simulated curves for LT power allocation are given in Fig. 3 for both the new bound and the bound in (24) (cf.[6]). The angular spread has been chosen to be 2 degrees at the transmitter and receiver (highly correlated scenario). For comparison, we have also plotted the curves for optimum instantaneous WF (e.g. [12]) and blind transmission.

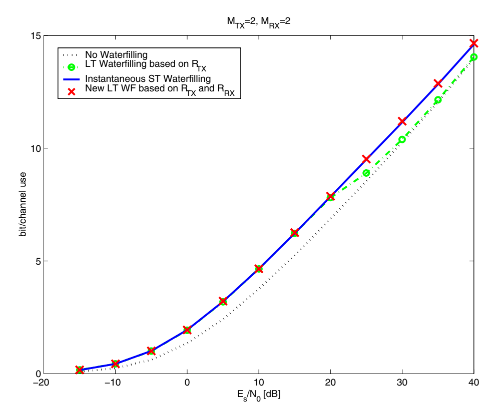

Fig. 3: Long-term waterfilling based on capacity bounds

There is no noticeable performance penalty in using the LT CSI based power allocation derived from the new bound in the given SNR range. Furthermore, the power allocation policy based only on the TX correlation matrix has a discontinuity in its slope, at that point where power is also assigned to the second subchannel. From this discontinuity point onwards, the performance is seen to be inferior to the new scheme and converges with the blind TX curve, while the waterfilling scheme based on the new bound still pours all power only on the single strongest LT Eigen mode and achieves a similar performance as optimum instantaneous WF. Obviously, if the RX correlation is not taken into account, power is poured on weaker Eigen modes and a capacity penalty results. However, if there is no RX correlation present, the power allocation based on (24) shows a very good performance. This is depicted in Fig. 4.

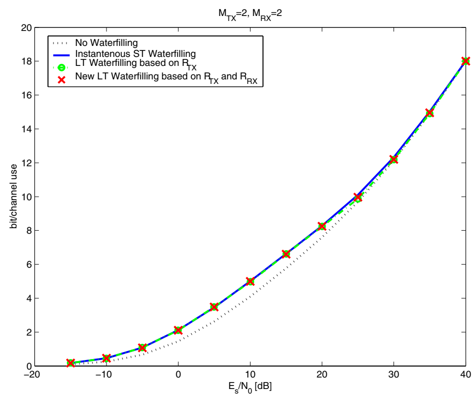

Fig. 4: Long-term waterfilling with TX correlation only

In the SCC there is apperently only a neglectable difference between instantanous WF and the stochastic WF schemes based on the various bounds.

Conclusion

We have derived a new bound on ergodic capacity taking into account both correlation at the TX and RX antenna arrays of the MIMO link. The new bound is used for the design of a long-term based linear transmit filter (the long-term analogue to waterfilling, stochastic waterfilling) that maximizes ergodic MIMO capacity. Simulation results show that long-term channel state information is very valuable and there is only a neglectable performance penalty in terms of ergodic capacity compared to the optimum instantaneous waterfilling case in semi-correlated as well as fully correlated channels. The results inspire the development of adaptive MIMO transceivers that make use of long-term channel state information ([14][15]).

Appendix

Let $\mathbf{x}_j$ be a m×1 vector of complex Gaussian entries (having elements with variance of the real and imaginary parts of ½ each) with covariance matrix $\mathbf{C}_{\mathbf{xx}}$, i.e. $\mathbf{x}_j \sim \mathbf{N}\left(0,\mathbf{C}_{\mathbf{xx}}\right)$. Arranging n of these vectors in a m×n matrix $\mathbf{X}=[\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_n]$ and letting $\mathbf{W}=\mathbf{X}\mathbf{X}^H$, we arrive at a complex Wishart distribution, denoted as $\mathbf{W} \sim \tilde{\mathbf{W}}_m\left(n,\mathbf{C}_{\mathbf{xx}}\right)$. Its so-called generalized variance is

$$E\left[\det \mathbf{W}\right] = \det \mathbf{C}_{\mathbf{xx}} \cdot \frac{n!}{\left(n-m\right)!}. \qquad \text{(A.1)}$$

For a general n×n matrix $\mathbf{X}$ we obtain the subdeterminant expansion ([10][11])

$$\det\left(\mathbf{I}_{n\times n} + \mathbf{X}_{n\times n}\right) = \sum_{k=0}^{n} \sum_{\hat{\alpha}_k} \left|\mathbf{X}\right|_{\hat{\alpha}_k}^{\hat{\alpha}_k}, \qquad \text{(A.2)}$$

$$\hat{\alpha}_k \subseteq \{1,2,\ldots,n\}, \quad \hat{\alpha}_k = \{\alpha_1,\alpha_2,\ldots,\alpha_k\}, \quad \left|\hat{\alpha}_k\right| = k$$

where the second sum is over all possible subsets of size k. Furthermore, for a k×k matrix $\mathbf{T}$ with $\mathbf{T}=\mathbf{A}\mathbf{B}\cdots\mathbf{R}\mathbf{S}$ we get [10][11]

$$\det \mathbf{T} = \sum_{\hat{\beta}_k}\sum_{\hat{\gamma}_k}\cdots\sum_{\hat{\sigma}_k} \left|\mathbf{A}\right|_{\hat{\beta}_k}^{\{1\,2\ldots k\}} \cdot \left|\mathbf{B}\right|_{\hat{\gamma}_k}^{\hat{\beta}_k} \cdots \left|\mathbf{R}\right|_{\hat{\sigma}_k}^{\hat{\rho}_k} \cdot \left|\mathbf{S}\right|_{\{1\,2\ldots k\}}^{\hat{\sigma}_k}. \qquad \text{(A.3)}$$

Finally, the following determinant identity is true for two quadratic matrices $\mathbf{A}$ and $\mathbf{B}$

$$\det\left(\mathbf{I}+\mathbf{A}\mathbf{B}\right) = \det\left(\mathbf{I}+\mathbf{B}\mathbf{A}\right). \qquad \text{(A.4)}$$

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