

# Impulse Compression for OFDM Transmission over Time-varying Multipath Channels

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**Abstract** – In today’s multicarrier modulation systems such as orthogonal frequency division multiplexing (OFDM) a circular extended guard interval is inserted between successive multicarrier symbols in order to reduce intersymbol and interchannel interference. The length of the guard interval is determined by the length of the channel impulse response. In this paper, we consider an equalizer to compress the channel impulse response for OFDM transmission. Due to the compression effect, the guard interval can be reduced and the efficiency of the OFDM system increases. The system performance is demonstrated for transmission over several time-varying multipath channels with additive white Gaussian noise (AWGN).

**Keywords**- OFDM; Impulse shortening; Equalization

## I. INTRODUCTION

Due to its insensitivity against multipath propagation, orthogonal frequency division multiplexing (OFDM) is well suited for wireless broadband access systems [1][2]. In such systems, intersymbol and interchannel interference (ISI, ICI) can be completely avoided by adding a circular prefix of length  $G$  for each multicarrier symbol of length  $M$ . In addition to that, equalization can be simplified by the guard interval insertion. Therefore, the guard interval has to be at least as long as the duration of the channel impulse response, which is of length  $L_K$ . So the following condition

$$G \geq L_K - 1 \quad (1)$$

has to be met. However, the guard interval reduces the efficiency

$$\eta = \frac{1}{1 + G/M} \quad (2)$$

of the OFDM system in comparison to a system without guard interval because within the guard interval no further information is transmitted. Thus, to keep the inefficiency low, the guard interval has to be chosen as small as possible. On the other hand, a channel impulse response exceeding the duration of a given guard interval will cause ISI and ICI [3]. To mitigate such effects efficiently, the idea is to feed the original channel impulse response  $h(\tau, t)$  into an equalizer such that the cascade of the channel and the equalizer yields an impulse response  $h_{eff}(\tau, t)$  shorter or equal to the guard interval length. After analog-to-digital conversion, the original channel impulse response is denoted as  $h[n, \Delta]$  and the shortened impulse response as

$h_{eff}[n, \Delta]$  respectively. Certainly, the length of  $h_{eff}[n, \Delta]$  is greater than the length  $L_K$  of  $h[n, \Delta]$ , but the intention is to design the discrete-time equalizer with impulse response  $a[n]$  and length  $L_F$  such, that  $h_{eff}[n, \Delta]$ , which is given by the convolution

$$h_{eff}[n, \Delta] = h[n, \Delta] * a[n] = \sum_{v=0}^{L_F-1} a[v] \cdot h[n-v, \Delta] \quad (3)$$

has samples of significant size only within a window of  $G + 1$  consecutive samples as shown schematically in Fig. 1 for a fixed time instant. The remaining samples of  $h_{eff}[n, \Delta]$  should be very small. We call an impulse  $h_{eff}[n, \Delta]$  with this property as compressed. Thus, the non-zero samples of  $h_{eff}[n, \Delta]$  are given for  $0 \leq n \leq L_F + L_K - 2$  by

$$\begin{aligned} &h_{eff}[0, \Delta], \dots, h_{eff}[\xi - 1, \Delta], \\ &h_{eff}[\xi, \Delta], \dots, h_{eff}[\xi + G, \Delta], \\ &h_{eff}[\xi + G + 1, \Delta], \dots, h_{eff}[L_K + L_F - 2, \Delta] \end{aligned} \quad (4)$$

## II. SYSTEM MODEL

The block diagram of the analyzed OFDM system is shown in Fig. 2. According to [4], a differential phase modulation (D-QPSK) is applied. Furthermore, a reference symbol is periodically transmitted to estimate the channel. Although D-QPSK is used, channel estimation is required to adapt the equalizer coefficients for impulse compression at the receiver. The system performance of the described system is investigated with respect to the bit error ratio (BER). Error correction coding is not considered.

In a multipath channel, the signal seen by the receiver is a sum of the delayed paths of the transmitted signal. As the re-

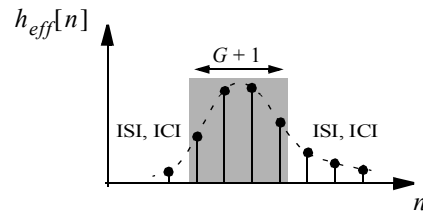


Figure 1: Principle of impulse compression

ceiver is moving, these paths are combining differently over time. Thus, the impulse response of the time-varying channel is given by

$$h(\tau, t) = \sum_{v=0}^{L_K-1} A_v \cdot e^{j\phi_v} \cdot \delta(\tau - \tau_v) \cdot e^{j2\pi f_{D_v} t} \quad (5)$$

where  $t$  denotes the time variation of the channel,  $f_{D_v}$  the Doppler shift and  $\tau_v$  the time delay of the  $v$ -th impulse with amplitude  $A_v$  and phase  $\phi_v$ . The distribution function of the Doppler shift is given by

$$p_{f_D}(f) = \begin{cases} \frac{1}{\pi \cdot f_{D, \max} \cdot \sqrt{1 - (f/f_{D, \max})^2}}, & f \leq f_{D, \max} \\ 0 & \text{elsewhere} \end{cases} \quad (6)$$

For simulations, we have considered a sampled version  $h[n, \Delta]$  of the continuous time-function  $h(\tau, t)$ .

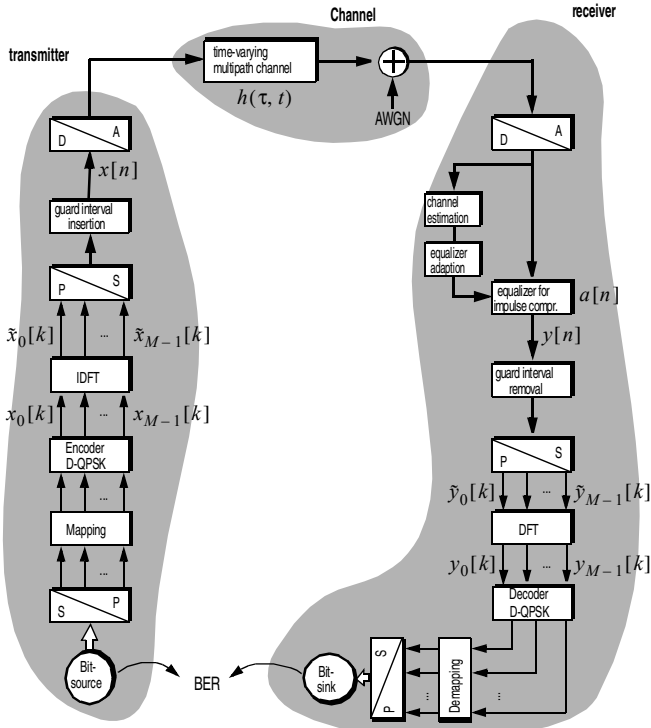


Figure 2: OFDM transmission system with impulse compression

In the following, a Hilly Terrain I delay profile is applied. Because the channel impulse response firstly exceeds the guard interval of length  $G$ , pilot-aided channel estimation is not applicable. To overcome this problem, channel estimation is performed using a periodically transmitted reference symbol containing 2 identical symbols of size  $M$ . Such a reference symbol allows to estimate the original channel impulse response up to a length of  $L_{K, \max} = M + 1$ . The applied frame structure of the transmitted signal is shown in Fig. 3, whereby the total frame length is of size  $\Gamma \cdot (M + G)$ . The overhead due

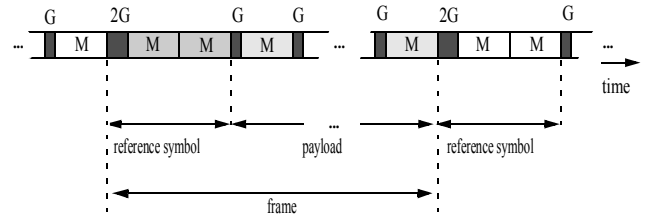


Figure 3: Frame structure of the transmitted signal

to the reference symbol decreases with increasing frame length. The optimization of the filters for channel estimation is done via a Wiener filter approach given in [5].

In order to compress the channel impulse response, we apply an algorithm [6] to calculate the equalizer coefficients of  $a[n]$  depending on estimated  $h[n]$ , that maximizes the ratio

$$\gamma = \frac{E_{in}(\xi)}{E_{out}(\xi)} \quad (7)$$

between the energy  $E_{in}(\xi)$  inside a specified time interval of  $G + 1$  consecutive samples, given by  $h_{eff}[\xi] \dots h_{eff}[\xi + G]$ , and the energy  $E_{out}(\xi)$  of the remaining samples outside this interval. For a fixed time instant, it follows from (3) and (4)

$$E_{in}(\xi) = \sum_{n=\xi}^{\xi+G} |h_{eff}[n]|^2 \quad (8)$$

$$E_{out}(\xi) = \sum_{n=0}^{\xi-1} |h_{eff}[n]|^2 + \sum_{n=\xi+G+1}^{L_K+L_F-2} |h_{eff}[n]|^2 \quad (9)$$

where the window position  $\xi$  is optimized in order to maximize (7).

### III. SIMULATION RESULTS

For simulations, we consider a DFT size of  $M = 256$  and a guard interval length of  $G = 64$ . The time base of the samples at the output of the transmitter is  $T_A = 0.488 \mu s$  and the frame length is defined by  $\Gamma = 40$  and  $\Gamma = 10$  concerning the results in Fig. 5 and Fig. 6, respectively.

Fig. 4 shows the original as well as the compressed time-discrete channel impulse response for a fixed time instant. The compressing effect of the equalizer of length  $L_F = 400$  is clearly visible in Fig. 4. According to (1), the OFDM system without impulse compression requires a guard interval length of about  $G > 180$  to avoid interference as can be seen from the original channel impulse response in Fig. 4. In contrast to that, the guard interval is reduced to  $G = 64$  with impulse compression. As a consequence, the efficiency  $\eta$  of the OFDM system increases.

Finally, the overall system performance in presence of additive white Gaussian noise (AWGN) on the channel is investigated. Fig. 5 and Fig. 6 show the BER versus the signal-to-noise ratio (SNR) at the receiver for an OFDM system with and without an equalizer for impulse compression.

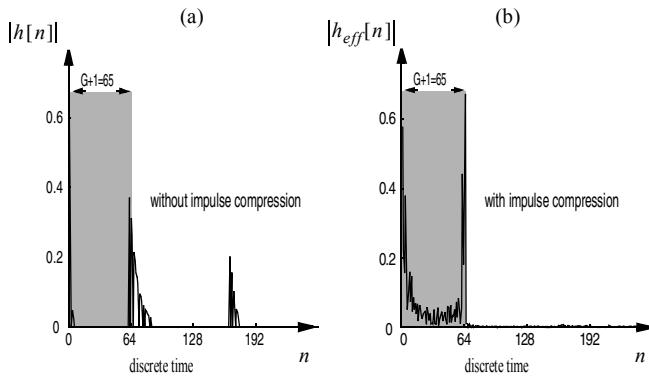


Figure 4: Original (a) and compressed (b) channel impulse response (Hilly Terrain I delay profile) considering a guard interval length of  $G = 64$

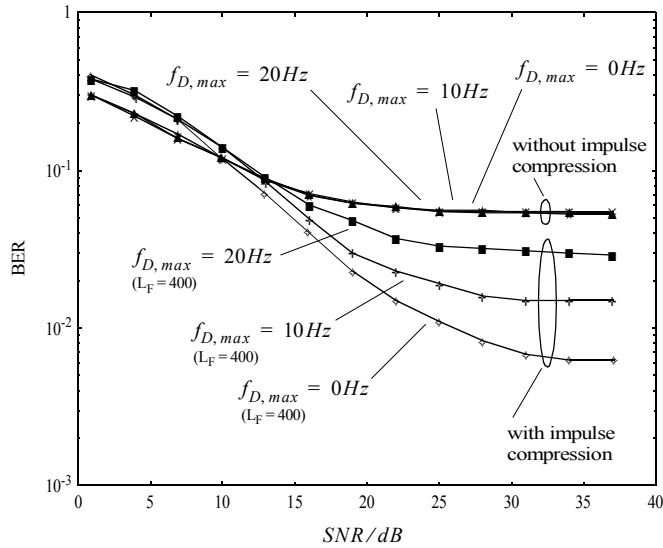


Figure 5: BER versus SNR as a function of the maximum Doppler shift  $f_{D,max}$  with and without impulse compression. A fixed guard interval length of  $G = 64$  is considered. The frame length is determined by  $\Gamma = 40$ .

It can be easily observed, that the BER of the OFDM system decreases with increasing SNR and results in a lower BER in comparison to a system without impulse compression. The improvement of BER depends on the time-varying characteristic of the channel determined by the maximum Doppler shift  $f_{D,max}$ . With increasing frequency of the channel estimation further reduction of the BER can be achieved because the adaption of the equalizer coefficients is improved as it can be seen by comparing Fig. 5 with Fig. 6. But the overhead due to the reference symbol increases simultaneously.

Note, that time-invariant multipath propagation is given by  $f_{D,max} = 0 \text{ Hz}$ . Since the remaining part of the channel impulse response outside the window of  $G + 1$  consecutive samples causes still some interference, an error floor occurs in Fig. 5 and Fig. 6 for high SNR.

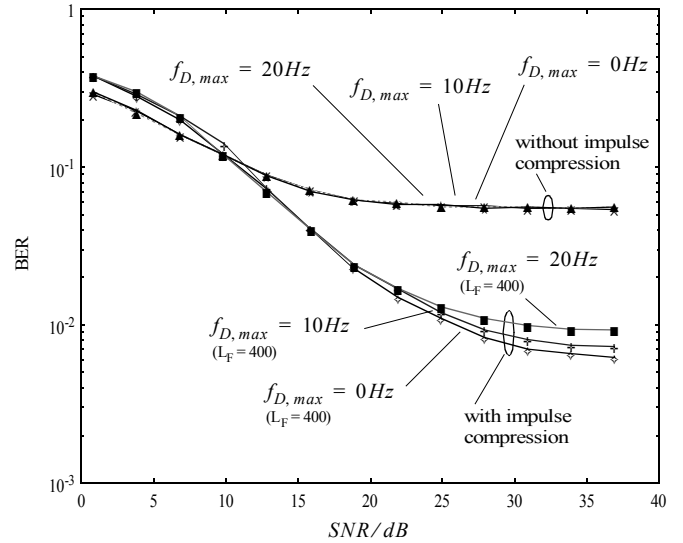


Figure 6: BER versus SNR as a function of the maximum Doppler shift  $f_{D,max}$  with and without impulse compression. A fixed guard interval length of  $G = 64$  is considered. The frame length is determined by  $\Gamma = 10$ .

#### IV. CONCLUSION

This paper examines impulse compression for OFDM transmission over time-varying multipath channels. In consequence of impulse compression, the guard interval can be reduced significantly and the efficiency of the OFDM system increases. Since the equalizer coefficients are adapted to the channel, the quality of the compressing effect is affected significantly by the quality of channel estimation, which is performed periodically by a reference symbol. Thus, a high efficiency and low BER can be achieved simultaneously in particular for time-invariant channels. Overall, if long channel estimation periods can be provided, the proposed OFDM system with impulse compression achieves superior performance in comparison with conventional OFDM systems.

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