Performance Analysis of MIMO Maximum Likelihood Receivers with Channel Correlation, Colored Gaussian Noise, and Linear Prefiltering

Mario Kiessling and Joachim Speidel
Institute of Telecommunications, University of Stuttgart
Pfaffenwaldring 47, 70569 Stuttgart, Germany

Norbert Geng and Markus Reinhardt
Siemens AG, Information and Communication Mobile
Munich and Ulm, Germany

Abstract— **We present an analytical performance evaluation of a Rayleigh fading MIMO link with matrix transmit prefiltering, channel correlation at transmitter and receiver, and spatially colored Gaussian noise for arbitrary two-dimensional signal constellations based on a tight union bound of the pairwise error probabilities. Asymptotic results for the high SNR region allow a simple characterization of the correlation effects and a quantification of the SNR penalty. It is shown that the diversity level of ML detection is unaffected by fading correlation and demonstrated that the effects of transmit and receive correlation may be assessed independently with the standard simplified channel model. Prefiltering algorithms based on long-term stable channel correlation characteristics are derived using the framework at hand. Simulation results illustrate the effectiveness of transmit prefilter designs based on the performance bound.**

Keywords-maximum likelihood; channel correlation; multiple antenna

I. INTRODUCTION

The detrimental effects of fading correlation between the elements of transmit and receive antennas on the capacity of multiple-input multiple-output (MIMO) links have been studied intensively in recent years ([1][2][10]). However, it has been shown that transmit prefiltering of the TX symbol vector is an adequate means to alleviate the performance degradation due to TX fading correlation. Moreover, the adaptation of the TX filter matrix may be controlled statistically, i.e. only by information on the long-term stable TX correlation matrix ([1][10]).

This paper is motivated by the objective to characterize and better understand the influence of fading correlation on the performance of practical MIMO receivers. In addition, abovementioned theoretical results on capacity inspire the research on statistical prefiltering algorithms for standard MIMO transceivers ([3][11][12]). To this end, a performance measure like symbol or bit error rate (SER, BER) has to be quantified as a function of the TX matrix filter. Having this function available, the optimum filter can be found.

In this paper we are analyzing the performance of a MIMO link with optimum maximum likelihood (ML) receiver, which has reasonable complexity for a small number of TX antennas and low-order constellations ([4][5]). We are extending the results presented in [6][7][8] for uncorrelated fading and white Gaussian noise such that we take into account matrix prefiltering, fading correlation at both transmitter and receiver, as well as colored Gaussian interference.

Using a widely accepted MIMO channel model, where the overall channel correlation matrix is split into independent TX and RX correlation matrices, our analysis reveals that the performance penalty at high SNR due to TX and RX correlation may be quantified independently of each other for MIMO ML receivers. Moreover, it turns out that the performance degradation due to RX correlation is a function of the geometric mean of the singular values of the RX correlation matrix, independent of the deployed modulation scheme. On the other hand, TX correlation causes a degradation that depends on the deployed modulation alphabet.

However, the modulation alphabet can be directly shaped by an adequate TX matrix filter. We study the design of such a filter that is capable of totally eliminating the performance degradation due to TX correlation for the considered propagation scenario. It can even slightly outperform the corresponding MIMO system with uncorrelated channel at lower SNR - an interesting result that was not necessarily intuitively expected.

In section II we give an overview of the signal and channel model of the MIMO link. In section III we derive the ML detection metrics and shortly review the union bound of the symbol error probability, which is basically the scaled sum of all possible pairwise error events. A characteristic function approach is used in section IV to derive the pairwise error probability (PEP). Using a Taylor series expansion we give the high SNR approximation of the PEP in section V, which is used for a quantitative assessment of the influence of TX and RX correlation. Using these results, we outline the design of a linear transmit matrix prefilter. We demonstrate the validity of the analysis and the tightness of the bound in Monte-Carlo simulations in section VI.

II. SIGNAL AND CHANNEL MODEL

We consider a flat fading MIMO link with linear transmit prefiltering modeled by

$$\mathbf{y} = \mathbf{HFs} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is the L×1 TX symbol vector, $\mathbf{F}$ is a M_{TX}×L TX linear matrix filter, $\mathbf{H}$ is the M_{RX}×M_{TX} MIMO channel matrix, $\mathbf{n}$ is the M_{RX}×1 noise vector, $\mathbf{y}$ is the M_{RX}×1 receive vector (see Fig. 1).

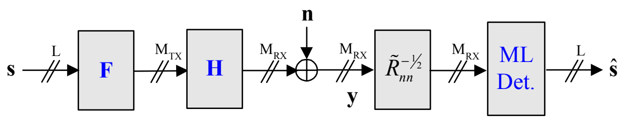

Figure 1. System model with linear transmit prefiltering

Furthermore, L is the number of independent subchannels, M_{RX} is the number of RX antennas and M_{TX} is the number of TX antennas. In the following we denote the noise covariance matrix by $\mathbf{R}_{nn}$, where we let

$$\mathbf{R}_{nn} = E\left[\mathbf{nn}^H\right] = N_0 \cdot \tilde{\mathbf{R}}_{nn}. \tag{2}$$

We have normalized the covariance matrix in order to simplify the expressions in the white Gaussian noise case. Note that the received signal vector $\mathbf{y}$ is first filtered by the noise whitening filter $\tilde{\mathbf{R}}_{nn}^{-1/2}$. After that, the ML processor detects the symbol vector $\hat{\mathbf{s}}$.

With the common simplifying assumptions on the MIMO propagation model given e.g. in [1][2][9], the correlated channel can mathematically be expressed as the matrix product

$$\mathbf{H} = \mathbf{A}^H \mathbf{GB}, \tag{3}$$

where $\mathbf{G}$ is a M_{RX}×M_{TX} matrix of complex i. i. d. Gaussian variables of zero mean and unity variance (i.e. we assume Rayleigh fading) and

$$\mathbf{A}^H \mathbf{A} = \mathbf{R}_{RX} \qquad \mathbf{B}^H \mathbf{B} = \mathbf{R}_{TX}, \tag{4}$$

where $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ is the long-term (LT) stable receive and transmit correlation matrix, respectively. In this paper, we consider the case of full rank of $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$.

III. MAXIMUM LIKELIHOOD DETECTION

A. Detection Metric

With TX matrix prefiltering and noise whitening at the receiver, the detection metric $\mu_{\nu;\mu}$ associated with TX vector hypothesis $\mathbf{s}_\nu$ (under the assumption that actually $\mathbf{s}_\mu$ was transmitted) is given by

$$\mu_{\nu;\mu} = \left\| \tilde{\mathbf{R}}_{nn}^{-1/2}\mathbf{y} - \tilde{\mathbf{R}}_{nn}^{-1/2}\mathbf{HFs}_\nu \right\|^2. \tag{5}$$

The ML receiver performs a minimization of the detection metrics over all TX vector hypotheses $\hat{\mathbf{s}} = \arg\underbrace{\min}_{\mathbf{s}_\nu} \mu_{\nu;\mu}$.

We then reformulate the detection metric in (5) to bring it into a form that appeared already in [8]. Introducing the singular value decompositions (SVD)

$$\begin{aligned} \tilde{\mathbf{R}}_{nn}^{-1/2}\mathbf{A}^H &= \mathbf{U}_A \mathbf{\Sigma}_A \mathbf{V}_A^H \\ \mathbf{B} &= \mathbf{U}_B \mathbf{\Sigma}_B \mathbf{V}_B^H \end{aligned}, \tag{6}$$

we get from (5) after left-multiplication with $\mathbf{U}_A^H$ (this leaves the norm unaffected)

$$\mu_{\nu;\mu} \cong \sum_{r=1}^{M_{RX}} \left| \tilde{y}_r - \sigma_{A,r}\mathbf{g}_r^H \tilde{\mathbf{\Sigma}}_B \mathbf{s}_\nu \right|^2, \tag{7}$$

where we have introduced the definitions

$$\tilde{\mathbf{y}} = \mathbf{U}_A^H \tilde{\mathbf{R}}_{nn}^{-1/2}\mathbf{y} = \left(\tilde{y}_1 \quad \tilde{y}_2 \quad \cdots \quad \tilde{y}_{M_{RX}}\right)^T, \tag{8}$$

$\tilde{\mathbf{\Sigma}}_B = \mathbf{\Sigma}_B \mathbf{\Phi}_f$, $\mathbf{G} = \left(\mathbf{g}_1 \quad \mathbf{g}_2 \quad \cdots \quad \mathbf{g}_{M_{RX}}\right)^H$ and the singular values $\sigma_{A,r}$ on the diagonal of the matrix $\mathbf{\Sigma}_A$. We emphasize again that we consider full rank matrices $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ in this paper. Furthermore, we have used ('$\cong$' means 'have the same distribution') the property $\mathbf{G} \cong \mathbf{VG} \cong \mathbf{GV}$ for a unitary matrix $\mathbf{V}$. The matrix prefilter $\mathbf{F}$ is decomposed as

$$\mathbf{F} = \mathbf{V}_B \mathbf{\Phi}_f \tag{9}$$

with general matrix $\mathbf{\Phi}_f$.

B. Symbol and Bit Error Probability

For the analysis of the symbol and bit error probability we introduce the following definitions. Let Q be the constellation size (e.g. Q=4 for QPSK), whereas we assume in this paper that the modulation on each of the L parallel subchannels is the same for simplicity (however, the generalization for different modulation schemes is straightforward). Let s_q be a symbol out of the signal constellation ($q \in \{0 \ldots Q-1\}$), $\{\mathbf{s}\}$ the set of all Q^L possible TX vectors, $\{\mathbf{s}_i\}$ the set of TX vectors with s_q as their mth element (m is the subchannel index), $\{\mathbf{s}_j\}$ the set of TX symbol vectors with their mth element differing from s_q, $\mu_{i;i}$ the metric associated with TX symbol vector hypothesis $\mathbf{s}_i$ (assuming that in fact $\mathbf{s}_i$ was transmitted), $\mu_{j;i}$ the metric associated with hypothesis $\mathbf{s}_j$ (again assuming that actually $\mathbf{s}_i$ was transmitted).

Furthermore, we denote the difference between the metrics associated with the correct and incorrect hypotheses as

$$D_{s_q,ij}(m) = \mu_{j;i} - \mu_{i;i}. \tag{10}$$

A pairwise error occurs if (for brevity in the following we use the short-hand notation D_{ij})

$$D_{ij} < 0. \tag{11}$$

Let the pairwise error probability (PEP) be defined as

$$P_{s_q,ij}(m) = P\left(D_{ij} < 0 \middle| s_q, \mathbf{s}_i, \mathbf{s}_j\right), \tag{12}$$

i.e. the probability, that actually $\mathbf{s}_i$ was transmitted with s_q as its mth element and the receiver erroneously decides in favor of the incorrect hypothesis $\mathbf{s}_j$. It can be calculated from

$$P_{s_q,ij}(m) = \int_{-\infty}^{0} p(D_{ij})\,dD_{ij}\ , \tag{13}$$

i.e. we have to find the PDF of D_{ij}, namely $p(D_{ij})$, and carry out the integration.

The union bound of the SER for transmit antenna m (or subchannel m, respectively, in the general case with TX prefiltering) reads

$$P_s(m) \le Q^{-L} \sum_q \sum_j \sum_i P_{s_q,ij}(m)\,. \tag{14}$$

It should be mentioned that simplifications are possible by exploiting symmetry of the constellation diagram (cf. [8]). An approximate calculation of the bit error probability is possible via

$$P_b(m) \approx \frac{1}{Q} P_s(m) \tag{15}$$

by assuming that each symbol error causes just exactly one bit error.

IV. Pairwise Error Probability

A. Characteristic Function of the Detection Metric

We are using a characteristic function approach similar to [8] for the evaluation of the PEP. First, we rewrite (7) as

$$\mu_{v;\mu} \cong \sum_r \left| \tilde{y}_r - \tilde{\mathbf{g}}_r^H \tilde{\mathbf{s}}_v \right|^2 \tag{16}$$

with $\tilde{\mathbf{g}}_r = \sigma_{A,r}\mathbf{g}_r$ and $\tilde{\mathbf{s}}_v = \tilde{\mathbf{\Sigma}}_B \mathbf{s}_v$. Note the similarity of (16) and equation (3) in [8]. Omitting details due to the space limitations, we can derive the characteristic function of D_{ij} using well-known results on the expectation of quadratic forms of complex Gaussian vectors

$$\Phi_{D_{ij}}(s) = \prod_{r=1}^{M_{RX}} \frac{1}{(1+\lambda_{r,1}s)(1+\lambda_{r,2}s)}\,. \tag{17}$$

It turns out that $\lambda_{r,1/2}$ are the two non-zero eigenvalues of a matrix given as

$$\lambda_{r,1/2} = \frac{\tilde{T}_{1,r} \pm \sqrt{\tilde{T}_{1,r}^2 + 4N_0\tilde{T}_{1,r}}}{2}\,, \tag{18}$$

where we have introduced the scaled TX symbol vector distance

$$\tilde{T}_{1,r} = \sigma_{A,r}^2 \cdot \left\| \tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j \right\|^2 . \tag{19}$$

It remains now to determine the PDF of D_{ij} and perform the integration in (13).

B. Calculation of the Pairwise Error Probability

Case I: Same singular values $\sigma_{A,r}=\sigma_A$ for all r

This case occurs for spatially white Gaussian noise and vanishing RX correlation. It has been studied in [6][7][8] and we summarize the results for completeness. With

$$r_{s_q,ij} = -\frac{\lambda_1}{\lambda_2} = \frac{\tilde{T}_1}{2N_0} + \sqrt{\left(\frac{\tilde{T}_1}{2N_0}\right)^2 + \frac{\tilde{T}_1}{N_0} + 1} \tag{20}$$

the resulting pairwise error probability is

$$P_{s_q,ij}(m) = \frac{1}{\left(1+r_{s_q,ij}\right)^{2M_{RX}-1}} \sum_{r=0}^{M_{RX}-1} \binom{2M_{RX}-1}{r} r_{ij}^r\,. \tag{21}$$

Note that $\tilde{T}_{1,r} = \tilde{T}_1$, $\lambda_{r,1/2}=\lambda_{1/2}$ for all r, and equations (18)(19)(20)(21) are valid for case I.

Case II: Different singular values $\sigma_{A,r}$

This case occurs for colored Gaussian noise and/or non-vanishing RX correlation. In order to be able to perform an inverse Laplace transform we rewrite (17)

$$\Phi_{D_{ij}}(s) = \prod_{r=1}^{2M_{RX}} \frac{1}{(1+\lambda_r s)}\,, \tag{22}$$

where we combine positive and negative eigenvalues from (18) in a single vector of length $2 \cdot M_{RX}$

$$\begin{pmatrix}\lambda_{1,1} & \lambda_{2,1} & \cdots & \lambda_{M_{RX},1} & \lambda_{1,2} & \lambda_{2,2} & \cdots & \lambda_{M_{RX},2}\end{pmatrix} = \begin{pmatrix}\lambda_1 & \lambda_2 & \cdots & \lambda_{2M_{RX}}\end{pmatrix}. \tag{23}$$

Note that in (23) the positive eigenvalues are arranged first. Now we apply a partial fractional expansion and write (22) as

$$\Phi_{D_{ij}}(s) = \prod_{r=1}^{2M_{RX}} \frac{1}{(1+\lambda_r s)} = \sum_{r=1}^{2M_{RX}} \left(\prod_{\substack{i=1\\ i\neq r}}^{2M_{RX}} \frac{\lambda_r}{\lambda_r - \lambda_i} \right) \frac{1}{1+\lambda_r s}\,. \tag{24}$$

This is the characteristic function of a weighted chi-squared distributed variable. Omitting details, we calculate the PDF of D_{ij} and carry out the straightforward integration in (13). We finally arrive at

$$P_{s_q,ij}(m) = 1 - \sum_{r=1}^{M_{RX}} \prod_{\substack{i=1\\ i\neq r}}^{2M_{RX}} \frac{\lambda_r}{\lambda_r - \lambda_i}\,. \tag{25}$$

Summarizing, for the general case with RX correlation and/or colored noise and arbitrary TX correlation, the PEP is given by equations (18)(19)(25).

V. Pairwise Error Probability Asymptotics

Application of a series expansion of the square root in (18) leads to the high SNR approximation

$$\bar{\lambda}_{r,1/2} = \begin{cases} \sigma^2_{\mathrm{A},r} \left\| \tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j \right\|^2 + N_0 \\ -N_0 \end{cases} . \tag{26}$$

Using the short-hand notation $d_r = \sigma^2_{\mathrm{A},r} \left\| \tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j \right\|^2$ we find from (23), (25), and approximation (26)

$$P_{s_q,ij}(m) \approx 1 - \sum_{r=1}^{M_{RX}} \left[\prod_{\substack{k=1 \\ k\neq r}}^{M_{RX}} \frac{1}{d_r - d_k} \right] \left(\frac{1}{d_r + 2N_0} \right)^{M_{RX}} (d_r + N_0)^{2M_{RX}-1} . \tag{27}$$

Omitting lengthy details, we can show via a Taylor series expansion that the high SNR approximation of (27) is

$$\bar{P}_{s_q,ij}(m) = \frac{1}{\left\| \bar{d}\left(\tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j \right) \right\|^{2M_{RX}}} \binom{2M_{RX}-1}{M_{RX}-1} N_0^{M_{RX}} , \tag{28}$$

where we have introduced the geometric mean

$$\bar{d} = \sqrt[M_{RX}]{\det\left(\mathbf{\Sigma}_A \right)} . \tag{29}$$

Equation (28) explicitly shows a diversity level of M_{RX} (independent of fading correlation and the number of input subchannels L) that can be achieved by ML detection. Furthermore, the SNR penalty induced by fading correlation at the RX array is a function of the geometric mean of the singular values of the RX correlation matrix. Note that (28) reduces to the results presented in [6][7][8] in case of vanishing RX correlation and white Gaussian noise.

The SNR penalty in dB compared to a system with vanishing RX correlation is given by

$$\Delta S_{RX} = 10 \log_{10} \frac{1}{\sqrt[M_{RX}]{\det\left(\mathbf{R}_{RX} \right)}} . \tag{30}$$

Moreover, the degradation is independent of the modulation scheme. With the underlying channel model (3), the effects of RX and TX correlation can be separated. To this end, note that the SNR penalty in dB due to TX correlation (again compared to an uncorrelated system) is [from (14) and (28)]

$$\Delta S_{TX} = 10 \log_{10} \left(\frac{\sum_q \sum_j \sum_i \left\| \mathbf{\Sigma}_B \mathbf{\Phi}_f \left(\mathbf{s}_i - \mathbf{s}_j \right) \right\|^{-2M_{RX}}}{\sum_q \sum_j \sum_i \left\| \left(\mathbf{s}_i - \mathbf{s}_j \right) \right\|^{-2M_{RX}}} \right)^{\frac{1}{M_{RX}}} . \tag{31}$$

Now the SNR penalty is a direct (yet complicated) function of the deployed modulation scheme and the TX matrix filter. It is thus possible to improve the overall performance of the MIMO system by adequate TX filtering. We can minimize the performance degradation via

$$\begin{gathered} \mathbf{\Phi}_{f,opt} = \arg \underbrace{\min}_{\mathbf{\Phi}_f} \sum_q \sum_j \sum_i \left\| \mathbf{\Sigma}_B \mathbf{\Phi}_f \left(\mathbf{s}_i - \mathbf{s}_j \right) \right\|^{-2M_{RX}} \\ s.t. \quad tr\left(\mathbf{\Phi}_f \mathbf{\Phi}_f^H \right) = \rho \end{gathered} , \tag{32}$$

where ρ is the total transmitted energy per channel use. Numerical optimization algorithms can be used to find the optimum filter of the constrained optimization problem in (32). Lower complexity schemes may be developed by exploiting symmetry properties of the symbol constellation and will be a topic of future research.

VI. Simulation Results

In the simulations of this paper, we use a propagation environment (affecting $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$) and a SNR definition that is described in [10][11][12]. We assume a single main direction of departure and arrival at TX and RX, respectively, and an angular spread (AS) characterized by a Laplacian power distribution.

A. *Validation of the BER bound*

In Fig. 2 we have depicted BER curves for a (4, 4) MIMO system. Results for the uncorrelated reference case and semi-correlated channels (RX correlation only) with 10 degrees AS and 2 degrees AS are shown (from left to right). The solid curves result from Monte-Carlo simulations, while the dashed-dotted curves are calculated bounds and the dotted curves are the high SNR approximations according to (28).

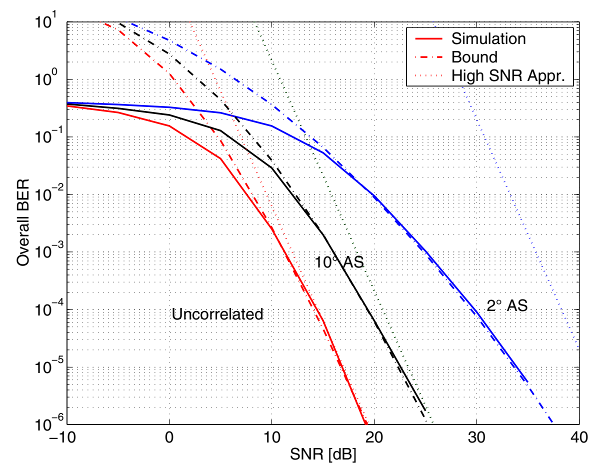

Figure 2. BER uncorrelated, 10° AS RX, 2° AS RX

Note that the performance penalty calculated from (30) is 6.3 dB for 10° AS (at RX) and 23.8 dB for 2° AS (at RX), respectively. One observation from Fig. 2 is that with high RX correlation the ML receiver achieves full diversity only at high SNR. To this end note that there is a noticeable gap between the high SNR approximation and the bound for 2° AS.

Similar curves are plotted for the fully correlated channel with RX and TX correlation in Fig. 3. There is again a good match between the theoretical bound and the simulated curves up to a BER of more than 10^{-2}. Due to TX correlation an additional asymptotic performance penalty of 4.2 dB and 16.8

dB, respectively, results. Small deviations of the bound and the simulated curves result from the approximation (15).

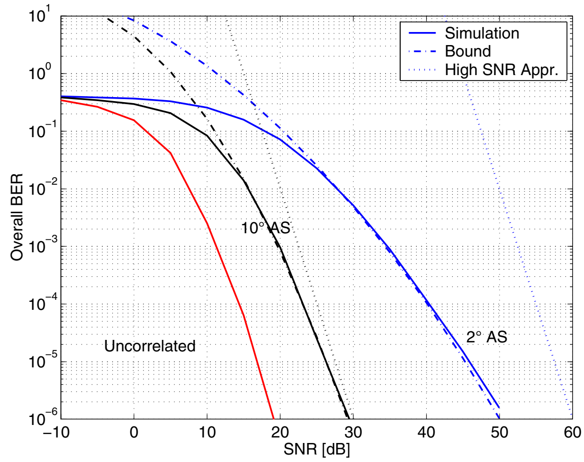

Figure 3. BER uncorrelated, 10° AS RX/TX, 2° AS RX/TX

B. Linear prefiltering based on the BER bound

A standard gradient-based numerical optimization has been used to design TX prefilters for (2, 2) MIMO systems with QPSK modulation via (32). Simulation results of the BER performance of the system with prefiltering are depicted in Fig. 4 for a semi-correlated channel with TX correlation.

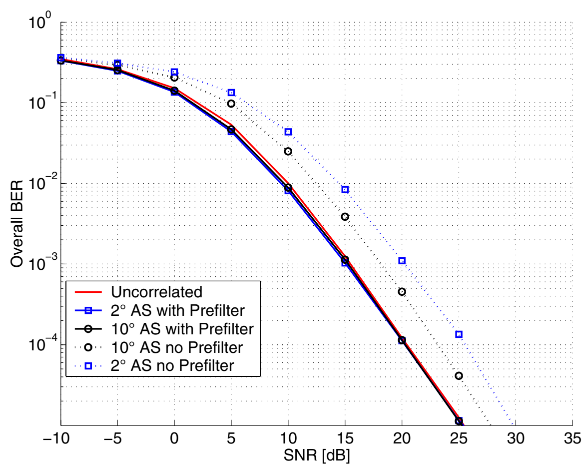

Figure 4. BER with prefiltering (10° and 2° AS at TX)

It is difficult to differentiate between the results for the uncorrelated reference case and those with TX prefiltering and TX correlation corresponding to 10°/2° AS. Obviously, for this particular scenario we can completely eliminate the SNR penalty due to TX correlation by statistical long-term based TX prefiltering and even slightly outperform the uncorrelated system at lower SNR. This is an observation that has already been noted in [1] for the ergodic MIMO capacity. For comparison, we have also plotted the curves with TX correlation but without TX prefiltering that are subject to an asymptotic SNR penalty of 2.5 dB and 4.6 dB (31), respectively.

Further insight may be obtained by looking at the virtual signal constellation (Fig. 5) that results from prefiltering and TX channel correlation ($\tilde{\mathbf{s}}_j = \mathbf{\Sigma}_B \mathbf{\Phi}_f \mathbf{s}_j$ for all j).

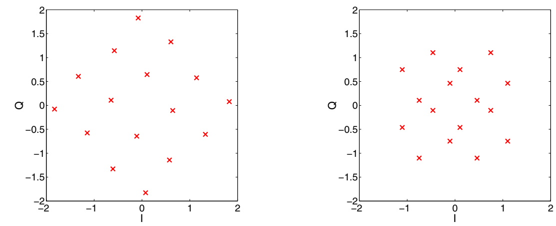

Figure 5. RX constellation with and without prefiltering

We have plotted the first element of the constellation vector $\tilde{\mathbf{s}}_j$ for all j (a superposition of two scaled QPSK constellations) and an AS of 10° at the transmitter including TX prefiltering (left) and without prefiltering (right).

It is clear from (32) that the optimum TX filter arranges the virtual signal constellation more regularly, as otherwise the symbol pair with the minimum distance would dominate the performance penalty at high SNR. Thus the prefilter automatically leads to an increase of the minimum constellation distance, a design criteria that was used in [12].

REFERENCES

[1] M. T. Ivrlac, T. P. Kurpjuhn, C. Brunner, W. Utschick, “Efficient use of fading correlations in MIMO systems”, VTC, October 2001

[2] D. Shiu, G. J. Foschini, M. J. Gans, J. M. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems”, *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 502-513, March 2000

[3] D. Gore, R. Heath, A. Paulraj, “Statistical antenna selection for spatial multiplexing systems”, ICC, pp. 450-454, May 2002

[4] A. van Zelst, R. van Nee, G. Awater, “Maximum likelihood decoding in a space division multiplexing system”, VTC, pp. 6-10, May 2000

[5] A. van Zelst, R. van Nee, G. Awater, “Reduced complexity space division multiplexing receivers”, VTC, pp. 11-15, May 2000

[6] S. J. Grant, J. K. Cavers, “Performance enhancement through joint detection of cochannel signals using diversity arrays”, *IEEE Transactions on Communications*, vol. 46, no. 8, August 1998

[7] S. J. Grant, J. K. Cavers, “Further analytical results on the joint detection of cochannel signals using diversity arrays”, *IEEE Transactions on Communications*, vol. 48, no. 11, November 2000

[8] X. Zhu, R. D. Murch, “Performance analysis of maximum likelihood detection in a MIMO antenna system”, *IEEE Transactions on Communications,* vol. 50, no. 2, February 2002

[9] Viering I., Reinhardt M., Frey T., “Statistical modelling of spatially correlated MIMO channels”, IEEE Int. Symposium on Intelligent Signal Processing, 2001

[10] Kiessling M., Viering I., Reinhardt M., Speidel J., “A closed form bound on correlated MIMO channel capacity”, VTC, September 2002

[11] Kiessling M., Viering I., Reinhardt M., Speidel J., “Short-term and long-term diagonalization of correlated MIMO channels with adaptive modulation,” PIMRC, September 2002

[12] Kiessling M., Viering I., Reinhardt M., Speidel J., “Long-term linear prefiltering for MMSE and ML receivers in correlated MIMO channels”, WCNC, March 2003