Analytical Performance of MIMO MMSE Receivers in Correlated Rayleigh Fading Environments

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Abstract—Closed-form analytical symbol error rate expressions are derived for a MIMO link with linear matrix transmit prefilter and minimum mean squared error receiver in a Rayleigh fading environment. Specifically, using a well-known simplified MIMO channel model, where correlation between the transmit and receive antenna elements is modeled independently, we present exact expressions for vanishing fading correlation as well as the case of correlation between the transmit antenna elements with long-term eigenmode transmission for arbitrary array sizes. Furthermore, exact results are given for a 2×2 system with fading correlation at both transmitter and receiver. High SNR approximations allow a simple quantification of the influence of fading correlation. The accuracy of the analysis is demonstrated via Monte-Carlo simulations.

I. Introduction

We consider the symbol error rate (SER) performance of a MIMO link with linear prefilter at the transmitter and minimum mean squared error (MMSE) receiver. It turns out that there is a close connection to the analysis of optimum combining in the context of smart antenna signal processing. Performance of optimum combining is very well analyzed in case of vanishing fading correlation, e.g. in [1], where the authors base their analysis on a special random quadratic form that was studied in the statistical literature [2]. Recently, an exact analysis based on the eigenvalue distribution of a complex Wishart matrix [3] was presented in [4].

However, only little is known about the exact performance in correlated fading environments. Available results in literature are semi-analytic or use approximations [5][6]. In this paper, we derive exact analytical expressions of the SER with a Rayleigh fading channel assumption for various scenarios. Using a well-known simplified channel model, results are given for the uncorrelated case as well as for the case of transmit correlation with long-term eigenmode transmission and arbitrary array sizes. Furthermore, closed-form expressions are given for the case of a 2×2 system with both transmit and receive correlation. High signal-to-noise-ratio (SNR) approximations give interesting insights in the effects of correlation. Monte-Carlo simulations demonstrate the close match of the analysis. We also mention that the analytical results can effectively be applied for the design of statistical prefiltering algorithms, as is demonstrated in [13].

II. Signal and Channel Model

We consider a flat fading MIMO link modeled by

$$\boldsymbol{y} = \boldsymbol{HFs} + \boldsymbol{n}, \tag{1}$$

where $\mathbf{s}$ is the L×1 TX symbol vector, i.e. there are L independent data streams (subchannels), $\mathbf{F}$ is a M_{TX}×L linear matrix transmit prefilter, $\mathbf{H}$ is the $M_{RX}\times M_{TX}$ MIMO channel matrix with correlated Rayleigh fading elements, $\mathbf{n}$ is the M_{RX}×1 noise vector, and $\mathbf{y}$ is the M_{RX}×1 receive vector (see Fig. 1). By $M_{RX}\geq L$ we denote the number of RX antennas and M_{TX} is the number of TX antennas. Note that L can in general be smaller than the number of transmit antennas.

In the remainder of the paper, by $\mathbf{I}$ we denote an identity matrix, $\mathbf{0}$ is a matrix with all elements equal to 0, diag($x_1,\ldots x_N$) is a diagonal matrix with elements $x_1,\ldots x_N$, vec($\mathbf{X}$) stacks the columns of matrix $\mathbf{X}$, $\otimes$ is the Kronecker product, $\mathbf{X}^*$ means complex conjugate, $\mathbf{X}^T$ means transpose, $\mathbf{X}^H$ means Hermitian, $[\mathbf{X}]_{kk}$ is the kth diagonal element of $\mathbf{X}$, x~ means 'random variable (RV) x is distributed as', x|y~ means 'conditioned on y, RV x is distributed as', x≅ means 'RV x has the same distribution as', and $E_x[\,]$ denotes expectation with respect to RV x.

We define the linear L×M_{RX} receive matrix $\mathbf{G}$ (Fig. 1) and the L×1 vector $\mathbf{z}$ that is used for the subsequent symbol detection. In the following we assume additive white Gaussian noise (AWGN), i.e. the noise covariance matrix is given by $\mathbf{R}_{nn}=N_0\cdot\mathbf{I}$, where N_0 is the noise power. Furthermore, we assume that without loss of generality the symbol energy is normalized such that the covariance matrix of the symbol vector $\mathbf{s}$ is $\mathbf{R}_{ss}=\mathbf{I}$.

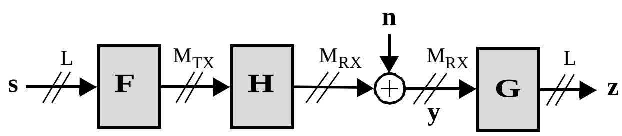

Fig. 1: System model

Using a widely accepted channel model, the correlated MIMO channel can be described by the matrix product

$$\boldsymbol{H} = \boldsymbol{A}^H\boldsymbol{H}_w\boldsymbol{B}, \tag{2}$$

where $\mathbf{H}_w$ is a $M_{RX}\times M_{TX}$ matrix of complex i.i.d. Gaussian variables of unity variance and

$$AA^H = R_{RX} \qquad BB^H = R_{TX}, \tag{3}$$

where $\mathbf{R}_{\mathrm{RX}}$ and $\mathbf{R}_{\mathrm{TX}}$ is the long-term stable (normalized) receive and transmit correlation matrix, respectively. Furthermore, we introduce the eigenvalue decompositions (EVD)

$$R_{RX} = V_{\mathrm{RX}}\Sigma V_{\mathrm{RX}}^H \qquad R_{TX} = V_{\mathrm{TX}}\Lambda V_{\mathrm{TX}}^H \tag{4}$$

with diagonal eigenvalue matrices $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_{M_{RX}})$ and $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_{M_{TX}})$.

III. Subchannel SNR Expression

In case of the MMSE receiver, the receive matrix filter $\mathbf{G}$ is given by ($\mathbf{R}_{ss}$=$\mathbf{I}$)

$$G = (F^H H^H HF + N_0 \cdot I)^{-1} F^H H^H. \tag{5}$$

The SNR γ_1 on subchannel 1 is given by (an extension to an arbitrary subchannel is straightforward)

$$\gamma_1 = \frac{1}{N_0[(F^H H^H HF + N_0 \cdot I)^{-1}]_{11}} - 1. \tag{6}$$

Partitioning the compound matrix HF as $HF = \begin{bmatrix} h_1 & \tilde{H} \end{bmatrix}$ with column vector $\mathbf{h}_1$, defining $\mathbf{C}=(\mathbf{F}^{\mathrm{H}}\mathbf{R}_{\mathrm{TX}}\mathbf{F})^*$, and exploiting properties of partitioned inverses (54) (see also [7]) as well as the matrix inversion lemma, the SNR on subchannel 1 can be reformulated

$$\gamma_1 = h_1^{\mathrm{H}}(\tilde{H}\tilde{H}^H + N_0 \cdot I)^{-1} h_1. \tag{7}$$

With the given channel model and results on matrix variate normal distributions (see appendix, [8], [14]) we find

$$\tilde{H} \sim \mathcal{N}_{M_{RX}, M_{TX-1}}(\mathbf{0}, R_{RX} \otimes C_{22}). \tag{8}$$

and equivalently (with scalar $C_{11 \cdot 2}$ for the given problem)

$$h_1 \Big| \tilde{H} \sim \mathcal{N}_{M_{RX}}\Big(\tilde{H}(C_{22})^{-1} C_{21}, C_{11 \cdot 2} \cdot R_{RX}\Big), \tag{9}$$

where we have used the notation of (53) and (54) for the partitioning of $\mathbf{C}$. For brevity, let $a = (C_{22})^{-1} C_{21}$ and introduce a new random vector $x \sim \mathcal{N}_{M_{RX}, 1}(\mathbf{0}, C_{11 \cdot 2} \cdot R_{RX})$. Then note that we can equivalently express (7) by

$$\gamma_1 \cong (x + \tilde{H}a)^H (\tilde{H}\tilde{H}^H + N_0 \cdot I)^{-1} (x + \tilde{H}a). \tag{10}$$

A generalization to an arbitrary subchannel is straightforward and left to the reader.

IV. Symbol Error Rate

Consider the conditional SER approximation for square M-QAM constellations

$$P_{e,c}(\gamma_1) = b \cdot \mathrm{erfc}(\sqrt{c\gamma_1}) \tag{11}$$

with parameters b and c chosen according to the modulation alphabet size M. Obviously, the unconditional SER P_e averaged over Rayleigh fading can be determined via the expectation $P_e = \mathrm{E}_{\gamma_1}[P_{e,c}(\gamma_1)]$. We note that the bit error rate BER P_b can be approximated via $P_b \approx P_e / \log_2(M)$. In the general case of arbitrary receive and transmit correlation the analytical calculation of the expectation seems to very involved. However, for special cases the SNR statistics simplify and closed-form SER expressions can be determined.

A. Transmit Correlation with Eigenmode Transmission

Note that for long-term eigenmode (EM) transmission the transmit matrix prefilter reads

$$F = V_{\mathrm{TX}}\Phi \tag{12}$$

with real diagonal power allocation matrix Φ, resulting in real diagonal $\mathbf{C}$=diag(c_1,…,c_L)=$\Phi^2\Lambda$. Given this choice of prefilter, the columns of the compound matrix HF are uncorrelated, thus resulting in $\mathbf{C}_{21}$=$\mathbf{0}$ ($\mathbf{a}$=$\mathbf{0}$, respectively) and from (10) the SNR expression reduces to

$$\begin{aligned} \gamma_{1,EM} &\cong x^H(\tilde{H}\tilde{H}^H + N_0 \cdot I)^{-1} x \\ &\cong c_1 \cdot u^H(\tilde{H}_w C_{22} \tilde{H}_w^H + N_0 \cdot I)^{-1} u \\ &= u^H\Big(\tilde{H}_w \Theta \tilde{H}_w^H + \frac{1}{\gamma} \cdot I\Big)^{-1} u \end{aligned} \tag{13}$$

with $\Theta = \mathrm{diag}(\theta_1, \ldots, \theta_{L-1}) = C_{22}/c_1$, $\gamma = c_1/N_0$, $\tilde{H}_w \sim \mathcal{N}_{M_{RX}, L-1}(\mathbf{0}, I)$, and a vector of i.i.d. Gaussian random variables $\mathbf{u}$. The reliability function R($\gamma_{1,EM}$) of the random variable $\gamma_{1,EM}$ was given in [1] in the context of optimum combining (corresponding to the case of multiple uncorrelated users with different transmit powers)

$$R(\gamma_{1,EM}) = e^{-\frac{\gamma_{1,EM}}{\gamma}} \cdot \left(\sum_{i=1}^{M_{RX}} \bar{\beta}_{i-1} \cdot \gamma_{1,EM}^{i-1}\right) \Bigg/ \left(\prod_{j=1}^{L-1} (1 + \theta_j \gamma_{1,EM})\right), \tag{14}$$

where the $\bar{\beta}_i$ are the coefficients of z^i in the series expansion of (we note that there is a recursive formula for the calculation)

$$\exp(z/\gamma) \cdot \prod_{j=1}^{L-1} (1 + \theta_j z). \tag{15}$$

Note that for a random variable u the reliability function is defined by R(u_0)=Prob(u>u_0), i.e. it is the complementary distribution function, and for $\gamma_{1,EM}$ obviously R(0)=1 and R(∞)=0. Applying partial integration and noting $P_{e,c}(\infty)$=0 and $P_{e,c}(0)$=b, one can derive in general

$$P_e = b + \int_0^\infty \Big(\frac{d}{d\gamma} P_{e,c}(\gamma)\Big) R(\gamma) d\gamma. \tag{16}$$

For the conditional symbol error rate formula (11) we find

$$\frac{d}{d\gamma} P_{e,c}(\gamma) = -b\sqrt{\frac{c}{\pi}} \cdot \gamma^{-1/2} \cdot e^{-c\gamma}. \tag{17}$$

For evaluating the integral (16) we make use of the expansion into partial fractions

$$\prod_{n=1}^{N} \frac{1}{1+\theta_n z} = \sum_{n=1}^{N} \left(\prod_{j=1, j \neq n}^{N} \frac{\theta_n}{\theta_n - \theta_j} \right) \frac{1}{1+\theta_n z} \tag{18}$$

and the integration result [9] 3.383.10 yielding for constants a, b, and c

$$\int_0^{\infty} \frac{z^{a-\frac{3}{2}}}{1+bz} \cdot e^{-cz}\, dz = \Gamma\left(a-\frac{1}{2}\right) \cdot b^{\frac{1}{2}-a} \cdot e^{c/b} \cdot \Gamma\left(\frac{3}{2}-a, \frac{c}{b}\right). \tag{19}$$

The resulting symbol error probability reads

$$P_e = b \cdot \left[\frac{1}{2} - \sqrt{\frac{c}{\pi}} \sum_{j=1}^{M_{RX}} \bar{\beta}_{j-1} \Gamma\left(j-\frac{1}{2}\right) \left(\sum_{n=1}^{L-1} c_n \theta_n^{\frac{1}{2}-j} e^{\delta/\theta_n} \Gamma\left(\frac{3}{2}-j, \delta/\theta_n\right) \right) \right], \tag{20}$$

where δ=c+N_0 and constants

$$c_n = \prod_{j=1, j\neq n}^{L-1} \frac{\theta_n}{\theta_n - \theta_j}. \tag{21}$$

$\Gamma(z)$ and $\Gamma(a,z)$ denote the Gamma and incomplete Gamma function (cf. [10]), respectively.

B. Uncorrelated Case

Following the proceeding in the previous paragraph, for uncorrelated fading without prefiltering (Γ=**I**), we can use the integral identity [9] 3.383.5 resulting with constants a, c, and N in

$$\int_0^{\infty} \frac{z^{a-\frac{3}{2}}}{(1+z)^N} \cdot e^{-cz}\, dz = \Gamma\left(a-\frac{1}{2}\right) \cdot \Psi\left(a-\frac{1}{2}, a-N+\frac{1}{2}, c\right) \tag{22}$$

and obtain the error probability expression

$$P_e = b \cdot \left[\frac{1}{2} - \sqrt{\frac{c}{\pi}} \sum_{j=1}^{M_{RX}} \bar{\beta}_{j-1} \Gamma\left(j-\frac{1}{2}\right) \Psi\left(j-\frac{1}{2}, j-L+\frac{3}{2}, \delta\right) \right]. \tag{23}$$

with Tricomi function $\Psi(a,c,z)$ defined by

$$\Psi(a,c,z) = \frac{\Gamma(1-c)}{\Gamma(1+a-c)}\, {}_1F_1(a,c,z) + \frac{\Gamma(c-1)}{\Gamma(a)} z^{1-c}\, {}_1F_1(1+a-c, 2-c, z) \tag{24}$$

and the so-called Kummer function (cf. [10])

$${}_1F_1(a,c,z) = \sum_{k=0}^{\infty} \frac{1}{k!} \cdot \frac{(a)_k}{(c)_k} \cdot z^k = 1 + \frac{a}{c} \cdot z + \frac{1}{2} \cdot \frac{a \cdot (a+1)}{c \cdot (c+1)} z^2 + \dots \tag{25}$$

with Pochhammer symbol $(a)_k$=a·(a+1)·...·(a+k-1).Introducing the auxiliary function

$$Z_1(x) = e^x \cdot \mathrm{erfc}(\sqrt{x}) \tag{26}$$

for brevity, it can be shown that in a 2×2 system (23) reduces to

$$P_e = b\left(1 + N_0\sqrt{c\pi} \cdot Z_1(c+N_0) - (N_0+1)\sqrt{\frac{c}{c+N_0}}\right). \tag{27}$$

This result can be confirmed by methods applied in the following paragraph for analyzing 2×2 systems.

C. Receive Correlation only in a 2×2 System

It appears that there are no results available on the probability density or reliability function of the SNR in case of receive correlation. However, for the 2×2 case, the average SER may be calculated by expressing the conditional SER as a function of the eigenvalues of a special random matrix and then averaging over the statistics of the random eigenvalues. We note that a similar technique has also been used in [4] for uncorrelated fading. The same proceeding presented here for the 2×2 case is possible for arbitrary array sizes, as soon as a joint eigenvalue probability density function is available. It appears, however, that this is a mathematically challenging task.

Now we have **C**=diag(c_1,c_2) and **S**=diag(σ_1,σ_2). We first focus on the case **C**=**I** resulting similarly to (13) after simple manipulations in the subchannel SNR expression

$$\gamma_1 \cong \boldsymbol{u}^{\mathrm{H}} (\boldsymbol{h}_w \boldsymbol{h}_w^H + N_0 \cdot \Sigma^{-1})^{-1} \boldsymbol{u} \tag{28}$$

with 2×1 column vector $\mathbf{h}_{\mathrm{w}}$ of uncorrelated Gaussian elements. If we introduce the EVD of the rank 2 matrix in (28)

$$\boldsymbol{Y} = \boldsymbol{h}_w \boldsymbol{h}_w^H + N_0 \cdot \Sigma^{-1} = \boldsymbol{V} \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} \boldsymbol{V}^H, \tag{29}$$

it is clear that by inserting (29) in (28) we arrive at a quadratic form in random variables and can express the SNR as

$$\gamma_1 \cong \frac{x}{\lambda_1} + \frac{y}{\lambda_2} \tag{30}$$

with exponentially distributed random variables x and y. Letting

$$T_1 = \mathrm{tr}(\boldsymbol{Y}) \qquad T_2 = \mathrm{tr}(\boldsymbol{Y}^2), \tag{31}$$

it can be shown that we get an explicit expression for the eigenvalues of the 2×2 matrix **Y** as a function of T_1 and T_2

$$\lambda_{1,2} = \frac{T_1 \pm \sqrt{2T_2 - T_1^2}}{2}. \tag{32}$$

We first focus on the conditional error probability approximation based on the exponential function to simplify the derivation

$$P_{e,c}(\gamma_1) = b \cdot \exp(-c\gamma_1). \tag{33}$$

For clarity we emphasize that the error rate can be calculated by calculating the expectation

$$P_e = \mathrm{E}_{\gamma_1}[P_{e,c}(\gamma_1)] = b \cdot \mathrm{E}_{x,y,\lambda_1,\lambda_2}\left[\exp\left(-c \cdot \left(\frac{x}{\lambda_1} + \frac{y}{\lambda_2}\right)\right)\right]. \tag{34}$$

Straightforward integration over x and y yields

$$P_e = b \cdot E_{\lambda_1,\lambda_2}\left[\frac{\lambda_1\lambda_2}{(\lambda_1\lambda_2 + c(\lambda_1+\lambda_2+c))}\right]. \tag{35}$$

Plugging (32) in (35), one can derive after tedious math

$$P_{e,c}(\gamma_1) = b \cdot E_{\boldsymbol{h}_w}\left[\frac{N_0 \cdot \boldsymbol{h}_w^H \Sigma \boldsymbol{h}_w + N_0^2}{\boldsymbol{h}_w^H[N_0 \cdot \Sigma + c \cdot \det(\Sigma) \cdot \boldsymbol{I}]\boldsymbol{h}_w + N_0^2 + cN_0 \cdot \mathrm{tr}(\Sigma) + c^2 \cdot \det(\Sigma)}\right]. \tag{36}$$

Note that the expression within the expectation in (36) is a ratio of quadratic forms in complex normally distributed random variables and can be written in the form (see also [11])

$$r = \frac{a_1 x + a_2 y + a_3}{b_1 x + b_2 y + b_3}, \tag{37}$$

again with exponentially distributed variables x, y and constants a_i, b_i. Omitting lengthy details of the integrations for determining the expectation, where we can exploit the Gamma integral

$$z^{-\beta} = \frac{1}{\Gamma(\beta)} \int_0^{\infty} x^{\beta - 1} e^{-zx} dx \tag{38}$$

for rewriting the denominator of (37), we finally arrive with the auxiliary function

$$Z_2(x) = e^x \cdot \mathrm{Ei}(x), \tag{39}$$

where Ei denotes the exponential integral [10], at the symbol error probability expression based on (33)

$$P_e = b \cdot \left[c \cdot \frac{(\det(\Sigma) + \sigma_2 N_0) \cdot Z_2\left(c + \frac{N_0}{\sigma_2}\right) - \left((\det(\Sigma) + \sigma_1 N_0) \cdot Z_2\left(c + \frac{N_0}{\sigma_1}\right)\right)}{N_0(\sigma_1 - \sigma_2)} + 1 \right] . \tag{40}$$

In order to refine the result using the conditional error expression in (11), we can make use of the integral representation of the erfc function [12]

$$\mathrm{erfc}(\sqrt{x}) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x}{\sin^2\theta}\right) d\theta . \tag{41}$$

Comparing (11) together with (41) and (33), the reader can readily check that we get a more accurate symbol error rate expression via replacing c by $c/\sin^2\theta$ and integrating

$$\tilde{P}_e = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} P_e\left(c \to \frac{c}{\sin^2\theta}\right) d\theta . \tag{42}$$

Using the integration result

$$\int_0^{\frac{\pi}{2}} \frac{c}{\gamma \cdot \sin^2\phi} Z_2\left(\frac{1}{\gamma} + \frac{c}{\sin^2\phi}\right) d\phi = \frac{\pi}{2} \cdot \frac{1}{\gamma} \cdot \sqrt{\pi c} \cdot Z_1\left(c + \frac{1}{\gamma}\right) \tag{43}$$

we finally arrive at

$$\tilde{P}_e = b \cdot \left[\sqrt{\pi c} \frac{(\det(\Sigma) + \sigma_2 N_0) \cdot Z_1\left(c + \frac{N_0}{\sigma_2}\right) - (\det(\Sigma) + \sigma_1 N_0) \cdot Z_1\left(c + \frac{N_0}{\sigma_1}\right)}{N_0(\sigma_1 - \sigma_2)} + 1 \right] . \tag{44}$$

Calculating the limit $\sigma_1 \to 1$, $\sigma_2 \to 1$, i.e. no RX correlation, it can be shown that (44) reduces to (27), as expected. Furthermore, via a series expansion it can be shown that the high SNR approximation (which is also valid for vanishing correlation with $\mathbf{R}_{RX}=\mathbf{I}$) reads

$$P_e \approx b \cdot \frac{1}{\det(\boldsymbol{R}_{RX})} \cdot \left(\sqrt{c\pi} \cdot Z_1(c) + \frac{1}{2c} - 1\right) \cdot N_0 . \tag{45}$$

As expected for a linear receiver (cf. [14]), the diversity order of 1 is obvious in (45). Equivalently, it can be shown by deriving the asymptotics of (20) and (23) that the diversity of a system with MMSE receiver is given by $N=M_{RX}-L+1$, as in case of a ZF receiver [14]. Moreover, we note that the determinant is a Schur-concave function of the eigenvalues, i.e. the symbol error rate is a Schur-convex function of the RX correlation matrix. Thus, (45) is a formal proof of the intuition that higher correlation (resulting in a higher spread of the eigenvalues of $\mathbf{R}_{RX}$) leads to higher symbol error rate.

D. Transmit Correlation in a 2×2 System

For a 2×2 system with transmit correlation and EM transmission (13) reduces to

$$\gamma_1 \cong \frac{c_1}{c_2} \cdot \boldsymbol{u}^{\mathrm{H}} \left(\boldsymbol{h}_w \boldsymbol{h}_w^H + \frac{N_0}{c_2} \cdot \Sigma^{-1} \right)^{-1} \boldsymbol{u} . \tag{46}$$

Now compare (28) and (46) together with (33). The reader can readily check that replacing c by $c \cdot c_1/c_2$ and N_0 by N_0/c_2 in equations (27), (40), (44) and (45) provides the corresponding symbol error rate results for the case of transmit correlation.

V. Simulation Results

In the following simulations we use the same setup as in [13][14] and assume QPSK modulation. In presence of correlation, we assume a single main direction of departure and arrival, respectively. Furthermore, there is a Laplacian power distribution over the angular spread (AS) of 10 degrees, corresponding to a RX correlation matrix with element-wise absolute values

$$\mathrm{abs}(\boldsymbol{R}_{RX}) = \mathrm{abs}\begin{bmatrix} 1 & 0.43 - j \cdot 0.78 \\ 0.43 + j \cdot 0.78 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.89 \\ 0.89 & 1 \end{bmatrix} . \tag{47}$$

Simulation results and theoretical curves including the asymptotics closely agree in Fig. 2 (2×2 system uncorrelated and with RX correlation).

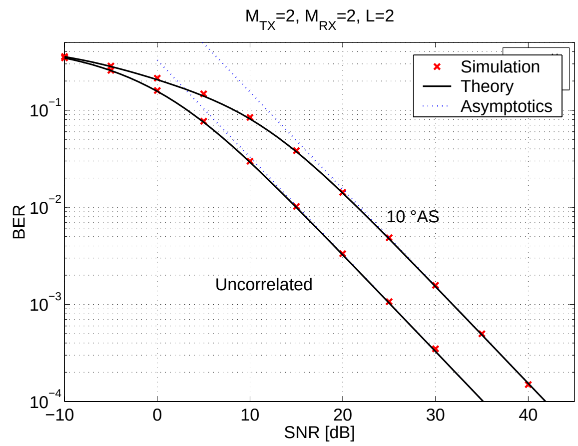

Fig. 2: BER performance (unc., RX corr., $L=M_{RX}=M_{TX}=2$)

Fig. 3 shows a 4×4 system with vanishing fading correlation and with TX correlation and eigenmode transmission. Note that in Fig. 3 we have plotted the BERs of all L=4 subchannels in case of TX correlation, as the different strength of the 4 long-term eigenmodes lead to significantly different BERs, while in the uncorrelated case all subchannels exhibit same BERs.

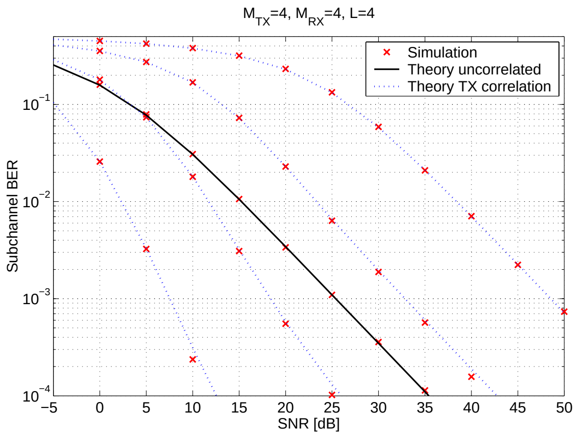

Fig. 3: BER performance (unc., TX corr., L=M_{RX}=M_{TX}=4)

Finally we present results for a 2×2 system with both RX and TX correlation with EM transmission in Fig. 4. Both correlation matrices are chosen according to (47).

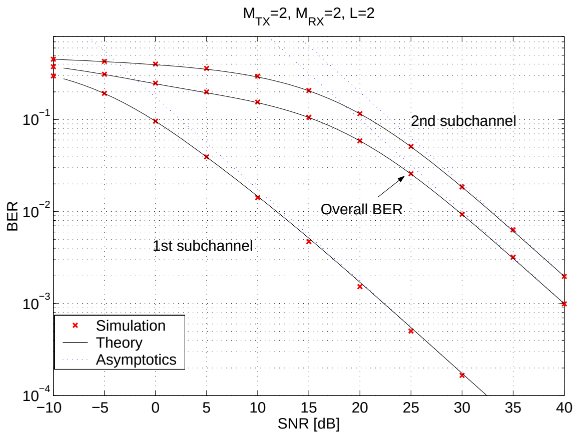

Fig. 4: BER performance (RX/TX corr., L=M_{RX}=M_{TX}=2)

VI. Appendix

Let the vector-valued complex normal distribution with m elements, covariance matrix Σ, and mean μ be denoted by $\mathcal{N}_m(\mu, \Sigma)$. A m×n matrix **X** is said to have a matrix variate complex normal distribution [8], denoted by $\mathcal{N}_{m,n}(\mathbf{M}, \Sigma\otimes\Psi)$, with mean **M** and covariance $\Sigma\otimes\Psi$, if

$$vec(\boldsymbol{X}^T) \sim \mathcal{N}_{mn}(vec((\boldsymbol{M}^T), \Sigma \otimes \Psi)) . \qquad (48)$$

Now introduce the following partitions

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{X}_2 \end{bmatrix} \quad \boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{M}_2 \end{bmatrix} \quad \Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} . \qquad (49)$$

with $\mathbf{X}_1$ of size m×r, $\mathbf{X}_2$ of size m×(n-r), equivalent partitions for **M,** Ψ_{11} of size r×r, Ψ_{22} of size (n-r)×(n-r), Ψ_{12} of size r×(n-r), Ψ_{21} of size (n-r)×r, and the definition

$$\Psi_{11 \cdot 2} = \Psi_{11} - \Psi_{12}\Psi_{22}^{-1}\Psi_{21} . \qquad (50)$$

It can then be shown that

$$\boldsymbol{X}_2 \sim \mathcal{N}_{m,\, n-r}(\boldsymbol{M}_2, \Sigma \otimes \Psi_{22}) . \qquad (51)$$

and conditioned on $\mathbf{X}_2$, the distribution of $\mathbf{X}_1$ is given by

$$\boldsymbol{X}_1 \big| \boldsymbol{X}_2 \sim \mathcal{N}_{m,\, r}(\boldsymbol{M}_1 + (\boldsymbol{X}_2 - \boldsymbol{M}_2)\Psi_{22}^{-1}\Psi_{21}, \Sigma \otimes \Psi_{11 \cdot 2}) . \qquad (52)$$

Partition an arbitrary matrix **A** and its inverse $\mathbf{A}^{-1}$ as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \qquad \boldsymbol{A}^{-1} = \begin{bmatrix} \boldsymbol{A}^{11} & \boldsymbol{A}^{12} \\ \boldsymbol{A}^{21} & \boldsymbol{A}^{22} \end{bmatrix} . \qquad (53)$$

It is then well-known that

$$\boldsymbol{A}^{11} = (\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21})^{-1} = (\boldsymbol{A}_{11 \cdot 2})^{-1} . \qquad (54)$$

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