ANALYTICAL PERFORMANCE OF MIMO ZERO-FORCING RECEIVERS IN CORRELATED RAYLEIGH FADING ENVIRONMENTS

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ABSTRACT

Simulation results show that fading correlation between the antenna elements of a wireless system can seriously affect the symbol error rate of MIMO receivers, especially in case of linear processing. In order to gain a better understanding of the fading effects, we present an analytical framework to assess the performance of a system with zero-forcing receiver in a correlated Rayleigh fading scenario. Closed-form expressions are derived for the symbol error rate with arbitrary array sizes in case of transmit correlation only. In the presence of a fully correlated channel, we study the 2×2 case. High SNR asymptotics allow an insightful comparison of systems with transmit and receive correlation, respectively. Monte-Carlo simulations evidence the accuracy of the analysis.

1. INTRODUCTION

The influence of fading correlation on the capacity of multiple-input multiple-output (MIMO) wireless systems has been subject to active research recently. However, due to the mathematical complexity of the problem, one has to resort to asymptotic analysis [1] or approximations [2][10].

On the other hand, relatively little is known about the performance of practical receiver types like e.g. linear zero-forcing (ZF) and minimum mean squared error (MMSE). This holds also for the more specialized single-input multiple-output (SIMO) and multiple-input single-output (MISO), respectively, cellular beamforming applications. While the uncorrelated Rayleigh fading case is well analyzed [3], only few results are available on correlated scenarios [4][5]. Again, the mathematical analysis is greatly complicated in the presence of fading correlation.

Motivated by the need to better characterize and quantify the influence of channel correlation on practical systems (e.g. for the design of adaptive systems with statistical TX prefiltering [11]), we study the performance of a MIMO ZF receiver in a correlated Rayleigh fading environment with arbitrary RX and TX correlation. To this end, we use results from the theory of complex matrix variate normal distributions [6] and ratios of random quadratic forms [7]. In this context, we present an alternative derivation for the subchannel SNR statistics in case of arbitrary TX correlation and unrestricted array sizes given in [8]. Furthermore, we derive closed-form expressions for the symbol error rate (SER) of a 2×2 MIMO system in the presence of a fully correlated channel.

Asymptotic results for the high SNR region allow a simple quantification of the influence of channel correlation. Interestingly, we can prove for the 2×2 case that two semi-correlated systems with the same RX or TX correlation matrix, respectively, yield the same asymptotic performance. Finally, we show via Monte-Carlo simulations the close agreement of theoretical and simulation results.

2. SIGNAL AND CHANNEL MODEL

We consider a flat fading MIMO link modeled by

$$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n} \ , \qquad (1)$$

where $\mathbf{s}$ is the $M_{TX}\times 1$ TX symbol vector, $\mathbf{H}$ is the $M_{RX}\times M_{TX}$ MIMO channel matrix with correlated Rayleigh fading elements, $\mathbf{n}$ is the $M_{RX}\times 1$ noise vector, and $\mathbf{y}$ is the $M_{RX}\times 1$ receive vector (see Fig. 1). By $M_{RX}\geq M_{TX}$ we denote the number of RX antennas, and M_{TX} is the number of TX antennas. In this paper, we abstain from including a linear prefilter in the system model due to the space limitation, however, we mention that it can be taken into account with only minor modifications.

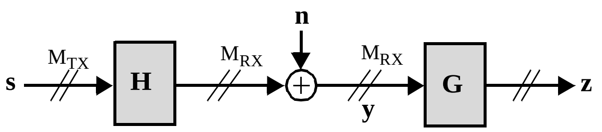

Fig. 1: System model

In the remainder of the paper, by $\mathbf{I}$ we denote an identity matrix, $\mathbf{0}$ is a matrix with all elements equal to 0, vec($\mathbf{X}$) stacks the columns of matrix $\mathbf{X}$, $\otimes$ is the Kronecker product, $\mathbf{X}^*$ means complex conjugate, $\mathbf{X}^T$ means transpose, $\mathbf{X}^H$ means Hermitian, $[\mathbf{X}]_{kk}$ is the kth diagonal element of $\mathbf{X}$, x~ means 'random variable (RV) x is distributed as', x|y~

means 'conditioned on y, RV x is distributed as', x≅ means 'RV x has the same distribution as', and $E_x[\ ]$ denotes expectation with respect to RV x.

We define the linear zero-forcing $M_{TX}\times M_{RX}$ matrix **G** and the $M_{TX}\times 1$ vector **z** that is used for the subsequent symbol detection. In the following we assume additive white Gaussian noise (AWGN), i.e. the noise covariance matrix is given by $\mathbf{R}_{nn}=N_0\cdot\mathbf{I}$, where N_0 is the noise power. Furthermore, we assume that the symbol energy is normalized such that the covariance matrix of the symbol vector **s** is $\mathbf{R}_{ss}=\mathbf{I}$.

Using a widely accepted channel model, the correlated MIMO channel can be described by the matrix product

$$\boldsymbol{H} = \boldsymbol{A}^H \boldsymbol{H}_w \boldsymbol{B}, \tag{2}$$

where $\mathbf{H}_w$ is a $M_{RX}\times M_{TX}$ matrix of complex i.i.d. Gaussian variables of unity variance and

$$\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{R}_{RX} \qquad \boldsymbol{B}\boldsymbol{B}^H = \boldsymbol{R}_{TX}, \tag{3}$$

where $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ is the long-term stable (normalized) receive and transmit correlation matrix, respectively. Exploiting the properties of the vec and Kronecker operators and using results on complex matrix variate normal distributions (see appendix), it can be seen that the statistics of **H** are given by

$$\boldsymbol{H} \sim \mathcal{N}_{M_{RX}, M_{TX}}(\mathbf{0}, \boldsymbol{R}_{RX} \otimes \boldsymbol{R}_{TX}^*), \tag{4}$$

which has zero mean, as we assume a vanishing Ricean component.

3. SUBCHANNEL SNR STATISTICS

The receiver zero-forcing matrix filter is given by the pseudo-inverse

$$\boldsymbol{G} = \boldsymbol{H}^\dagger = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H, \tag{5}$$

with **GH**=**I**. Under the AWGN assumption it is then straightforward to show that the signal-to-noise ratio on subchannel 1 can be expressed as

$$\gamma_1 = \frac{1}{N_0[\boldsymbol{H}^\dagger(\boldsymbol{H}^\dagger)^H]_{11}} = \frac{1}{N_0[(\boldsymbol{H}^H\boldsymbol{H})^{-1}]_{11}}. \tag{6}$$

For brevity, we will focus on subchannel 1 in the following. However, the extension to an arbitrary subchannel is straightforward and left to the reader. Partitioning **H** as $\boldsymbol{H} = \begin{bmatrix}\boldsymbol{h}_1 & \tilde{\boldsymbol{H}}\end{bmatrix}$, with column vector $\mathbf{h}_1$, and using (41) in the appendix, we can rewrite (6) as

$$\gamma_1 = \frac{1}{N_0}\boldsymbol{h}_1^H(\boldsymbol{I} - \tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H)\boldsymbol{h}_1. \tag{7}$$

However, from (38) and (39) we know that (using the same notation as in the appendix for partitioning the TX covariance matrix with r=1 according to the partitioning of **H**)

$$\tilde{\boldsymbol{H}} \sim \mathcal{N}_{M_{RX}, M_{TX-1}}(\mathbf{0}, \boldsymbol{R}_{RX} \otimes \boldsymbol{R}_{TX,22}^*). \tag{8}$$

and equivalently (with scalar $\boldsymbol{R}_{TX,11\cdot 2}^*$)

$$\boldsymbol{h}_1 \Big| \tilde{\boldsymbol{H}} \sim \mathcal{N}_{M_{RX}}\Big(\tilde{\boldsymbol{H}}(\boldsymbol{R}_{TX,22}^*)^{-1}\boldsymbol{R}_{TX,21}^*, \boldsymbol{R}_{TX,11\cdot 2}^* \cdot \boldsymbol{R}_{RX}\Big). \tag{9}$$

We emphasize that the covariance matrix of $\mathbf{h}_1$ is just a scaled version of the RX correlation matrix and with equality (41) we find that

$$\boldsymbol{R}_{TX,11\cdot 2}^* = \frac{1}{[(\boldsymbol{R}_{TX}^*)^{-1}]_{11}} = \frac{1}{[(\boldsymbol{R}_{TX})^{-1}]_{11}}. \tag{10}$$

For brevity, let $\boldsymbol{a} = (\boldsymbol{R}_{TX,22}^*)^{-1}\boldsymbol{R}_{TX,21}^*$ and introduce a new random vector $\boldsymbol{x} \sim \mathcal{N}_{M_{RX},1}(\mathbf{0}, \boldsymbol{R}_{TX,11\cdot 2}^* \cdot \boldsymbol{R}_{RX})$. Using the definitions above we find from (7) that

$$\gamma_1 \cong \frac{1}{N_0}(\boldsymbol{x} + \tilde{\boldsymbol{H}}\boldsymbol{a})^H(\boldsymbol{I} - \tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H)(\boldsymbol{x} + \tilde{\boldsymbol{H}}\boldsymbol{a}). \tag{11}$$

However, noting that

$$\begin{array}{c} \boldsymbol{a}^H\tilde{\boldsymbol{H}}^H(\boldsymbol{I} - \tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H) = 0 \\ (\boldsymbol{I} - \tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H)\tilde{\boldsymbol{H}}\boldsymbol{a} = 0 \end{array}, \tag{12}$$

and inserting this result in (11) we get

$$\gamma_1 \cong \frac{1}{N_0}\boldsymbol{x}^H(\boldsymbol{I} - \tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H)\boldsymbol{x}. \tag{13}$$

We now explicitly model $\tilde{\boldsymbol{H}}$ and **x** with (2), (8) and (10) as

$$\tilde{\boldsymbol{H}} = \boldsymbol{A}^H\tilde{\boldsymbol{H}}_w\boldsymbol{R}_{TX,22}^{1/2} \qquad \boldsymbol{x} = \frac{1}{\sqrt{[(\boldsymbol{R}_{TX})^{-1}]_{11}}}\boldsymbol{A}^H\boldsymbol{u}, \tag{14}$$

where $\tilde{\boldsymbol{H}}_w$ and **u** have unit variance i.i.d. complex Gaussian elements. Then we arrive at (note that $\boldsymbol{R}_{TX,22}^{1/2}$ is invertible) the quadratic form of random variables

$$\begin{aligned} \gamma_1 &\cong \alpha \cdot \boldsymbol{u}^H\boldsymbol{A}(\boldsymbol{I} - \boldsymbol{A}^H\tilde{\boldsymbol{H}}_w(\tilde{\boldsymbol{H}}_w^H\boldsymbol{A}\boldsymbol{A}^H\tilde{\boldsymbol{H}}_w)^{-1}\tilde{\boldsymbol{H}}_w^H\boldsymbol{A})\boldsymbol{A}^H\boldsymbol{u} \\ &\cong \alpha \cdot \boldsymbol{u}^H\boldsymbol{Q}\boldsymbol{u} \end{aligned}, \tag{15}$$

where we have introduced a scaling factor $\alpha = 1/(N_0[(\boldsymbol{R}_{TX})^{-1}]_{11})$ and the matrix **Q** as short-hand notation.

3.1. Transmit correlation only

This special case was considered in [8], where statistical results on complex Wishart matrices were applied to find the subchannel SNR. In this paper, we present an alternative approach. Without RX correlation, we get from (15) using **A**=**I**

$$\gamma_1 \cong \alpha \cdot \boldsymbol{u}^H(\boldsymbol{I} - \tilde{\boldsymbol{H}}_w(\tilde{\boldsymbol{H}}_w^H\tilde{\boldsymbol{H}}_w)^{-1}\tilde{\boldsymbol{H}}_w^H)\boldsymbol{u}\,. \tag{16}$$

However, it can easily be seen that $\tilde{\boldsymbol{H}}_w(\tilde{\boldsymbol{H}}_w^H\tilde{\boldsymbol{H}}_w)^{-1}\tilde{\boldsymbol{H}}_w^H$ is an idempotent matrix with $\mathrm{rk}(\tilde{\boldsymbol{H}}_w) = M_{TX} - 1$ eigenvalues of value 1 and all other eigenvalues of value 0. Thus, the number of eigenvalues of value one of $\boldsymbol{I} - \tilde{\boldsymbol{H}}_w(\tilde{\boldsymbol{H}}_w^H\tilde{\boldsymbol{H}}_w)^{-1}\tilde{\boldsymbol{H}}_w^H$ is $N = M_{RX} - M_{TX} + 1$. Obviously, γ_1 in (16) has a chi-squared distribution with N degrees of freedom, which can be explicitly written as

$$\mathrm{f}(\gamma_1) = \frac{\exp\left(-\frac{\gamma_1}{\alpha}\right)}{\alpha \cdot \Gamma(N)}\left(\frac{\gamma_1}{\alpha}\right)^{N-1}. \tag{17}$$

This result perfectly agrees with the conclusion given in [8], which is restricted to the case of TX correlation only. Now consider the conditional SER approximation for square M-QAM constellations

$$P_{e,c}(\gamma_1) = b \cdot \mathrm{erfc}(\sqrt{c\gamma_1}) \tag{18}$$

with the parameters

$$b = 2\left(1 - \frac{1}{\sqrt{M}}\right) \qquad c = \frac{3}{2(M-1)}. \tag{19}$$

Omitting details, by integrating over the subchannel SNR density function and using an AWGN approximation after ZF processing one gets

$$P_e = b\left(1 - \frac{2\sqrt{\alpha c} \cdot \Gamma(N+1/2)}{\sqrt{\pi} \cdot \Gamma(N)}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2} + N; \frac{3}{2}; -\alpha c\right)\right) \tag{20}$$

with the hypergeometric function ${}_2F_1$. For the special 2×2 case (20) reduces to ($\mathbf{R}_{TX}$ has ones on its diagonal)

$$P_e = b\left(1 - \sqrt{\frac{\alpha c}{1 + \alpha c}}\right) = b\left(1 - \sqrt{\frac{c\det(\boldsymbol{R}_{TX})}{N_0 + c\det(\boldsymbol{R}_{TX})}}\right). \tag{21}$$

Equation (21) nicely shows the dependency of the SER on the determinant (and indirectly the eigenvalues) of the transmit correlation matrix.

3.2. Transmit and receive correlation

In the general case of a fully correlated channel, the SNR in (15) has - conditioned on the eigenvalues of $\mathbf{Q}$ - a weighted chi-squared distribution. However, for arbitrary correlations and antenna array sizes, it is at least challenging if not impossible to find closed-form expressions of reasonable complexity for the eigenvalues and their joint probability density function, respectively. Therefore, we are restricting the analysis in this paper to the 2×2 MIMO case. Obviously, in this case the matrix $\tilde{\boldsymbol{H}}$ reduces to a column vector that we denote by $\boldsymbol{v} = \tilde{\boldsymbol{H}}$. Furthermore, the rank of the matrix $\mathbf{Q}$ reduces to 1, i.e. there is only one non-vanishing eigenvalue λ (we mention that this is true for all $\tilde{\boldsymbol{H}}$ of size M_{RX}×(M_{RX}-1), i.e. for systems with the same number of TX and RX antennas). Note that now λ is given by

$$\begin{aligned}\lambda &= \mathrm{tr}(\boldsymbol{Q}) = \mathrm{tr}(\boldsymbol{A}(\boldsymbol{I} - \boldsymbol{A}^H\boldsymbol{v}(\boldsymbol{v}^H\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{v})^{-1}\boldsymbol{v}^H\boldsymbol{A})\boldsymbol{A}^H) \\ &\cong \mathrm{tr}(\boldsymbol{D}) - \mathrm{tr}\left(\frac{\boldsymbol{v}^H\boldsymbol{D}^2\boldsymbol{v}}{\boldsymbol{v}^H\boldsymbol{D}\boldsymbol{v}}\right) = \mathrm{tr}(\boldsymbol{D}) - \frac{\boldsymbol{v}^H\boldsymbol{D}^2\boldsymbol{v}}{\boldsymbol{v}^H\boldsymbol{D}\boldsymbol{v}}\end{aligned}, \tag{22}$$

where we have introduced the diagonal matrix $\mathbf{D}$ of eigenvalues d_1 and d_2 of the eigenvalue decomposition

$$\boldsymbol{R}_{RX} = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^H = \boldsymbol{V}\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}\boldsymbol{V}^H. \tag{23}$$

We emphasize that - conditioned on λ – γ_1 has a simple exponential distribution, i.e. from (15) we find

$$\gamma_1 \,|\, \lambda \cong \alpha\lambda u\,, \tag{24}$$

where u is an exponentially distributed RV. In order to find the unconditional SER, we integrate the conditional SER (18) over γ_1, i.e.

$$P_e = E_{\gamma_1}[b \cdot \mathrm{erfc}(\sqrt{c\gamma_1})] = E_\lambda\Big[E_u[b \cdot \mathrm{erfc}(\sqrt{c\alpha\lambda u})]\Big]. \tag{25}$$

Using the integration result

$$\int_0^\infty \mathrm{erfc}(\sqrt{ay})\exp(-y)dy = 1 - \sqrt{\frac{a}{1+a}} \tag{26}$$

we find from (25)

$$P_e = b\left(1 - E_\lambda\left[\sqrt{\frac{c\alpha\lambda}{1 + c\alpha\lambda}}\right]\right). \tag{27}$$

Using (22) in (27) one arrives at

$$P_e = b\left(1 - E_\lambda\left[\sqrt{\frac{c\alpha\boldsymbol{v}^H(tr(\boldsymbol{D}) \cdot \boldsymbol{D} - \boldsymbol{D}^2)\boldsymbol{v}}{\boldsymbol{v}^H(\boldsymbol{D} + c\alpha(tr(\boldsymbol{D}) \cdot \boldsymbol{D} - \boldsymbol{D}^2))\boldsymbol{v}}}\right]\right), \tag{28}$$

where we now have to calculate the expectation of the square root of a ratio of quadratic forms of RVs. Note that the nominator has a special structure, as

$$tr(\boldsymbol{D}) \cdot \boldsymbol{D} - \boldsymbol{D}^2 = \det(\boldsymbol{R}_{RX}) \cdot \boldsymbol{I}\,. \tag{29}$$

Exploiting the property of the Gamma integral (see also [7])

$$z^{-\beta} = \frac{1}{\Gamma(\beta)}\int_0^\infty x^{\beta-1}e^{-zx}dx\,, \tag{30}$$

in order to simplify the square root in (28), one can find after lengthy calculations for exponentially distributed x and y with constants b and c

$$E_x\left[E_y\left[\sqrt{\frac{x+y}{bx+cy}}\right]\right] = \frac{2(\sqrt{c}-\sqrt{b})}{c-b}. \quad (31)$$

Omitting details, application of (31) to (28) and simplifying the result leads to the closed-form expression of the sub-channel SER

$$P_e = b\left(1 - \frac{2\sqrt{c\det(\boldsymbol{R}_{RX}\boldsymbol{R}_{TX})}}{\sqrt{N_0 d_1 + c\det(\boldsymbol{R}_{RX}\boldsymbol{R}_{TX})} + \sqrt{N_0 d_2 + c\det(\boldsymbol{R}_{RX}\boldsymbol{R}_{TX})}}\right). \quad (32)$$

Via a series expansion the asymptotics for high SNR

$$\bar{P}_e = \frac{bN_0}{4c} \cdot \frac{[(\boldsymbol{R}_{TX}^{*})^{-1}]_{11} \cdot \mathrm{trace}(\boldsymbol{R}_{RX})}{\det(\boldsymbol{R}_{RX})} = \frac{bN_0}{2c} \cdot \frac{1}{\det(\boldsymbol{R}_{RX}\boldsymbol{R}_{TX})} \quad (33)$$

can be derived. The reader can readily check that the asymptotic SER of a 2×2 system with RX correlation only and a system that experiences the same TX correlation are exactly the same. Moreover, we emphasize that (32) and (33) reduce to the known results in case of vanishing fading correlation with $\mathbf{R}_{RX}=\mathbf{R}_{TX}=\mathbf{I}$.

4. SIMULATION RESULTS

In the simulations of this paper, a new random channel matrix is determined via (2) for each channel use, with the long-term stable correlation matrices $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ (**A** and **B**, respectively) held constant. Both RX and TX arrays have an antenna element spacing of 0.5 wavelengths, assuming a uniform linear array (ULA) for both.

In the presence of fading correlation, we assume at transmit and receive side a mean direction of departure (DOD) and direction of arrival (DOA), respectively, of 20 degrees with respect to the array perpendicular and a root mean square angular spread (AS) characterized by a Laplacian power distribution. $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ are chosen according to these assumptions.

We study an AWGN (i.e. no colored interference) scenario with the SNR given by

$$SNR = 10 \cdot \log_{10}\left(\frac{M_{TX} \cdot E_b}{N_0}\right) \quad [dB], \quad (34)$$

where E_b is the energy per information bit. Throughout our simulations we normalize the total transmitted energy to $\rho=M_{TX}$ and assume QPSK modulation.

In Fig. 2 we depict the SER performance of a 2×2 MIMO system with ZF receiver in the presence of an uncorrelated channel, with transmit correlation corresponding to an AS of 10 degrees, and finally with an AS of 2 degrees. We note that in the special 2×2 case the SER on both subchannels of the MIMO system agree.

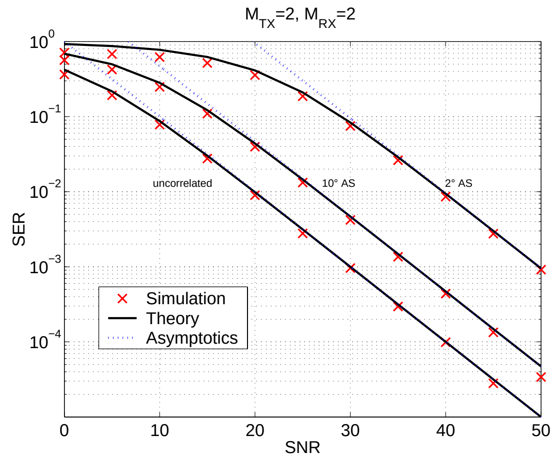

Fig. 2: Transmit correlation (2×2)

The close agreement between theoretical results (cf. (20) and (21), respectively) and simulation are obvious. As expected, there is a slight mismatch in the low SNR region due to the SER approximation in (18). Moreover, it can be seen that the curves quickly converge with the high SNR asymptotics from (33).

Simulation results are plotted in Fig. 3 for a system with RX correlation only and TX correlation only, whereas the correlation matrices are chosen to be the same in both cases.

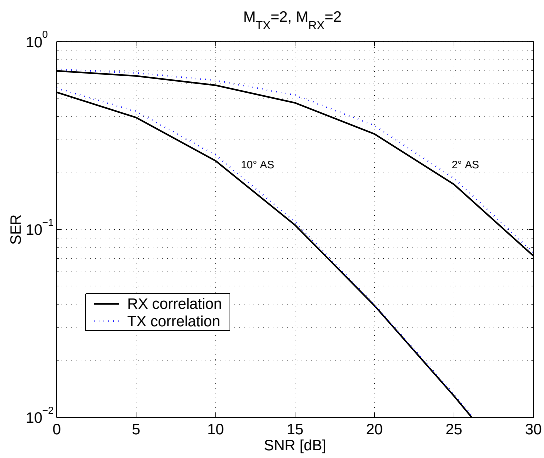

Fig. 3: Comparison of RX and TX correlation (2×2)

It can be seen that the curves differ in the low SNR region due to the differing SNR statistics in case of RX and TX correlation, respectively. The theoretical results in (33) are confirmed by the simulations - the asymptotic performance in the high SNR region agrees in both cases.

For the case of both receive and transmit correlation (Fig. 4) we again can observe a close match between the theoretical analysis and Monte-Carlo simulations. As expected, the performance of the linear receiver is inacceptably degraded in the fully correlated channel.

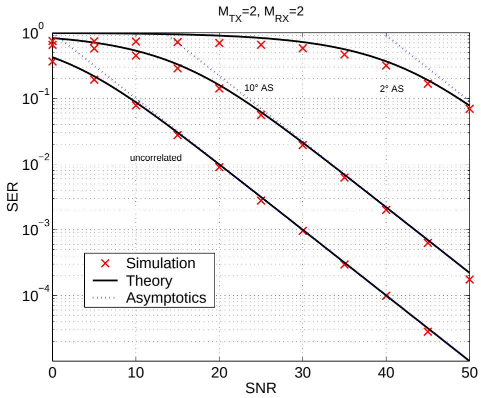

Fig. 4: Fully correlated channel (2×2)

5. CONCLUSION

Based on results on complex matrix variate normal statistics we have presented closed-form expressions for the SER of a MIMO ZF receiver in a correlated Rayleigh fading environment. For the fully correlated channel, the analysis is restricted to the simpler yet in practice important 2×2 case due to the considerable mathematical complexity of the problem. Simulations have shown the closeness of the theoretical analysis. The results can serve as a basis for the design of flexible signal processing algorithms that can adapt to the prevailing fading conditions. Further research is necessary to find tractable solutions for arbitrary array sizes.

6. APPENDIX

Let the vector-valued complex normal distribution with m elements, covariance matrix $\boldsymbol{\Sigma}$, and mean $\boldsymbol{\mu}$ be denoted by $\mathcal{N}_m(\mu, \Sigma)$. A m×n matrix $\mathbf{X}$ is said to have a matrix variate complex normal distribution [6], denoted by $\mathcal{N}_{m,n}(\mathbf{M}, \Sigma\otimes\Psi)$, with mean $\mathbf{M}$ and covariance $\Sigma\otimes\Psi$, if

$$vec(\boldsymbol{X}^T) \sim \mathcal{N}_{mn}(vec((\boldsymbol{M}^T), \boldsymbol{\Sigma}\otimes\boldsymbol{\Psi})) . \qquad (35)$$

Now introduce the following partitions

$$\boldsymbol{X} = \begin{bmatrix}\boldsymbol{X}_1 & \boldsymbol{X}_2\end{bmatrix} \quad \boldsymbol{M} = \begin{bmatrix}\boldsymbol{M}_1 & \boldsymbol{M}_2\end{bmatrix} \quad \boldsymbol{\Psi} = \begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} \\ \boldsymbol{\Psi}_{21} & \boldsymbol{\Psi}_{22}\end{bmatrix} . \qquad (36)$$

with $\mathbf{X}_1$ of size m×r, $\mathbf{X}_2$ of size m×(n-r), equivalent partitions for $\mathbf{M}$, $\boldsymbol{\Psi}_{11}$ of size r×r, $\boldsymbol{\Psi}_{22}$ of size (n-r)×(n-r), $\boldsymbol{\Psi}_{12}$ of size r×(n-r), $\boldsymbol{\Psi}_{21}$ of size (n-r)×r, and the definition

$$\boldsymbol{\Psi}_{11\cdot 2} = \boldsymbol{\Psi}_{11} - \boldsymbol{\Psi}_{12}\boldsymbol{\Psi}_{22}^{-1}\boldsymbol{\Psi}_{21} . \qquad (37)$$

It can then be shown that

$$\boldsymbol{X}_2 \sim \mathcal{N}_{m,\,n-r}(\boldsymbol{M}_2, \boldsymbol{\Sigma}\otimes\boldsymbol{\Psi}_{22}) . \qquad (38)$$

and conditioned on $\mathbf{X}_2$, the distribution of $\mathbf{X}_1$ is given by

$$\boldsymbol{X}_1 \big| \boldsymbol{X}_2 \sim \mathcal{N}_{m,\,r}(\boldsymbol{M}_1 + (\boldsymbol{X}_2 - \boldsymbol{M}_2)\boldsymbol{\Psi}_{22}^{-1}\boldsymbol{\Psi}_{21}, \boldsymbol{\Sigma}\otimes\boldsymbol{\Psi}_{11\cdot 2}) . \qquad (39)$$

Partition an arbitrary matrix $\mathbf{A}$ and its inverse $\mathbf{A}^{-1}$ as

$$\boldsymbol{A} = \begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix} \qquad \boldsymbol{A}^{-1} = \begin{bmatrix}\boldsymbol{A}^{11} & \boldsymbol{A}^{12} \\ \boldsymbol{A}^{21} & \boldsymbol{A}^{22}\end{bmatrix} . \qquad (40)$$

It is then well-known [9] that

$$\boldsymbol{A}^{11} = (\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21})^{-1} = (\boldsymbol{A}_{11\cdot 2})^{-1} . \qquad (41)$$

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