

Statistical Transmit Processing for Enhanced MIMO Channel Estimation in Presence of Correlation

Mario Kiessling^{1,2} and Joachim Speidel¹

¹Institute of Telecommunications, University of Stuttgart, Germany
²Siemens AG, Information and Communication Mobile, Ulm, Germany
 mario.kiessling@gmx.net, speidel@inue.uni-stuttgart.de

Abstract—A novel pilot aided channel estimation scheme is considered for wireless MIMO systems in presence of fading correlation. Assuming a linear minimum mean squared error based channel estimator, it is demonstrated that adequate statistical shaping of the training sequence depending on the correlation properties of the channel can minimize the estimation mean squared error. The new scheme is a general concept that can also readily be applied to smart antenna multiple-input single-output systems. With long-term update intervals of the correlation properties of the channel, the complexity of the concept is moderate and allows for an implementation in FDD systems. Simulation results for various MIMO transmission systems including realistic channel estimation demonstrate the effectiveness of the proposed scheme with a significant SNR gain.

I. INTRODUCTION

The assumption of a rich scattering environment leading to uncorrelated fading between the antenna elements of a wireless MIMO system has to be abandoned in typical cellular scenarios with an exposed antenna array at the base station and quasi-line-of-sight propagation conditions [1][2]. If the system is not properly designed to adapt to this situation, fading correlation can have a catastrophic impact on performance. This is especially true for linear spatial signal processing at the receiver [10].

However, knowledge of the correlation properties of the channel (e.g. in form of the correlation matrices) at the transmitter can beneficially be exploited for improving transmission quality. While on one hand this knowledge can be used to improve data transmission (e.g. [11]), the focus of this paper is on channel estimation aspects. Increasing array sizes require a larger overhead for channel estimation purposes, thus limiting the overall spectral efficiency [3]. Advanced channel estimation (CE) schemes are therefore a critical part of coherently modulated MIMO systems [4][5].

Assuming a linear minimum mean squared error (MMSE) channel estimator [6] at the receiver and orthogonal training sequences, we derive the optimum linear transmit prefilter for the training block that minimizes the mean squared error (MSE) of the channel estimator. Alternatively, the statistical prefiltering scheme can also be considered as optimal training sequence design for correlated MIMO channels. Our derivations are based on a standard flat fading channel model,

whereas a generalization to arbitrary channel models is straightforward.

Monte-Carlo bit error rate (BER) simulations comprising CE effects for various signal processing approaches, e.g. transmission on the maximum eigenmode of the MIMO channel, demonstrate the effectiveness of the proposed scheme.

II. SIGNAL AND CHANNEL MODEL

In the remainder of the paper, bold lowercase letters denote column vectors, bold uppercase letters describe matrices, by \mathbf{I} we denote an identity matrix, $[\mathbf{X}]_{i,j}$ is the element in row i and column j of matrix \mathbf{X} , $\text{vec}(\mathbf{X})$ stacks the columns of matrix \mathbf{X} in a column vector, $\text{diag}(\mathbf{x})=\text{diag}(\mathbf{x}^T)$ returns a diagonal matrix with the elements of \mathbf{x} on the diagonal, \otimes is the Kronecker product, \mathbf{X}^* means complex conjugate, \mathbf{X}^T means transpose, and \mathbf{X}^H means Hermitian (conjugate transpose).

We consider the transmission of a training sequence over a flat fading MIMO link in Fig.1 with noise whitening at the receiver modeled by

$$\mathbf{Y} = \mathbf{R}_{nn}^{-1/2} \mathbf{H} \mathbf{F} \mathbf{S} + \mathbf{R}_{nn}^{-1/2} \tilde{\mathbf{N}} = \mathbf{R}_{nn}^{-1/2} \mathbf{H} \mathbf{F} \mathbf{S} + \mathbf{N}, \quad (1)$$

where \mathbf{S} is a $M_{\text{TX}} \times N_t$ training sequence of length N_t . M_{TX} is the number of TX antennas. We assume orthogonal training sequences, such that \mathbf{S} is a matrix fulfilling

$$\mathbf{S} \mathbf{S}^H = \mathbf{S}^H \mathbf{S} = N_t \cdot \mathbf{I}. \quad (2)$$

For example, a possible choice for a training sequence fulfilling (2) could be a standard DFT matrix with elements

$$[\mathbf{S}]_{k,l} = e^{-j2\pi \frac{(k-1)(l-1)}{N_t}} \quad \left\{ \begin{array}{l} 1 \leq k \leq M_{\text{TX}} \\ 1 \leq l \leq N_t \end{array} \right. \quad (3)$$

\mathbf{F} is a $M_{\text{TX}} \times M_{\text{TX}}$ linear matrix transmit prefilter. We mention that the product $\mathbf{F} \mathbf{S}$ could also be interpreted as a new training sequence $\tilde{\mathbf{S}}$, however, due to the invertibility of \mathbf{S} , both formulations are mathematically equivalent.

\mathbf{H} is the $M_{\text{RX}} \times M_{\text{TX}}$ MIMO channel matrix with correlated Rayleigh fading elements, $\tilde{\mathbf{N}}$ is the $M_{\text{RX}} \times N_t$ noise matrix before noise whitening, \mathbf{N} is the $M_{\text{RX}} \times N_t$ noise matrix after noise whitening, and \mathbf{Y} is the noisy $M_{\text{RX}} \times N_t$ receive sequence (see Fig.1). By M_{RX} we denote the number of RX antennas. Furthermore, if we decompose $\tilde{\mathbf{N}}$ into its column vectors

$$\tilde{\mathbf{N}} = [\tilde{\mathbf{n}}_1 \dots \tilde{\mathbf{n}}_{N_t}], \quad (4)$$

the covariance matrix of the column vectors is

$$\mathbb{E}[\tilde{\mathbf{n}}_i \tilde{\mathbf{n}}_i^H] = \mathbf{R}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}}} \quad , \quad 1 \leq i \leq N_t. \quad (5)$$

It is then obvious that the matrix \mathbf{N} in (1) models white Gaussian noise with identity covariance matrix. By appropriate processing of the received training sequence \mathbf{Y} , the MIMO receiver is capable of producing a channel estimate $\hat{\mathbf{H}}$ in Fig.1.

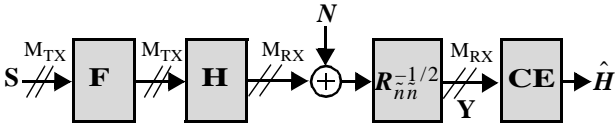


Fig. 1: System model

Using a widely accepted simplified channel model (see e.g. [1]), the correlated MIMO channel can be described by the matrix product

$$\mathbf{H} = \mathbf{A}^H \mathbf{H}_w \mathbf{B}, \quad (6)$$

where \mathbf{H}_w is a $M_{RX} \times M_{TX}$ matrix of complex i.i.d. Gaussian variables of unity variance and

$$\mathbf{A} \mathbf{A}^H = \mathbf{R}_{RX} \quad \mathbf{B} \mathbf{B}^H = \mathbf{R}_{TX}, \quad (7)$$

where \mathbf{R}_{RX} and \mathbf{R}_{TX} is the long-term stable (normalized) receive and transmit correlation matrix, respectively.

III. LINEAR MMSE CHANNEL ESTIMATION

In order to derive the MMSE MIMO channel estimator, we rewrite (1) in vector form to apply standard results from estimation theory

$$\underbrace{\text{vec}(\mathbf{Y})}_{\mathbf{y}} = \underbrace{((\mathbf{F}\mathbf{S})^T \otimes \mathbf{R}_{nn}^{-1/2})}_{\mathbf{X}} \cdot \underbrace{\text{vec}(\mathbf{H})}_{\mathbf{h}} + \underbrace{\text{vec}(\mathbf{N})}_{\mathbf{n}}, \quad (8)$$

where we have used [7]

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \cdot \text{vec}(\mathbf{B}). \quad (9)$$

Denoting the covariances of \mathbf{h} and \mathbf{n} by \mathbf{R}_{hh} and \mathbf{R}_{nn} , respectively, the linear MMSE estimator of \mathbf{h} is given by the well-known equation [8]

$$\hat{\mathbf{h}} = (\mathbf{R}_{hh}^{-1} + \mathbf{X}^H \mathbf{R}_{nn}^{-1} \mathbf{X})^{-1} \mathbf{X}^H \mathbf{R}_{nn}^{-1} \mathbf{y}. \quad (10)$$

Obviously, the receiver needs to know the channel covariance \mathbf{R}_{hh} and \mathbf{X} , which is also a function of \mathbf{R}_{hh} (see below), in the estimation process. The noise vector is white Gaussian with $\mathbf{R}_{nn} = \mathbf{I}$ and with the channel model (6), we find via application of (9)

$$\mathbf{R}_{hh} = \mathbf{R}_{TX}^* \otimes \mathbf{R}_{RX} \quad (11)$$

the typical Kronecker correlation structure of the given channel model. The covariance matrix of the resulting estimation error vector $\hat{\mathbf{h}} - \mathbf{h} = \mathbf{e}$ is then [8]

$$\boldsymbol{\varsigma} = \mathbb{E}[\mathbf{e}\mathbf{e}^H] = (\mathbf{R}_{hh}^{-1} + \mathbf{X}^H \mathbf{R}_{nn}^{-1} \mathbf{X})^{-1}. \quad (12)$$

It is already clear from (12) that the channel covariance \mathbf{R}_{hh} dominates at low SNR, while the second term dominates in the high SNR region. We can therefore expect that statistical pre-filtering based on the correlation properties of the channel given in \mathbf{R}_{hh} is only effective at lower SNR. Simulation results will confirm this perception.

Plugging the definitions from (8) in (12) results in

$$\begin{aligned} \boldsymbol{\varsigma} &= ((\mathbf{R}_{TX}^*)^{-1} \otimes \mathbf{R}_{RX}^{-1} + ((\mathbf{F}\mathbf{S})^T \otimes \mathbf{R}_{nn}^{-1/2})^H ((\mathbf{F}\mathbf{S})^T \otimes \mathbf{R}_{nn}^{-1/2}))^{-1} \\ &= ((\mathbf{R}_{TX}^*)^{-1} \otimes \mathbf{R}_{RX}^{-1} + N_t (\mathbf{F}^* \mathbf{F}^T \otimes \mathbf{R}_{nn}^{-1}))^{-1} \end{aligned} \quad (13)$$

where we have used (2) and

$$\begin{aligned} (\mathbf{A} \otimes \mathbf{B}) \cdot (\mathbf{C} \otimes \mathbf{D}) &= \mathbf{AB} \otimes \mathbf{CD} \\ (\mathbf{A} \otimes \mathbf{B})^{-1} &= \mathbf{A}^{-1} \otimes \mathbf{B}^{-1} \end{aligned} \quad (14)$$

We deploy the overall MSE ε as a measure of the quality of the MIMO channel estimator, namely from (13)

$$\varepsilon = \text{tr}((\mathbf{R}_{TX}^*)^{-1} \otimes \mathbf{R}_{RX}^{-1} + N_t (\mathbf{F}^* \mathbf{F}^T \otimes \mathbf{R}_{nn}^{-1}))^{-1}. \quad (15)$$

Assuming that the transmitter has information on the long term stable channel correlation properties given in \mathbf{R}_{TX} and \mathbf{R}_{RX} , it is obvious from (15) that by appropriately designing the linear prefilter \mathbf{F} , the overall MSE ε of the channel estimator may be minimized. Due to the space limitation (see [12] for extensions), we are restricting the following analysis to the case of additive white Gaussian noise (AWGN), i.e. $\mathbf{R}_{nn} = N_0 \cdot \mathbf{I}$, resulting in

$$\varepsilon = \text{tr}\left((\mathbf{R}_{TX}^*)^{-1} \otimes \mathbf{R}_{RX}^{-1} + \frac{N_t}{N_0} (\mathbf{F}^* \mathbf{F}^T \otimes \mathbf{I})\right)^{-1}. \quad (16)$$

Now, based on (16), the optimum prefilter \mathbf{F} may be designed for various propagation scenarios. After introducing the eigenvalue decompositions (EVD) with matrices Λ_{RX} and Λ_{TX} , which contain the sorted (descending) eigenvalues

$$\mathbf{R}_{RX} = \mathbf{V}_{RX} \Lambda_{RX} \mathbf{V}_{RX}^H \quad \mathbf{R}_{TX}^* = \mathbf{V}_{TX} \Lambda_{TX} \mathbf{V}_{TX}^H, \quad (17)$$

we find from (16) using (14)

$$\varepsilon = \text{tr}\left(\Lambda_{TX}^{-1} \otimes \Lambda_{RX}^{-1} + \frac{N_t}{N_0} (\mathbf{V}_{TX}^H \mathbf{F}^* \mathbf{F}^T \mathbf{V}_{TX} \otimes \mathbf{I})\right)^{-1}. \quad (18)$$

Then setting without loss of generality

$$\mathbf{F}^* = \mathbf{V}_{TX} \Phi_f \quad (19)$$

with diagonal Φ_f (see [12] for a proof of the diagonality) and diagonal elements $\phi_{f,l}$ we get from (18)

$$\varepsilon = \text{tr}\left(\Lambda_{TX}^{-1} \otimes \Lambda_{RX}^{-1} + \frac{N_t}{N_0} (\Phi_f \Phi_f^H \otimes \mathbf{I})\right)^{-1}. \quad (20)$$

We use the term long-term eigenmode transmission for the special prefilter-design in (19), due to the fact that we essentially apply beamforming for each single-dimensional training sequence along the eigenvectors of the transmit correlation matrix with power allocation via the matrix elements $\phi_{f,l}$.

A. Both Receive and Transmit Correlation

Minimization of (20) via matrix filter \mathbf{F} (Φ_f , respectively) under a power constraint ρ results in the optimization problem

$$\underbrace{\min}_{\Phi_f} \text{tr} \left(\Lambda_{TX}^{-1} \otimes \Lambda_{RX}^{-1} + \frac{N_t}{N_0} (\Phi_f \Phi_f^H \otimes \mathbf{I}) \right)^{-1}, \quad (21)$$

s.t. $\text{tr}(\Phi_f \Phi_f^H) = \rho$

where 's.t.' stands for 'subject to'. However, the argument of the trace operator in (21) is diagonal, such that with $\Phi_f = \text{diag}(\phi_{f,1}, \phi_{f,2}, \dots, \phi_{f,M_{TX}})$ and alike definitions for the elements of Λ_{TX} and Λ_{RX} we get after simple transformations an equivalent diagonalized, i.e. scalar problem.

We emphasize that the diagonalization due to the prefilter structure also simplifies the estimation process in the receiver according to (10), where only diagonal matrices have to be inverted (for further details see [12]).

Using the Lagrange method for constrained optimization problems with Lagrange multiplier μ , we find the necessary condition

$$-\lambda_{TX,l}^2 \cdot \frac{N_t}{N_0} \cdot \sum_{k=1}^{M_{RX}} \frac{\lambda_{RX,k}^2}{1 + \frac{N_t}{N_0} \cdot \lambda_{TX,l} \lambda_{RX,k} \phi_{f,l}^2} + \mu = 0. \quad (22)$$

for all $1 \leq l \leq M_{TX}$. A general closed-form solution of (22) can not be given for arbitrary choices of Λ_{TX} and Λ_{RX} . In this case, numerical optimization techniques may be deployed.

However, we mention that the optimization problem can be solved in the low and high SNR region, which is outlined in [12]. Summarizing the results, for high SNR the optimum prefilter turns out to be a (scaled) identity matrix, i.e. there is essentially no prefilter at high SNR. This agrees with the statement above on the MSE in (12). On the other hand, at lower SNR the optimum prefilter pours all available power on the strongest long-term eigenmode of the channel, such that standard training sequences are clearly suboptimum in this SNR region.

B. Transmit Correlation only

We further specialize the problem by assuming a semi-correlated channel with transmit correlation only. This is a typical downlink scenario with an exposed base station transmit antenna array and a mobile station surrounded by a huge number of scatterers. Now $\Lambda_{RX} = \mathbf{I}$ and (22) reads

$$-\lambda_{TX,l}^2 \cdot \frac{N_t}{N_0} \cdot \frac{M_{RX}}{\left(1 + \frac{N_t}{N_0} \cdot \lambda_{TX,l} \phi_{f,l}^2\right)^2} + \mu = 0. \quad (23)$$

Solving for $\phi_{f,l}$ and rewriting in matrix notation leads to

$$\Phi_f = \left[\frac{1}{M_{TX}} \left(\left(\frac{N_t}{N_0} \right)^{-1} \cdot \text{tr}(\Lambda_{TX}^{-1}) + \rho \right) \cdot \mathbf{I} - \left(\frac{N_t}{N_0} \right)^{-1} \Lambda_{TX}^{-1} \right]_+^{1/2}, \quad (24)$$

where we implicitly have eliminated the Lagrange multiplier via the power constraint. The '+' sign in (24) indicates that all $\phi_{f,l}$ have to be greater or equal to 0. This can be assured in an iterative procedure, where the weakest eigenmodes are consecutively switched off by setting the corresponding $\phi_{f,l}$ to 0. The switching points are studied in [12].

C. Receive Correlation only

On the other hand, if the fading at the transmit antenna array is completely uncorrelated, i.e. $\Lambda_{TX} = \mathbf{I}$, and there is only correlation present at the receiver, (22) reads again for all $1 \leq l \leq M_{TX}$

$$\frac{N_t}{N_0} \cdot \sum_{k=1}^{M_{RX}} \frac{\lambda_{RX,k}^2}{1 + \frac{N_t}{N_0} \cdot \lambda_{RX,k} \phi_{f,l}^2} + \mu = 0. \quad (25)$$

This can only be fulfilled if all $\phi_{f,l}$ agree in size, i.e. for this special case we find in matrix notation $\Phi_f = \rho / M_{TX} \cdot \mathbf{I}$, implying that the training sequence is left unchanged. This result agrees with intuition. When there is no transmit correlation present, there are no prominent directions and the transmitter equally distributes power.

IV. CHANNEL ESTIMATION MEAN SQUARED ERROR

We study the effects of statistically shaping the training sequence according to the correlation properties of the channel with a prefilter \mathbf{F} designed according to (19) and matrix Φ_f according to (24), i.e. we are focusing on the case of TX correlation only. To this end, we investigate the overall MSE of the channel estimator given in (18) or (20), respectively.

In the simulations of this paper, a new random channel matrix is determined via (6) for each training sequence and subsequent data transmission, with the long-term stable correlation matrices \mathbf{R}_{TX} and $\mathbf{R}_{RX} = \mathbf{I}$ held constant for the total link level simulation. Again, we emphasize that they are both assumed to be known to RX as well as TX. The transmission of the training sequence is then modelled with (1). Via (10) the channel is estimated, whereas we assume that the channel is constant during the transmission of the training sequence. Both RX and TX arrays have an antenna element spacing of 0.5 wavelengths, assuming a uniform linear array (ULA) for both.

In the presence of fading correlation, we assume at the transmit side a mean direction of departure (DOD) of 20 degrees with respect to the array perpendicular and a root mean square angular spread (AS) characterized by a Laplacian power distribution. \mathbf{R}_{TX} is chosen according to these assumptions.

We study an AWGN (i.e. no colored interference) scenario with the SNR given by

$$\text{SNR} = 10 \cdot \log_{10} \left(\frac{M_{TX} \cdot E_b}{N_0} \right) \quad [dB], \quad (26)$$

where E_b is the energy per information bit. Throughout our simulations we normalize the total transmitted energy per channel use to $\rho=M_{TX}$ and assume QPSK modulation.

In Fig.2 we have plotted the resulting MSE of the channel estimator for a 4×4 system with TX correlation according to an AS of 2 degrees (strong correlation in order to highlight the effects) with and without prefiltering of a DFT training sequence, whereas the resulting transmit correlation matrix reads for the given propagation parameters

$$\text{abs}(\mathbf{R}_{TX}) \approx \begin{bmatrix} 1 & 0.99 & 0.98 & 0.95 \\ 0.99 & 1 & 0.99 & 0.98 \\ 0.98 & 0.99 & 1 & 0.99 \\ 0.95 & 0.98 & 0.99 & 1 \end{bmatrix}. \quad (27)$$

With a length $N_t=8$ training sequence one can observe a significant reduction of the MSE with prefiltering over a wide SNR range with a maximum in the range of -10 to 0 dB. As expected, a shorter training sequence leads to a shift to the right.

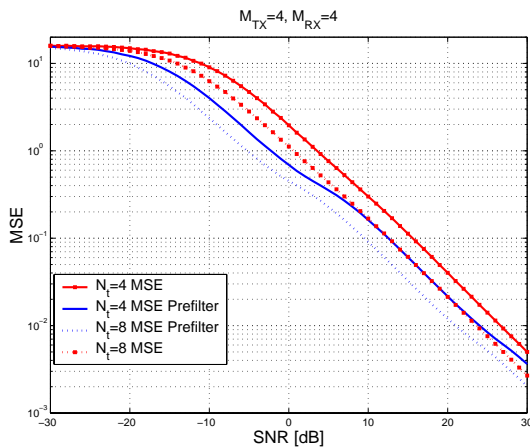


Fig. 2: MSE with TX AS 2° , $M_{TX}=4$, $M_{RX}=4$

If the channel is less correlated with an AS of 10 degrees at the TX in Fig.3, as expected, the SNR range with improved channel estimator MSE is reduced. Moreover, the improvement is less pronounced.

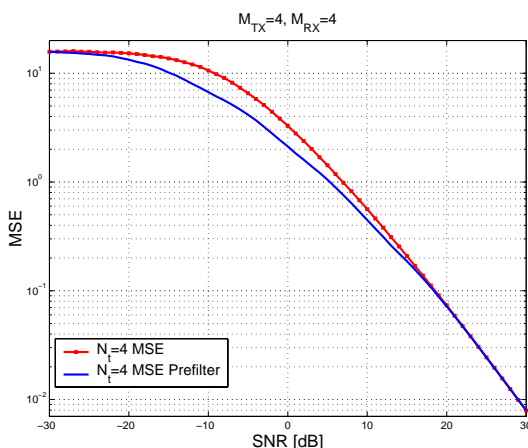


Fig. 3: MSE with TX AS 10° , $M_{TX}=4$, $M_{RX}=4$, $N_t=4$

The insights gained from the MSE simulations above are the key for effectively deploying the proposed CE scheme. Obviously, it can only improve the system performance, if the operating point of the system agrees with the range of MSE improvements (depicted for special cases in Fig.2 and Fig.3).

V. BER SIMULATIONS

We consider the application of the proposed CE scheme to various uncoded MIMO systems. As was mentioned above, the channel is assumed to be perfectly interleaved, i.e. it has no memory and consecutive instantiations are independent. We mention that in this case, CE can not be improved by appropriate filtering in time.

A. Maximum Eigenmode Transmission

With a low mobility mobile station and time division duplex (TDD) transmission, we can approximately assume reciprocity of the wireless channel. In this case, receiver as well as transmitter are aware of the instantaneous channel state. Under those conditions, we can deploy maximum eigenmode transmission in the MIMO system, which is especially suited for strongly correlated channels. Omitting details, transmitter as well as receiver essentially apply the eigenvector corresponding to the maximum eigenvalue of the equivalent channel (resulting from the combination of channel, noise whitening and matched filtering) as beamforming vectors for the transmission of a single data stream. Maximum eigenmode transmission can thus exploit the full beamforming gain of both transmit and receive antenna array.

In order to separate the effects of CE at RX and TX, we presume ideal CE at the TX, while RX CE is based on the novel CE scheme. In Fig.4 we have plotted BER simulation results for a 4×4 system with QPSK modulation. The performance improvement due to enhanced CE is obvious in this strongly semi-correlated scenario with an AS of 2 degrees at the TX. A gain of 1.0 to 1.7 dB in SNR can be observed in the given range for $N_t=8$, for $N_t=16$ the gain is 0.7 to 1 dB.

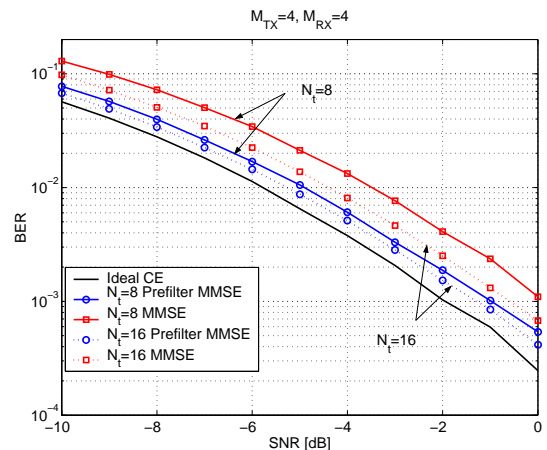


Fig. 4: BER of maximum eigenmode transmission with TX angular spread 2° , $M_{TX}=4$, $M_{RX}=4$

B. Maximum Likelihood Receiver

We apply the novel scheme to another system with non-adaptive transmitter and maximum likelihood (ML) receiver [13]. In general it can be stated that the ML receiver is very robust with respect to CE errors. However, one can still observe an improvement when deploying the novel concept (Fig.5). With a training sequence of minimum length $N_t=4$ the gain is in the range of 0.4 to 0.8 dB. A longer training sequence $N_t=8$ reduces the gain to 0.1 to 0.4 dB for the given SNR span.

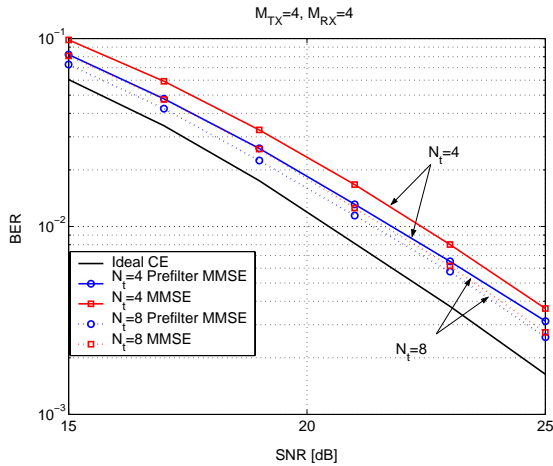


Fig. 5: BER of ML detection with TX angular spread 2° , $M_{TX}=4$, $M_{RX}=4$

C. Alamouti Scheme

In case of the Alamouti scheme [9], the full diversity of the MIMO channel can be exploited without the need for TX channel state information. For CE at the RX, we deploy the new scheme. Simulation results for a 2×2 system are given in Fig.6, again for a semi-correlated channel with an AS of 2 degrees at the TX.

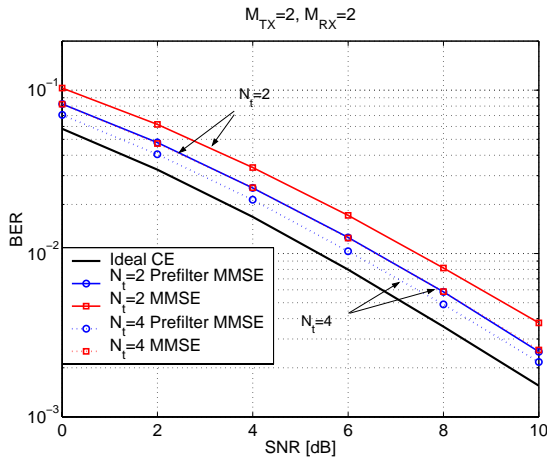


Fig. 6: BER of Alamouti scheme with TX AS 2° , $M_{TX}=2$, $M_{RX}=2$

While the Alamouti scheme is again very robust in the presence of CE errors, a small gain of 0.4 to 0.8 dB with $N_t=2$ and 0.1 to 0.4 dB with $N_t=4$, respectively, can be seen. Note that

the curves for $N_t=2$ with prefiltering and $N_t=4$ without prefiltering coincide and can not be differentiated in the figure.

VI. CONCLUSION

We have introduced an optimized channel estimation concept for correlated MIMO channels that adapts the training sequence to the prevailing correlation properties of the channel. Its application was demonstrated with standard MIMO signal processing algorithms, exhibiting a significant gain compared to standard MMSE channel estimation. Particularly, the improvements emerge at lower SNR, where channel estimation is a critical system aspect. The adaptive transmit processing of the training sequence is based on long-term stable statistical channel state information only. Therefore, the concept can readily be applied to frequency division duplex systems.

The investigation of system aspects and estimation of channels with frequency selective fading as well as memory are topics of future study.

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