Viterbi Decoding of a Convolutionally Encoded OCDMA System

Robert Fritsch
Institut für Nachrichtenübertragung, Pfaffenwaldring 47
Universität Stuttgart, +49 / (0)711 / 685-8016, -7929
fritsch@inue.uni-stuttgart.de

Abstract

I present a novel optical code division multiple access system (OCDMA), where the user's signature code is given by a convolutional code. Thus CDMA user separation and bit error correction coding are combined by this method and then mapped to optical quadrature QAM (OQ^2AM) symbols. A multiuser Viterbi Decoder is introduced, which operates on the samples of the electrical field strength detected by an OQ^2AM demodulation. The trellis structure of the whole system can be derived from the trellis structures of each user. The increase of complexity for increasing number of users and number of memories of the coders is analyzed. Furthermore the system performance is compared to synchronous OCDMA with Walsh-Hadamard codes. As a result the proposed scheme performs better than classical OCDMA, however the complexity of the decoder is a great challenge.

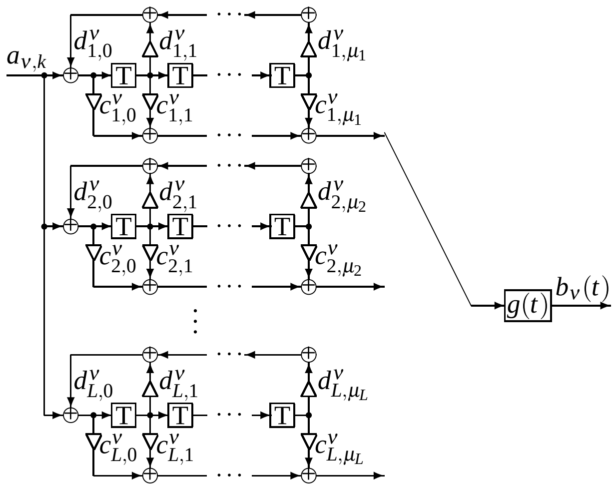

Figure 2. Principle encoder structure tx ν

1. Introduction

For upstream communications in a Passive Optical Network (PON) in Fig. 1 the access can be done by Time Division Multiple Access (TDMA), Code Division Multiple Access (CDMA) or Wavelength Division Multiple Access (WDMA). In case of CDMA normally block codes are taken for encoding [1, 2, 3]. In this paper I investigate the use of recursive convolutional codes. Thus there are no codewords anymore and the decoding can be done with the Viterbi algorithm. Differing from Fig. 1 decoding will not be done for each user separately. Instead a multiuser decoder which takes all users together and uses the Viterbi algorithm is applied.

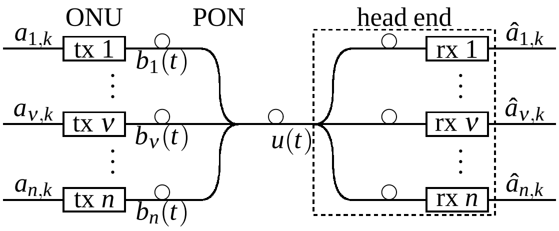

Figure 1. Principle communication system (tx transmitter, rx receiver)

2. Transmitter Structure

In Fig. 2 the principle structure of each transmitter tx ν is depicted. Each of the L branches of the encoder is a recursive convolutional structure. L is the spreading factor and

$$r = \frac{1}{L} \tag{1}$$

the code rate of the encoder. The coefficients

$$\begin{gathered} d_{i,j}^{\nu} \in \{0,1\}; \quad c_{i,j}^{\nu} \in \{0,1\}; \quad d_{i,0} = 1 \\ j = 0,1,\ldots,\mu_i; \quad i = 1,2,\ldots,L; \quad \nu = 1,2,\ldots,n \end{gathered} \tag{2}$$

define the particular code. $\oplus$ denotes addition modulo 2 and $g(t)$ is the impulse response of the impulse shaping filter including electro optical conversion.

Viterbi Decoding of a Convollutionally Encoded OCDMA System,
5. ITG-Fachtagung Photonische Netze, Fachhochschule der Deutschen Telekom in Leipzig, Mai 2004

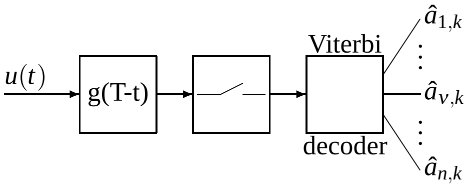

Figure 3. Principle structure of the multiuser receiver (head end)

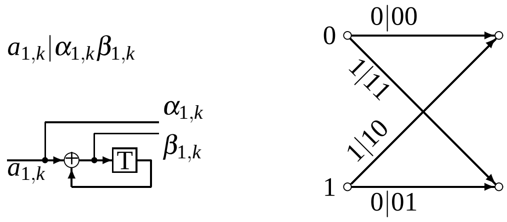

Figure 4. Example for an encoder of user 1

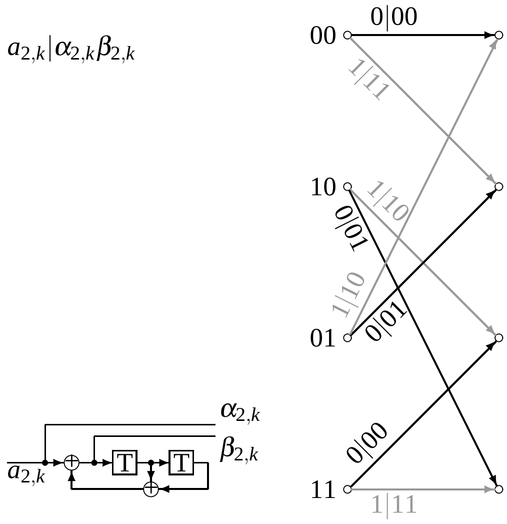

Figure 5. Example for an encoder of user 2

For the first branch

$$d^1_{1,j} = 0; \qquad j = 1,2,\ldots,\mu$$
$$c^1_{1,j} = \begin{cases} 1 & j = 0 \\ 0 & j = 1,2,\ldots,\mu \end{cases} \tag{3}$$

is defined, generating a systematic recursive convolutional code. In general the code given by Fig. 2 has a maximum of

$$2^{M_v}; \quad M_v = \sum_{i=1}^{L} \mu_i \tag{4}$$

number of states. M_v is the number of memories.

In the following, we focus on systematic codes with

$$d^v_j = d^v_{i,j}; \quad i = 1,2,\ldots,L; \quad j = 0,1,\ldots,\mu. \tag{5}$$

Thus all recursive parts in Fig. 2 are equal, except for the first systematic one. In this case the number of states will reduce to maximum

$$2^{\mu} \tag{6}$$

with $\mu = \mu_2 = \mu_3 = \ldots = \mu_L$. Hence the maximal effective number of memories is

$$M_v = \mu. \tag{7}$$

3. Receiver Structure

At receiver side the information bits of all n users have to be detected by one decoder. The principle structure is depicted in Fig. 3. $g(T-t)$ is the impulse response of the matched filter including optical electrical conversion. The demodulation is done by an Optical Quadrature QAM (OQ^{2}AM) receiver which detects the electrical field strength [4]. Subsequently the signal will be sampled and passed on to the Viterbi decoder. Its output is the vector

$$\vec{\hat{a}}_k = \begin{pmatrix} \hat{a}_{1,k} \\ \vdots \\ \hat{a}_{v,k} \\ \vdots \\ \hat{a}_{n,k} \end{pmatrix}. \tag{8}$$

Obviously the Viterbi algorithm has to work with vectors. In its trellis structure, each possible input vector

$$\vec{a}_k = \begin{pmatrix} a_{1,k} \\ \vdots \\ a_{v,k} \\ \vdots \\ a_{n,k} \end{pmatrix} \tag{9}$$

leads to a state transition with L OQ^{2}AM symbols as output values.

Fig. 4 and Fig. 5 depict two simple examples of coders and their associated trellis structures. Both arise from Fig. 2. The first one with the parameters

$$\begin{array}{lll} \mu_1 = 1; & & d^1_{0,1} = 0 \\ & c^1_{0,0} = 1; & c^1_{0,1} = 0 \\ & & d^1_{1,1} = 1 \\ & c^1_{1,0} = 1; & c^1_{1,1} = 0 \end{array} \tag{10}$$

and the second one with

$$\begin{array}{llll} \mu_2 = 2; & & d^2_{0,1} = 0; & d^2_{0,2} = 0 \\ & c^2_{0,0} = 1; & c^2_{0,1} = 0; & c^2_{0,2} = 0 \\ & & d^2_{1,1} = 1; & d^2_{1,2} = 1 \\ & c^2_{1,0} = 1; & c^2_{1,1} = 0; & c^2_{1,2} = 0 \end{array} \tag{11}$$

In [5] an octal representation of convolutional codes is introduced. The most right coefficient in Fig. 2 is the least significant bit (LSB), the most left coefficient the most significant bit (MSB). Applied to our example, the first encoder has

$$\begin{array}{lll} 1_8 & \text{recursive part} & \multirow{2}{*}{path 0} \\ 1_8 & \text{non recursive part} & \text{path 0} \\ 3_8 & \text{recursive part} & \\ 2_8 & \text{non recursive part} & \text{path 1} \end{array} \tag{12}$$

coefficients and the second one

$$\begin{array}{lll} 1_8 & \text{recursive part} & \\ 1_8 & \text{non recursive part} & \text{path 0} \\ 7_8 & \text{recursive part} & \\ 4_8 & \text{non recursive part} & \text{path 1.} \end{array} \tag{13}$$

The trellis structure can be derived as shown in [6].

Fig. 6 shows the total trellis structure, which is needed for decoding, and how it can be created out of Fig. 4 and Fig. 5. One trellis structure has to be included in each state of the other trellis structure. This is illustrated by the oval around two states, respectively. The different colors illustrate the transitions of the other trellis structure. Each branch in Fig. 5 gets a whole set of branches of Fig. 4. Thus the total amount of states is

$$2^M; \quad M = \sum_{\nu=1}^{n} M_\nu. \tag{14}$$

where M is the total number of memories. If

$$\tilde{M} = M_\nu; \quad \mu = 1, 2, \ldots, n \tag{15}$$

holds, like in case (5), equation (14) can be simplified to

$$2^{n \cdot \tilde{M}}; \quad M = n \cdot \tilde{M}. \tag{16}$$

In equation (18) it is obvious, that the number of states grows exponentially with the number of users and the number of memories. The number of transitions per state

$$2^n \tag{17}$$

also grows exponentially with the number of users n. Thus the number of transitions per column

$$2^n \cdot 2^M = 2^{n \cdot M} \tag{18}$$

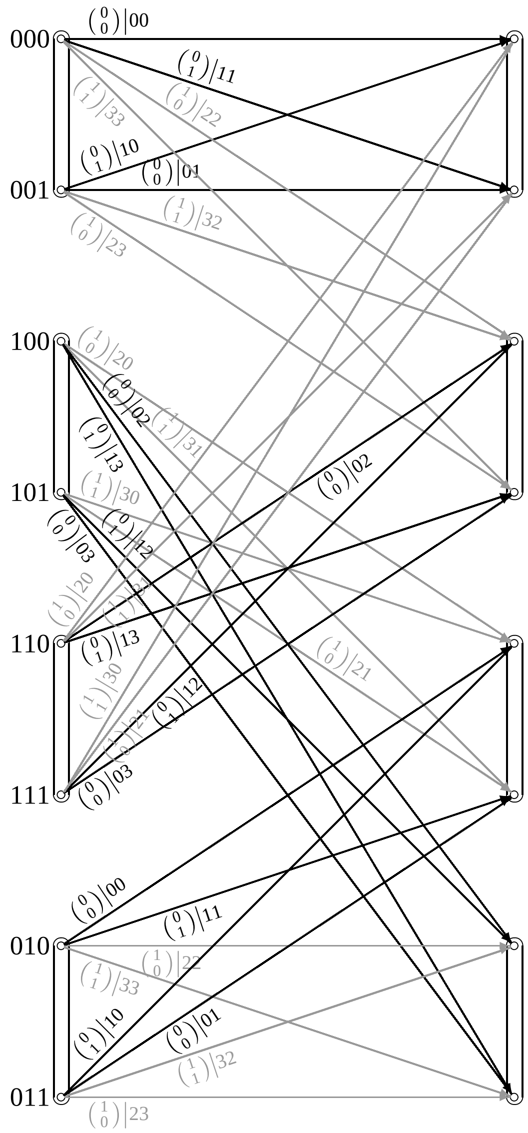

Figure 6. Exaple of a total trellis structure of two users with codes depicted in Fig. 4 and Fig. 5

grows extremly fast. In case of (15) equation (18) can be simplified to

$$2^n \cdot 2^{n \cdot \tilde{M}} = 2^{n^2 \tilde{M}}. \tag{19}$$

Thus the number of transitions per column grows quadratic exponentially with the number of users n and exponentially with the number of memories $\tilde{M}$.

4. Performance of OCDMA with Convolutional Encoder

I consider an OCDMA system with $n = 4$ users and spreading factor $L = 4$. Each user is coded by a systematic convolutional code with memory $M_v = 4$. Thus the Viterbi decoder has to handle a trellis structure with $2^{16} = 65536$ states with $2^4 \cdot 2^{16} = 1048576$ transitions per column. In octal notation apart from the systematic path the codes are

$$\begin{array}{lll} \text{user 1:} & 11_8 & \text{recursive part} \\ & \left.\begin{array}{l} 23_8 \\ 33_8 \\ 27_8 \end{array}\right\} & \text{non recursive parts} \end{array} \quad (20)$$

$$\begin{array}{lll} \text{user 2:} & 03_8 & \text{recursive part} \\ & \left.\begin{array}{l} 31_8 \\ 33_8 \\ 27_8 \end{array}\right\} & \text{non recursive parts} \end{array} \quad (21)$$

$$\begin{array}{lll} \text{user 3:} & 03_8 & \text{recursive part} \\ & \left.\begin{array}{l} 27_8 \\ 33_8 \\ 31_8 \end{array}\right\} & \text{non recursive parts} \end{array} \quad (22)$$

$$\begin{array}{lll} \text{user 4:} & 11_8 & \text{recursive part} \\ & \left.\begin{array}{l} 27_8 \\ 33_8 \\ 23_8 \end{array}\right\} & \text{non recursive parts.} \end{array} \quad (23)$$

As can be seen, user 3 and 4 have the same codes as 2 and 1 in vice versa order. Thus codes can be used several times with different order of the coded bits.

The OCDMA system with convolutional encoding is compared to OCDMA with Walsh-Hadamard codes (WHC). In contrast to [7] the Walsh-Hadamard coded symbols are modulated by optical binary phase shift keying (BPSK) instead of on off keying (OOK). The dominant noise is assumed to be from optical amplifiers. Because of the OQ^2AM demodulation this noise remains gaussian. A square root Nyquist filter is assumed to be the impulse shaping filter. Thus the sampled noise at the receiver is assumed to be uncorrelated. The average signal energy per sample of each user is normalized to 1. Hence I use

$$\mathrm{SNR} = \frac{1}{N_0} \quad (24)$$

as definition for the signal to noise ratio. N_0 is the normalized constant noise power density. A straightforward calculation yields the bit error probability (BER)

$$P_{\mathrm{b,WHC}} = Q\left(\frac{1}{\sqrt{N_0}}\right) = Q\left(\sqrt{10^{\mathrm{SNR_{dB}}/10}}\right) \quad (25)$$

of each WHC OCDMA user.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \mathrm{e}^{-\frac{u^2}{2}}\,\mathrm{d}u \quad (26)$$

is the Q-function and

$$\mathrm{SNR_{dB}} = 10\log(\mathrm{SNR}) \quad (27)$$

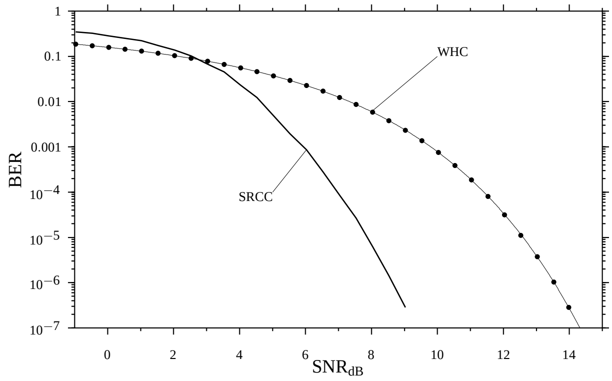

Figure 7. Comparison of a 4 user OCDMA system with Walsh Hadamard Codes (WHC) and systematic recursive convolutional codes (SRCC) if 1 user is active.

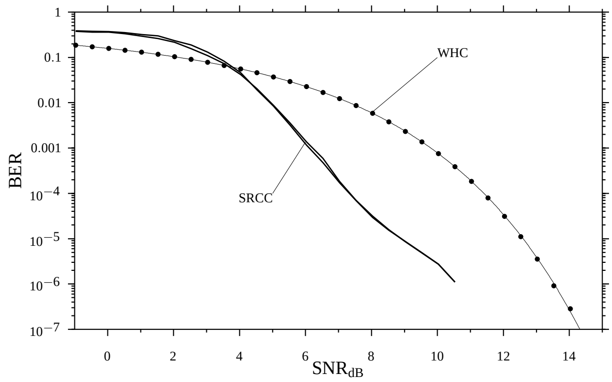

Figure 8. Comparison of a 4 user OCDMA system with WHC and SRCC if 2 users are active.

the signal to noise ratio in decibel. For verification of the simulation, this bit error ratio is simulated as well. Both match excellently, which can be seen in Fig. 7-10.

Fig. 7 shows the case of an OCDMA system with four users when only one user is sending data and the others are inactive. For reasonable low BER the systematic recursive convolutional code (SRCC) outperforms the WHC by some dB. Fig. 8 shows the result, if a second user also transmits information bits. The graphs for WHC are equal for all users. The graphs for SRCC are a litte bit different, but very close together. Fig. 9 and Fig. 10 show the BER for three and four active users. The more users are active, the worse SRCC is. This is different to WHC where the performance is independent of the number of active users. Fig. 11 shows Fig. 7-10 in one picture. The degradation of BER for increasing number of users is relatively low. Thus for high SNR SRCC still outperforms WHC. Furthermore SRCC are easy to expand with additional codes for addi-

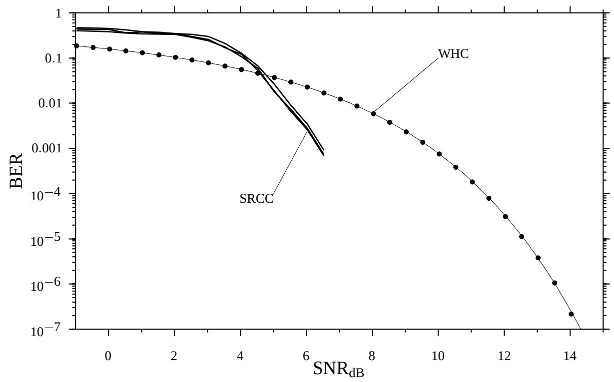

Figure 9. Comparison of a 4 user OCDMA system with WHC and SRCC if 3 users are active.

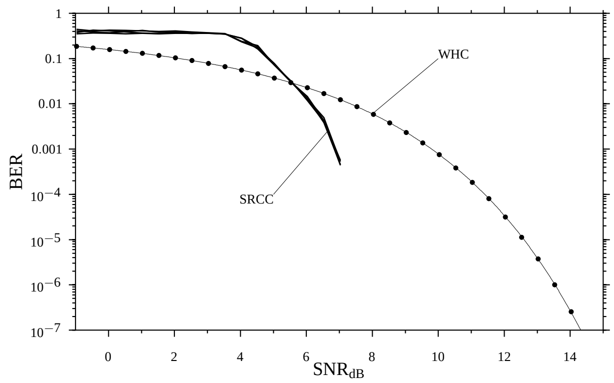

Figure 10. Comparison of a 4 user OCDMA system with WHC and SRCC if 4 users are active.

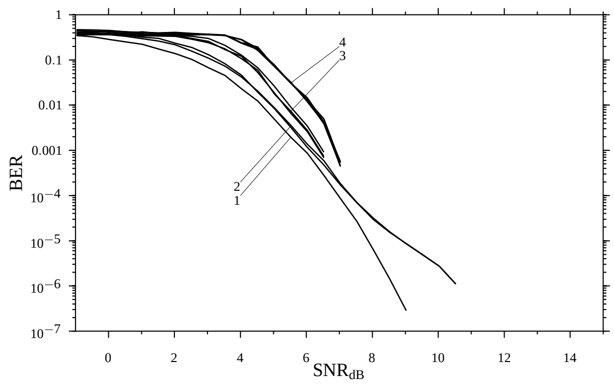

Figure 11. 4 user OCDMA system with SRCC if 1,2,3,4 users are active.

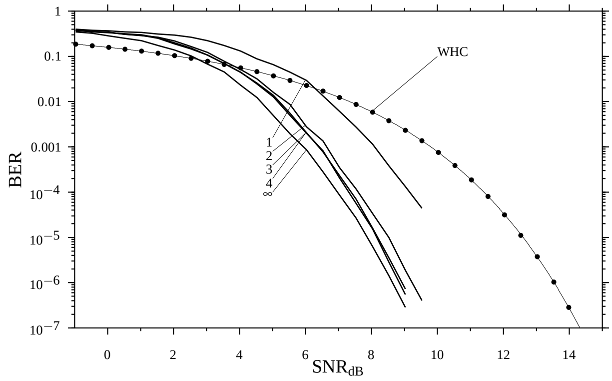

Figure 12. Dependency of the resolution of the analog to digital converter of a 4 user OCDMA system with SRCC if one user is active.

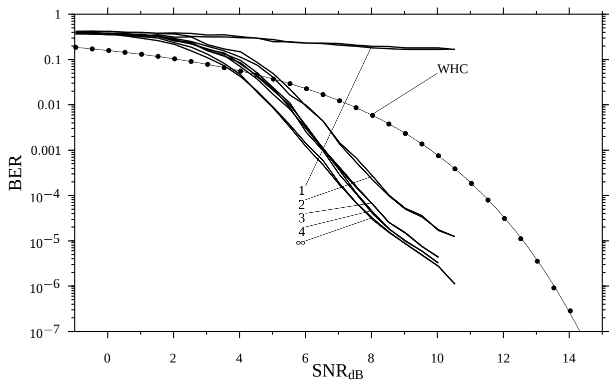

Figure 13. Dependency of the resolution of the analog to digital converter of a 4 user OCDMA system with SRCC if one user is active.

tional users. WHC have to be designed for the maximum possible number of users.

Fig. 12 shows the case of one active user, if the received signal is quantized to 1, 2, 3 and 4 bits. As a reference, the curves of Fig. 7 are plotted as well. The graphs for qantization to 3 and 4 bits are very close. Thus analog to digital conversion with 3-4 bits is adequate for the considered system. Fig. 13 shows the impact of quantization if two users are active. The prinziple characteristic is the same.

5. Conclusion

I have applied systematic recursive convolutional codes (SRCC) joinly for user separation and forward error correction coding in OCDMA. The proposed multiuser Viterbi algorithm shows good performance. However, the complexity is fast growing with the number of CDMA users and memory of the convolutional code. This could be overcome by a single user detection scheme combined with iterative (turbo) detection. For analog to digital conversion a resolution of 3-4 bits is adequate. Another benefit of SRCC is the possibility of a simple expansion of the code for additional OCDMA users, which is not the case with Walsh-Hadamard codes (WHC). The latter have to be designed for the maximum number of users. We have demonstrated the scheme together with OQ^2AM, however the proposed method of joint convolutional encoding for CDMA user separation and error correction coding can also be applied to other modulation techniques, including simple intensity modulation.

References

[1] Fan R. K. Chung, Jawad A. Salehi, Victor K. Wei, *Optical Orthogonal Codes: Design Analysis, and Applications*, IEEE Transactions on Information Theory, vol. 35, no. 3, May 1989.

[2] Jawad A. Salehi, *Code Division Multiple-Access Techniques in Optical Fiber Networks - Part I: Fundamental Principles*, IEEE Transactions on Communications, vol. 37, pp. 824-833, August 1989.

[3] Jawad A. Salehi, Charles A. Brackett, *Code Division Multiple-Access Techniques in Optical Fiber Networks - Part II: Systems Performance Analysis*, IEEE Transactions on Communications, vol. 37, pp. 834-842, August 1989.

[4] Robert Fritsch, Joachim Speidel, *OQ^2AM – Optical QAM scheme with orthogonal polarization*, 4. ITG-Fachtagung Photonische Netze, Leipzig, pp. 169-176, Mai 2003.

[5] John G. Proakis *Digital Communications*, 4th ed. 2001.

[6] Andrew J. Viterbi *Error bounds for convolutional codes and an asymptotically optimum decoding algorithm*, IEEE Transactions on Information Theory, vol. 13, no. 2, pp. 260-269, April 1967.

[7] Robert Fritsch, Joachim Speidel, *Comparison of OTDMA and Synchronous OCDMA with Optical and Electrical Decoding*, 3. ITG-Fachtagung Photonische Netze, Leipzig, pp. 137-143, April 2002.