# Asymptotically Tight Bound on SER of MIMO Zero-Forcing Receivers with Correlated Fading

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*Abstract* — **Analysis of the symbol error rate (SER) for linear MIMO receivers is a challenging task in the presence of spatial fading correlation at the antenna arrays, especially at the receiver side. In this paper, for the first time we present a tight bound on the SER of MIMO zero-forcing receivers in correlated Rayleigh fading environments. The novel bound is based on a moment generating function (MGF) approach and requires the evaluation of a single integral with finite integration limits only. It is tight in the SNR range of practical interest and exact at high SNR, thereby allowing for a concise characterization of the influence of fading correlation in terms of an asymptotic SNR shift. Specifically, the SNR penalty due to correlation at the receiver can be expressed simply via elementary symmetric functions of the eigenvalues of the receive correlation matrix. Furthermore, the novel bound can serve as a valuable tool for an efficient analysis of multi-user smart antenna systems deploying e.g. optimum combining.**

## I. Signal and Channel Model

We consider a general flat fading MIMO link with linear transmit prefilter modeled by

$$\boldsymbol{y} = \boldsymbol{HFs} + \boldsymbol{n}\,, \tag{1}$$

where **s** is the L×1 TX symbol vector, i.e. there are L independent data streams (subchannels) and **F** is a T×L linear matrix transmit prefilter. With the availability of adequate channel state information at the transmitter, a subchannel specific beamforming is possible via **F**, which can be used e.g. for minimizing the SER [4]. **H** is the R×T MIMO channel matrix with correlated Rayleigh fading elements, **n** is the R×1 noise vector, and **y** is the R×1 receive vector. By R≥L we denote the number of RX antennas and T is the number of TX antennas. Note that L can in general be smaller than the number of transmit antennas.

For the covariance matrices we assume without loss of generality $\mathbf{R}_{nn}=N_0\cdot\mathbf{I}$ and $\mathbf{R}_{ss}=E_s\cdot\mathbf{I}$. The signal to noise ratio (SNR) is defined by $\gamma = E_s/N_0$. Using a widely accepted channel model, the correlated MIMO channel can be described by the matrix product

$$\boldsymbol{H} = \boldsymbol{A}^H\boldsymbol{H}_w\boldsymbol{B}\,, \tag{2}$$

where $\mathbf{H}_w$ is a R×T matrix of complex i.i.d. Gaussian variables of unity variance, $\boldsymbol{AA}^H = \boldsymbol{R}_{RX}$ and $\boldsymbol{BB}^H = \boldsymbol{R}_{TX}$. $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ is the long-term stable (normalized) receive and transmit correlation matrix, respectively. We note that (2) generalizes the transmit correlated model analyzed in [1]. Finally, it can be shown that

$$\gamma_{SC,k} = \gamma/\left[\left((\boldsymbol{HF})^H\boldsymbol{HF}\right)^{-1}\right]_{kk}, \tag{3}$$

where $\gamma_{SC,k}$ is the SNR on subchannel k (k=1…L) after ZF processing.

## II. Symbol Error Rate Calculation

The SER on subchannel k $P_{s,k}(\gamma)$ can (approximately) be calculated via the subchannel SNR moment generating function (MGF)

$$M_{\gamma,k}(s) = E_{\gamma_{SC,k}}[\exp(-s\gamma_{SC,k})] \tag{4}$$

and the single integral [1]

$$P_{s,k}(\gamma) = \frac{2b}{\pi}\cdot\int_0^{\pi/2} M_{\gamma,k}\left(\frac{c}{\sin^2\theta}\right)d\theta\,, \tag{5}$$

where b and c are modulation dependent parameters. It remains to calculate the subchannel SNR MGF $M_{\gamma,k}(s)$. An exact solution to this problem could be found by the authors only recently [5]. However, due to the complexity of the exact analysis, we present a simple MGF approximation in this paper.

## III. Subchannel SNR Moment Generating Function

By reformulating (3) as a random quadratic form similar to the proceeding in [6], it can be shown that the subchannel SNR MGF can be expressed as a ratio of certain random determinants. The expected value of this ratio can be approximated by using a so-called Laplace approximation paralleling [3] and due to the space limitation we only present the final result.

**Theorem 1.** The MGF $M_{\gamma,k}(s)$ can be approximated by

$$M_{\gamma,k}(s) \approx M^A_{\gamma,k}(s) = \frac{1}{|\boldsymbol{I} + s\alpha_k\boldsymbol{O}|}\cdot\frac{\mathrm{tr}_{L-1}(\boldsymbol{O})}{\mathrm{tr}_{L-1}(\boldsymbol{O}(\boldsymbol{I}+s\alpha_k\boldsymbol{O})^{-1})}\,, \tag{6}$$

where we have introduced the eigenvalue decomposition (EVD) $\boldsymbol{O} = \mathrm{eig}(\boldsymbol{R}_{RX}) = \mathrm{diag}(o_1, \ldots, o_R)$, $\mathrm{tr_n}(\mathbf{X})$ is the elementary symmetric function of order n of the eigenvalues of matrix **X,** and

$$\alpha_k = \frac{\gamma}{[(\boldsymbol{F}^H\cdot\boldsymbol{R}_{TX}\cdot\boldsymbol{F})^{-1}]_{kk}} = \gamma\cdot\beta_k\,. \tag{7}$$

The approximation (6) is exact in the high SNR regime and reads

$$\bar{M}_{\gamma,k}(s) = \frac{\mathrm{tr}_{L-1}(\boldsymbol{O})}{|\boldsymbol{O}|\cdot\binom{R}{L-1}}\cdot(s\beta_k)^{-N}\cdot\gamma^{-N} \tag{8}$$

with diversity parameter N=R-L+1.❐

We emphasize that the approximate SER calculation now boils down to calculating the single integral in (5) together with (6). In the high SNR regime the integral can be solved in closed form and we find

**Theorem 2.** The asymptotic subchannel SER $\bar{P}_{s,k}(\gamma)$ in the high SNR region with fully correlated Rayleigh fading MIMO channel and ZF receiver is given by

$$\bar{P}_{s,k}(\gamma) = b\cdot\frac{\mathrm{tr}_{L-1}(\boldsymbol{O})}{|\boldsymbol{O}|\cdot\binom{R}{L-1}}\cdot\frac{1\cdot3\cdot5\ldots\cdot(2N-1)}{2^N\cdot\Gamma(N+1)}\cdot(c\beta_k)^{-N}\cdot\gamma^{-N}\,.❐ \tag{9}$$

We emphasize the explicit diversity of N in (9), which is independent of the correlation properties of the channel.

## IV. Conclusion

Based on a MGF approach, we have for the first time presented an asymptotically tight bound on the SER performance of MIMO ZF receivers in correlated Rayleigh fading environments with transmit as well as receive correlation. The bound requires the evaluation of a single integral with finite integration limits only and can be calculated in closed form in the high SNR regime.

## References

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