Exact Ergodic Capacity of MIMO Channels in Correlated Rayleigh Fading Environments

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***Abstract*—Using recent results on the moment generating function of mutual information, we derive exact formulas for the calculation of ergodic capacity of a fully correlated MIMO channel with transmit as well as receive correlation in a flat Rayleigh fading environment. The analysis is non-asymptotic, i.e. it is applicable without constraints to systems with a small number of antenna elements. It turns out that the ergodic capacity can be expressed in terms of a sum of determinants with elements that are a combination of polynomials, exponentials, and the exponential integral E_1 solely. Various Monte-Carlo simulations confirm the validity and accuracy of the analysis.**

I. Introduction

A unifying solution to the moment generating function (MGF) of mutual information of arbitrarily correlated MIMO channels in a Rayleigh fading environment was recently presented by the authors in [13] for the standard channel model with Kronecker product covariance structure. In this paper, we use the MGF to derive exact formulas for the ergodic MIMO capacity with uninformed transmitter, thus generalizing and complementing existing solutions in literature, comprising e.g. the seminal paper [1]. Telatar gave an expression for the ergodic capacity of a MIMO link with uncorrelated Rayleigh fading in an additive white Gaussian noise (AWGN) environment, which requires the evaluation of a single integral only. He predicted enormous capacity gains by combined spatial processing at transmitter and receiver, thus initiating immense research activities in this area. Later, Foschini and Gans presented numerical results and bounds on i.i.d. Rayleigh MIMO outage capacity in their fundamental work [2]. Bounds on the ergodic capacity of i.i.d. and correlated Rayleigh fading MIMO channels were given in [3][4][5][6][14]. Asymptotic results for large antenna arrays can be found in [7] and [8], where empirical eigenvalue probability density functions (PDF) of certain large dimensional random matrices are used for the derivation. However, even the asymptotic analysis is restricted to MIMO links, where only one side exclusively experiences fading correlation.

By integrating over the eigenvalue PDF of a complex i.i.d. Wishart matrix, in [9] the MGF of the mutual information of an i.i.d. Rayleigh fading MIMO channel was derived. A similar MGF approach is taken in [10], where the authors present results for various propagation scenarios including i.i.d. and one-side correlated Rayleigh fading, as well as Ricean fading. Again, a mathematically challenging integration over the eigenvalue PDF of certain (non-central) Wishart matrices is necessary, prohibiting a general solution for Rayleigh fading with both receive and transmit correlation.

It is demonstrated in the paper at hand, that the general ergodic capacity expression for fully correlated MIMO systems can be reduced to a sum of determinants, where the entries of the determinants are just polynomials, exponentials, and the only special function that occurs is the exponential integral E_1 [11].

Finally, the analysis is verified by Monte-Carlo simulations of different MIMO systems with varying correlation properties and variable number of transmit as well as receive antennas. Simulation results and analysis show a perfect match.

II. Signal and Channel Model

We consider a flat fading MIMO link modeled by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{1}$$

where $\mathbf{s}$ is the T×1 TX symbol vector, $\mathbf{H}$ is the R×T MIMO channel matrix with correlated Rayleigh fading elements, $\mathbf{n}$ is the R×1 noise vector, and $\mathbf{y}$ is the R×1 receive vector. By R we denote the number of RX antennas and T is the number of TX antennas. In the following we assume additive Gaussian noise, where the normalized noise covariance matrix is given by $N_0 \cdot \mathbf{R}_{nn}$. The normalized signal covariance matrix is given by $E_s \cdot \mathbf{R}_{ss}$. Both covariances can in general be colored.

Using a widely accepted channel model, the correlated MIMO channel can be described by the matrix product

$$\boldsymbol{H} = \boldsymbol{A}^H \boldsymbol{H}_w \boldsymbol{B}, \tag{2}$$

where $\mathbf{H}_w$ is a R×T matrix of complex i.i.d. Gaussian variables of unity variance and

$$\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{R}_{RX} \qquad \boldsymbol{B}\boldsymbol{B}^H = \boldsymbol{R}_{TX}, \tag{3}$$

where $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ is the long-term stable (normalized) receive and transmit correlation matrix, respectively.

In the remainder of the paper, by $\mathbf{I}_n$ we denote an identity matrix of size n×n (the index can be omitted, if the size of the matrix is clear from the context), $\mathrm{diag}(x_1,\ldots x_n)$ is a diagonal matrix with elements $x_1,\ldots x_n$, $|\boldsymbol{X}|$ is the determinant of the quadratic matrix $\mathbf{X}$, $[x_{ij}]$ is a matrix with element x_{ij} in row i and column j, $\mathrm{eig}(\mathbf{X})$ returns a diagonal matrix of eigenvalues of $\mathbf{X}$, $\mathbf{X}^H$ means Hermitian (conjugate transpose), x≅means 'random variable (RV) x has the same distribution as', and $E_x[\,]$ denotes expected value with respect to RV x.

III. MIMO MUTUAL INFORMATION

A. General expressions

For the given system model with flat fading, it is well known [1] that the mutual information between input and output vector of the MIMO channel is given by

$$I(\boldsymbol{s}, \boldsymbol{y}) = \log_2|\boldsymbol{I} + \gamma \cdot \boldsymbol{R}_{ss}\boldsymbol{H}^H\boldsymbol{R}_{nn}^{-1}\boldsymbol{H}| \quad (4)$$

with SNR definition $\gamma = E_s/N_0$. Plugging in the channel model from (2), we find

$$I(\boldsymbol{s}, \boldsymbol{y}) = \log_2|\boldsymbol{I} + \gamma \cdot \boldsymbol{R}_{ss}\boldsymbol{B}^H\boldsymbol{H}_w^H\boldsymbol{A}\boldsymbol{R}_{nn}^{-1}\boldsymbol{A}^H\boldsymbol{H}_w\boldsymbol{B}| . \quad (5)$$

Then introduce the following diagonal matrices of eigenvalues for brevity

$$\Omega = \mathrm{eig}(\boldsymbol{R}_{nn}^{-1}\boldsymbol{R}_{RX}) = \mathrm{diag}(\omega_1, \ldots, \omega_R) . \quad (6)$$

and

$$\Sigma = \mathrm{eig}(\boldsymbol{R}_{ss}\boldsymbol{R}_{TX}) = \mathrm{diag}(\sigma_1, \ldots, \sigma_T) . \quad (7)$$

By noticing that the distribution of the i.i.d. complex Gaussian distributed matrix $\mathbf{H}_w$ is invariant to left- or right multiplications with unitary matrices **U** and **V**, i.e.

$$\boldsymbol{U}\boldsymbol{H}_w\boldsymbol{V} \cong \boldsymbol{H}_w , \quad (8)$$

we find

$$I(\boldsymbol{s}, \boldsymbol{y}) \cong \log_2|\boldsymbol{I} + \gamma \cdot \Sigma\boldsymbol{H}_w^H\Omega\boldsymbol{H}_w| . \quad (9)$$

We note, that in the remainder of the paper we assume R≥T without loss of generality. To this end, we show that we can also reduce the case T>R to an equivalent problem by just switching Σ and Ω, as we have

$$I(\boldsymbol{s}, \boldsymbol{y}) \cong \log_2|\boldsymbol{I} + \gamma \cdot \Sigma\boldsymbol{H}_w^H\Omega\boldsymbol{H}_w| = \log_2|\boldsymbol{I} + \gamma \cdot \Omega\boldsymbol{H}_w\Sigma\boldsymbol{H}_w^H| . (10)$$

IV. MGF OF MUTUAL INFORMATION

For expressing the MGF of mutual information we need some definitions. First, the complex multivariate Gamma function is given by

$$\Gamma_m(r) = \prod_{i=1}^{m}\Gamma(r - i + 1) , \quad (11)$$

where Γ(x) is the standard Gamma function. The Vandermonde determinant of a diagonal matrix **X**=diag(x_1, …,x_m) can be expressed as

$$\alpha_m(\boldsymbol{X}) = \prod_{i<j}(x_i - x_j) . \quad (12)$$

Furthermore, define the auxiliary function

$$\psi_q^{(m)}(\boldsymbol{b}) = \prod_{i=1}^{m}\prod_{j=1}^{q}(b_j - i + 1)^{i-1} \quad (13)$$

with $\boldsymbol{b} = (b_1, b_2, \ldots, b_q)$.

A. MGF in terms of hypergeometric functions

We first state the main

Theorem 1. The MGF Z_{TR}(s) of the fully correlated MIMO channel mutual information according to (9) with T transmit and R receive antennas is given by

$$Z_{TR}(s) = \frac{\chi}{\gamma^{\frac{R\cdot(R-1)}{2}} \cdot |\Sigma|^{R-T}} \cdot \frac{|\Psi(s)|}{\alpha_T(\Sigma)\cdot\alpha_R(\Omega)\cdot\psi_2^{(R)}\left(-\frac{s}{\ln 2}, R\right)} , \quad (14)$$

with the auxiliary constant

$$\chi = \frac{\Gamma_R(R)}{(-1)^{\frac{R\cdot(R-1)}{2}} \cdot (-1)^{\frac{(R-T)\cdot(R-T-1)}{2}}} \quad (15)$$

and the R×R matrix

$$\Psi(s) = \begin{bmatrix}\Psi_1(s)\\ \Psi_2(s)\end{bmatrix} , \quad (16)$$

with the T×R matrix (i runs from 1 to T and j from 1 to R)

$$\Psi_1(s) = \left[{}_2F_0\left(-\frac{s}{\ln 2} - R + 1, 1;\ ;-\gamma\sigma_i\omega_j\right)\right] , \quad (17)$$

the (R-T)×R matrix (i runs from 1 to R-T and j from 1 to R)

$$\Psi_2(s) = \left[(-\gamma\omega_j)^{i-1}\left[-\frac{s}{\ln 2} - R + 1\right]_{i-1}\right] , \quad (18)$$

scalar hypergeometric function ${}_2F_0(a_1, a_2;\ ;z)$ [11], and Pochhammer's symbol

$$[a]_k = a\cdot(a+1)\cdot\ldots\cdot(a+k-1) \qquad [a]_0 = 1 . \quad (19)$$

Proof: See [13] for a detailed derivation.

B. Alternative representations of the MGF

In order to get concise and numerically stable expressions in the following calculation of ergodic capacity, we state a nontrivial alternative representation of the MGF in

Corollary 1. The MGF Z_{TR}(s) in (14) can be written as

$$Z_{TR}(s) = \frac{\chi}{\gamma^{\frac{R\cdot(R-1)}{2}}} \cdot \frac{|\tilde{\Psi}(s)|}{\alpha_T(\Sigma)\cdot\alpha_R(\Omega)\cdot\psi_2^{(R)}\left(-\frac{s}{\ln 2}, R\right)} , \quad (20)$$

with the R×R matrix

$$\tilde{\Psi}(s) = \begin{bmatrix}\tilde{\Psi}_1(s)\\ \Psi_2(s)\end{bmatrix} , \quad (21)$$

Ψ_2(s) from (18), χ from (15), and the T×R matrix (i runs from 1 to T and j from 1 to R)

$$\tilde{\Psi}_1(s) = \begin{cases}\displaystyle\sum_{k=R-T}^{R-2}(-1)^k\cdot(\gamma\omega_j)^k\cdot\sigma_i^{k-(R-T)}\cdot\left[-\frac{s}{\ln 2} - R + 1\right]_k + \\ (-1)^{R-1}\cdot\sigma_i^{T-2}\cdot(\gamma\omega_j)^{R-2}\cdot\left[-\frac{s}{\ln 2} - R + 1\right]_{R-1}\cdot U\left(1, \frac{s}{\ln 2} + 2, \frac{1}{\gamma\sigma_i\omega_j}\right)\end{cases} , \quad (22)$$

where U(a,b,z) is the Kummer U function ([11], chapter 13).

Proof: For the following analysis, we give a relation between the scalar hypergeometric function ${}_2F_0(a_1, a_2;\ ;z)$ and the Kummer U(a,b,z) function. To this end we have from [11], paragraph 13.1, the alternative notations for the Kummer U function

$$U(a, b, z) = z^{-a}\cdot{}_2F_0\left(a, 1 + a - b;\ ;-\frac{1}{z}\right) . \quad (23)$$

Furthermore, the Kummer transformation reads (cf. [11], equation 13.1.29)

$$U(a,b,z) = z^{1-b}\cdot U(1+a-b, 2-b, z)\,. \tag{24}$$

From (23), together with (24) we find

$${}_2F_0\left(a, 1+a-b;\ ;-\frac{1}{z}\right) = z^{1+a-b}\cdot U(1+a-b, 2-b, z)\,. \tag{25}$$

Equation (25) is important, as there are integral representations (in contrast to the infinite sum representation of the hypergeometric function, which could possibly exhibit convergency problems) available for the Kummer U function, whereas [11], equation 13.2.5 reads

$$U(a,b,z) = \frac{1}{\Gamma(a)}\cdot\int_0^\infty e^{-zt}\cdot t^{a-1}\cdot(1+t)^{b-a-1}dt\,. \tag{26}$$

We further state the following recurrence relation for the Kummer U function

$$U(1,b,z) = \frac{1}{z}+\frac{1}{z}(b-2)U(1,b-1,z)\,, \tag{27}$$

which can be derived from (26) via integration by parts. We find after iteratively applying (27)

$$\frac{1}{\gamma\sigma_i\omega_j}\cdot U\left(1,\frac{s}{\ln 2}+R+1,\frac{1}{\gamma\sigma_i\omega_j}\right) = \begin{cases}\sum_{k=0}^{R-2}(-1)^k\cdot(\gamma\sigma_i\omega_j)^k\cdot\left[-\frac{s}{\ln 2}-R+1\right]_k + \\ (-1)^{R-1}\cdot(\gamma\sigma_i\omega_j)^{R-2}\cdot\left[-\frac{s}{\ln 2}-R+1\right]_{R-1}\cdot U\left(1,\frac{s}{\ln 2}+2,\frac{1}{\gamma\sigma_i\omega_j}\right)\end{cases} \tag{28}$$

and the remaining Kummer U function has from (26) the integral representation

$$U\left(1,\frac{s}{\ln 2}+2,\frac{1}{\gamma\sigma_i\omega_j}\right) = \int_0^\infty e^{-\frac{1}{\gamma\sigma_i\omega_j}t}\cdot(1+t)^{\frac{s}{\ln 2}}dt\,. \tag{29}$$

Using (25) and (28) in (14), then subtracting properly scaled multiples of the rows of Ψ_2 from the rows of Ψ_1 (this does not alter the determinant), and finally factoring out σ_i^{R-T} from the resulting rows of Ψ_1 yields (20). *QED.*

V. Calculation of Ergodic Capacity

Note that from the MGF of mutual information we can derive the ergodic capacity C_{erg} with uninformed transmitter, which is the first moment of mutual information

$$C_{\mathrm{erg},TR} = E_{H_w}[I(\boldsymbol{s},\boldsymbol{y})] = E_{H_w}[\log_2|\boldsymbol{I}+\gamma\cdot\Sigma\boldsymbol{H}_w^H\Omega\boldsymbol{H}_w|]\,. \tag{30}$$

by

$$C_{\mathrm{erg},TR} = \left.\frac{d}{ds}Z_{TR}(s)\right|_{s=0}\,. \tag{31}$$

In the following we derive expressions for a fully correlated MIMO channel, while extensions to semi-correlated and uncorrelated channels can be found in [15].

We directly state

Theorem 2. The ergodic capacity of a fully correlated T×R MIMO system is given by

$$C_{erg,TR}(\gamma) = \frac{\Gamma_R(R)\cdot(-1)^{\frac{(R-T)\cdot(R-T-1)}{2}}}{\ln 2\cdot\alpha_T(\Sigma)\cdot\alpha_R(\Omega)\cdot\gamma^{\frac{R\cdot(R-1)}{2}}\cdot\prod_{k=1}^{R-1}k^R}\cdot\sum_{l=1}^{T}\begin{vmatrix}\Xi(l)\\ \Psi_2(0)\end{vmatrix} \tag{32}$$

with the T×R matrix (i runs from 1 to T and j from 1 to R)

$$\Xi(l) = \begin{cases}\Gamma(R)\cdot\sigma_i^{T-1}\cdot(\gamma\omega_j)^{R-1}\cdot e^{\frac{1}{\gamma\sigma_i\omega_j}}\cdot\mathrm{E}_1\left(\frac{1}{\gamma\sigma_i\omega_j}\right) & i=l\\ \sum_{k=R-T}^{R-1}(-1)^k\cdot(\sigma_i)^{k-(R-T)}\cdot(\gamma\omega_j)^k\cdot[1-R]_k & i\neq l\end{cases}\,. \tag{33}$$

Proof: By the quotient rule of differentiation we get from (20)

$$C_{erg,TR}(\gamma) = \varsigma\cdot\left.\frac{\frac{\partial}{\partial s}|\tilde{\Psi}(s)|\cdot\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)-|\tilde{\Psi}(s)|\cdot\frac{\partial}{\partial s}\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)}{\left[\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)\right]^2}\right|_{s=0} \tag{34}$$

with

$$\varsigma = \frac{\chi}{\gamma^{\frac{R\cdot(R-1)}{2}}}\cdot\frac{1}{\alpha_T(\Sigma)\cdot\alpha_R(\Omega)}\,. \tag{35}$$

Omitting the details, we find

$$\left.\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)\right|_{s=0} = (-1)^{\frac{R\cdot(R-1)}{2}}\cdot\prod_{k=1}^{R-1}k^R \tag{36}$$

and

$$\left.\frac{\partial}{\partial s}\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)\right|_{s=0} = \frac{R-1}{\ln 2}\cdot\left.\psi_2^{(R)}\left(-\frac{s}{\ln 2},R\right)\right|_{s=0}\,. \tag{37}$$

Moreover, we introduce the auxiliary function

$$\xi_k(s) = \left[-\frac{s}{\ln 2}-R+1\right]_k = \prod_{l=1}^{k}\left(-\frac{s}{\ln 2}-R+l\right) \qquad \xi_0(s) = 1\,, \tag{38}$$

with

$$\xi_k(0) = [-R+1]_k = \prod_{l=1}^{k}(-R+l) \tag{39}$$

and derivative

$$\left.\frac{\partial\xi}{\partial s}\right|_{s=0} = -\frac{1}{\ln 2}\xi_k(0)\cdot\sum_{l=1}^{k}\frac{1}{-R+l}\,. \tag{40}$$

For finding the derivative $\frac{\partial}{\partial s}|\tilde{\Psi}(s)|$, we can apply the following formula for differentiation of a determinant

$$\frac{\partial}{\partial s}|\boldsymbol{X}(s)| = \sum_i|\boldsymbol{X}_i(s)|\,, \tag{41}$$

where $|\boldsymbol{X}_i(s)|$ is the determinant of matrix **X**, where the ith column (or alternatively row) is differentiated with respect to s. Moreover, we can use

$$\left.U\left(1,\frac{s}{\ln 2}+2,\frac{1}{\gamma\sigma_i\omega_j}\right)\right|_{s=0} = \int_0^\infty e^{-\frac{1}{\gamma\sigma_i\omega_j}t}dt = \gamma\sigma_i\omega_j \tag{42}$$

and by exchanging the sequence of integration and differentiation by Lebesgue's dominated convergence theorem in (26), it is possible to derive

$$\left.\frac{\partial}{\partial s}U\left(1,\frac{s}{\ln 2}+2,\frac{1}{\gamma\sigma_i\omega_j}\right)\right|_{s=0} = \gamma\sigma_i\omega_j\cdot e^{\frac{1}{\gamma\sigma_i\omega_j}}\cdot\mathrm{E}_1\left(\frac{1}{\gamma\sigma_i\omega_j}\right)\,, \tag{43}$$

where $E_1(x)$ is the exponential integral defined by [11] 5.1.1

$$E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt . \quad (44)$$

Without going into the details, by the application of (36)-(43) for the calculation of (34), combination and simplification of the resulting determinants, we find (32). *QED.*

VI. Simulation Results

Without loss of generality, we study systems with white input signals of power E_s and additive white Gaussian noise of variance N_0 (other signal and noise covariances can easily be absorbed in an equivalent channel), i.e.

$$\boldsymbol{R}_{ss} = E_s \cdot \boldsymbol{I} \qquad \boldsymbol{R}_{nn} = N_0 \cdot \boldsymbol{I} . \quad (45)$$

In the following, we consider exponential correlation matrices at the receiver and the transmitter with

$$[\boldsymbol{R}_{\mathrm{RX/TX}}]_{ij} = (r_{\mathrm{RX/TX}})^{|i-j|} , \quad (46)$$

i.e. r_{RX} is the correlation coefficient between two neigboring receive antennas and r_{TX} models the correlation between two transmit antennas. Moreover, the correlation between two antennas decreases exponentially with their distance. The SNR in dB is defined by

$$SNR = 10 \cdot \log_{10}\left(\frac{T \cdot E_s}{N_0}\right) = 10 \cdot \log_{10}\left(\frac{\rho}{N_0}\right) \qquad [dB] \quad ,(47)$$

where ρ is the total transmitted power per channel use.

Simulation results and theoretical ergodic capacity curves closely agree in Fig. 1 for a 4×4 system with fully correlated MIMO channel, where the correlation parameter is the same at transmitter and receiver, i.e. r_{TX}=r_{RX}=r. As expected, the negative impact of channel correlation on ergodic capacity with uninformed transmitter can be observed. Moreover, it can be seen that for correlation parameter r values around 0.7 and higher, the decrease in capacity becomes very pronounced. A similar behavior can be observed for the dependence on correlation of symbol error rates of smart antenna systems.

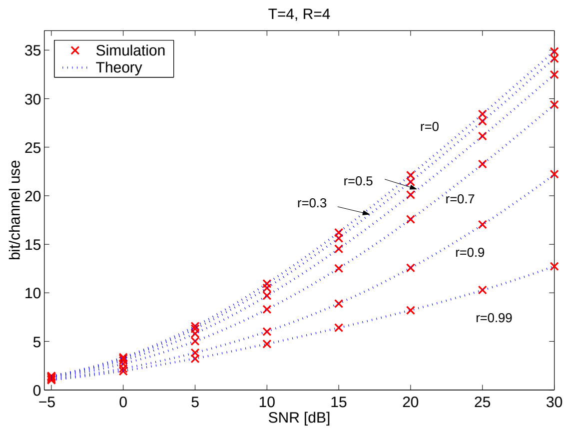

Fig. 1: Ergodic capacity of fully correlated MIMO system (T=R=4)

In Fig. 2 we study a correlated MIMO system, where the correlation matrix at the transmitter side has parameter r_{TX}=0.9 and at the receiver r_{RX}=0.5. We keep the total number of antenna elements constant T+R=8. As expected, in the high SNR region the (T=4,R=4) system achieves the highest capacity, as the slope of the capacity curves is proportional to min(R,T). However, in the low SNR region, the picture changes and the asymmetric (T=2, R=6) system yields highest capacity. We note that this principal behavior can also be observed for systems with uncorrelated fading at the antenna arrays.

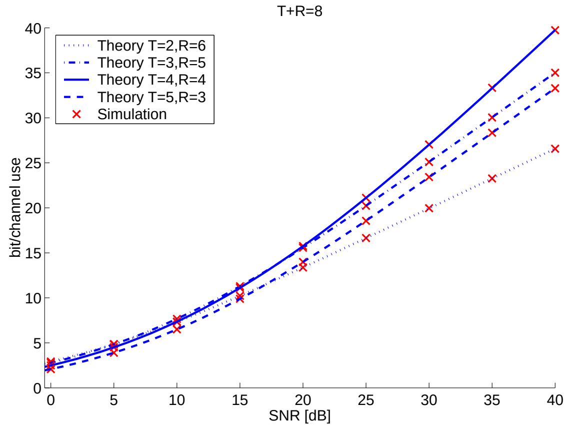

Fig. 2: Ergodic capacity, T+R=8, r_{TX}=0.9, r_{RX}=0.5

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