BIT ERROR RATE ANALYSIS OF ORTHOGONAL SPACE-TIME BLOCK CODES IN NAKAGAMI-M KEYHOLE CHANNELS

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ABSTRACT

We analyze the bit error rate (BER) performance of multiple-input multiple-output (MIMO) systems employing orthogonal space-time block codes (STBC) in Nakagami-m keyhole channels. We derive exact analytical closed-form expressions for the average BER of I-ary pulse amplitude modulation (I-PAM) and M-ary quadrature amplitude modulation (M-QAM) as well as a tight approximation for M-ary phase shift keying (M-PSK). These BER expressions are given as finite sums of weighted Meijer G-functions, which can be easily evaluated numerically. Furthermore, we determine the corresponding high SNR asymptotics, based on which we quantify the diversity order of the considered system. Numerical results illustrate the impact of several different parameters on the average BER and are shown to be in excellent agreement with simulated values.

1. INTRODUCTION

Orthogonal space-time block codes (STBC) represent a simple, but nevertheless effective means for mitigating the detrimental effects of multipath fading by exploiting the spatial diversity inherent to multiple antenna systems [1]. Being capable of providing full diversity advantage with only moderate encoding and decoding complexity, they are expected to find numerous applications in future wireless communication systems. In this paper, we investigate the bit error rate (BER) performance of multiple-input multiple-output (MIMO) systems employing orthogonal STBC in Nakagami-m keyhole channels. Keyhole channels generally characterize rank-deficient MIMO channels, which may have sufficient scattering around the transmitter and the receiver to obtain uncorrelated or at least only weakly correlated signals, but due to other propagation effects, such as diffraction or waveguiding, the channel matrix might nevertheless exhibit only low rank. For that reason, keyholes might have a significant impact on the performance of any MIMO system. The existence of this effect has been predicted theoretically in [2], for instance, and it has been verified by several different measurement campaigns, see for example [3].

In [4], the average symbol error rates (SER) of MIMO systems employing orthogonal STBC in Nakagami-m fading channels with keyhole have been calculated for various frequently used modulation schemes and the final results have been given as single integrals over generalized hypergeometric functions with finite integration limits. Herein, we consider the average BERs instead, which are often even more important for the analysis and design of communication systems than the corresponding SER expressions. In this regard, we derive exact analytical closed-form expressions for the average BER of I-PAM and M-QAM signal point constellations as well as a tight approximation for M-PSK signals, always assuming that constellations based on Gray mappings are used. Furthermore, we also determine the corresponding high SNR asymptotics, based on which we quantify the diversity order of the considered system.

The remainder of this paper is structured as follows: In Section 2, we outline our system and channel model and we determine the post-processing SNR statistics at the receiver-side. The actual BER analysis follows in Section 3 whereas the corresponding high SNR asymptotics are derived in Section 4. Finally, some numerical results are provided in Section 5 and the conclusions are given in Section 6.

2. SYSTEM MODEL AND SNR STATISTICS

2.1. System and Channel Model

We consider a frequency-flat MIMO system with N_{TX} and N_{RX} transmit and receive antennas, respectively. The discrete-time equivalent baseband representation of the channel is modeled by the $N_{\text{RX}} \times N_{\text{TX}}$ matrix $\mathbf{H}$, where the (i, j)-th element $[\mathbf{H}]_{i,j}$ corresponds to the channel coefficient between the j-th transmit and the i-th receive antenna. We assume that the channel does not change during the transmission of a block of $L \geq N_{\text{TX}}$ symbol vectors, but changes independently from block to block. Hence, the input-output relationship of our system can be expressed as $\mathbf{Y} = \sqrt{\bar{\gamma}/N_{\text{TX}}} \cdot \mathbf{HC} + \mathbf{N}$, where $\mathbf{C}$ is an $N_{\text{TX}} \times L$ codeword matrix with entries having unity average energy, $\mathbf{N}$ an $N_{\text{RX}} \times L$ additive white Gaussian noise (AWGN) matrix whose elements are i.i.d. $\mathcal{CN}(0, 1)$ distributed, and $\mathbf{Y}$ is the

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$N_{\text{RX}} \times L$ received signal matrix. The parameter $\bar{\gamma}$ denotes the average SNR per receive antenna. Generally, $W \leq L$ information symbols are transmitted during L consecutive time intervals, resulting in a code rate equal to $R_C = W/L$. By design, the row vectors of the codeword matrices **C** are always pairwise orthogonal. Due to this property, it can be shown that—at the output of the space-time decoder—a MIMO system employing orthogonal STBC is equivalent to a set of W parallel scalar channels with effective SNR [4]

$$\gamma = \frac{\bar{\gamma}}{R_C\, N_{\text{TX}}}\, ||\mathbf{H}||_F^2 = \gamma_0\, ||\mathbf{H}||_F^2, \quad (1)$$

where $\gamma_0 = \bar{\gamma}/(R_C\, N_{\text{TX}})$ and with $||\cdot||_F^2$ as the squared Frobenius norm of a matrix. We assume that both the transmitter and the receiver are located in rich-scattering environments, resulting in spatially uncorrelated transmit and receive signals. Furthermore, we assume a perfect keyhole channel, i.e., the only way for the radio waves to propagate from the transmitter to the receiver is to pass through a keyhole (e.g., a hallway acting as a single-mode waveguide), which ideally re-radiates all the captured energy. In this case, the channel can be considered as a concatenation of a multiple-input single-output (MISO) channel from the individual transmit antennas to the keyhole and a (statistically independent) single-input multiple-output (SIMO) channel from the keyhole to the various receive antennas [2], i.e., $\mathbf{H} = \mathbf{h}_{\text{SIMO}}\, \mathbf{h}_{\text{MISO}}^H$, where $\mathbf{x}^H$ denotes the conjugate-transpose of vector $\mathbf{x}$ and the N_{TX} and N_{RX} dimensional vectors $\mathbf{h}_{\text{MISO}}$ and $\mathbf{h}_{\text{SIMO}}$ represent the aforementioned MISO and SIMO channel, respectively. The phases of all entries of $\mathbf{h}_{\text{MISO}}$ and $\mathbf{h}_{\text{MISO}}$ are assumed to be uniformly distributed in $[0; 2\pi)$ whereas the corresponding magnitudes are modeled as Nakagami-m variates with fading parameters m_{TX} and m_{RX} and average power gains Ω_{TX} and Ω_{RX}, respectively. In addition, we assume that each entry of **H** has unity average power—or equivalently that $\Omega_{\text{TX}}\, \Omega_{\text{RX}} = 1$—and that the receiver has at all time instants perfect channel knowledge, thus facilitating ideal coherent detection.

2.2. SNR Statistics

Exploiting the special structure of keyhole channels outlined in the previous paragraph, it can easily be shown that

$$||\mathbf{H}||_F^2 = ||\mathbf{h}_{\text{SIMO}}\, \mathbf{h}_{\text{MISO}}^H||_F^2 = \Psi_1\, \Psi_2 =: \Xi \quad (2)$$

where $\Psi_1 = ||\mathbf{h}_{\text{MISO}}||_F^2$ and $\Psi_2 = ||\mathbf{h}_{\text{SIMO}}||_F^2$ are two gamma distributed random variables with shape parameters $\mu_{\text{TX}} = m_{\text{TX}}\, N_{\text{TX}}$ as well as $\mu_{\text{RX}} = m_{\text{RX}}\, N_{\text{RX}}$ and scale parameters $\Omega_{\text{TX}}/m_{\text{TX}}$ and $\Omega_{\text{RX}}/m_{\text{RX}}$, respectively. Hence, the corresponding probability density functions (PDFs) are given as

$$p_{\Psi_1}(\psi_1) = \frac{\psi_1^{\mu_{\text{TX}}-1}}{\Gamma(\mu_{\text{TX}})} \cdot \left(\frac{m_{\text{TX}}}{\Omega_{\text{TX}}}\right)^{\mu_{\text{TX}}} e^{-\frac{m_{\text{TX}}\, \psi_1}{\Omega_{\text{TX}}}} \quad (3)$$

$$p_{\Psi_2}(\psi_2) = \frac{\psi_2^{\mu_{\text{RX}}-1}}{\Gamma(\mu_{\text{RX}})} \cdot \left(\frac{m_{\text{RX}}}{\Omega_{\text{RX}}}\right)^{\mu_{\text{RX}}} e^{-\frac{m_{\text{RX}}\, \psi_2}{\Omega_{\text{RX}}}}, \quad (4)$$

where $\Gamma(\cdot)$ represents the well-known gamma function [5]. Since $\mathbf{h}_{\text{MISO}}$ and $\mathbf{h}_{\text{SIMO}}$ and consequently also Ψ_1 and Ψ_2 are statistically independent of each other, it can easily be shown that the PDF of Ξ according to (2) can be calculated based on (3) and (4) as

$$p_\Xi(\xi) = \int_{-\infty}^{\infty} p_{\Psi_1}(\psi_1)\, p_{\Psi_2}\left(\frac{\xi}{\psi_1}\right) \frac{1}{|\psi_1|}\, d\psi_1 \quad (5)$$

$$= \int_0^{\infty} \frac{m_{\text{TX}}^{\mu_{\text{TX}}}\, m_{\text{RX}}^{\mu_{\text{RX}}}\, \xi^{\mu_{\text{RX}}-1}}{\Gamma(\mu_{\text{TX}})\, \Gamma(\mu_{\text{RX}})\, \Omega_{\text{TX}}^{\mu_{\text{TX}}}\, \Omega_{\text{RX}}^{\mu_{\text{RX}}}} \times \psi_1^{\mu_{\text{TX}}-\mu_{\text{RX}}-1}\, e^{-\left(\frac{m_{\text{TX}}\, \psi_1}{\Omega_{\text{TX}}} + \frac{m_{\text{RX}}\, \xi}{\psi_1\, \Omega_{\text{RX}}}\right)}\, d\psi_1. \quad (6)$$

Making use of [6] eq. (3.471,9) and performing the simple transformation $\gamma = \gamma_0\, \Xi$, we then obtain the PDF of the instantaneous post-processing SNR γ according to (1) as

$$p_\gamma(\gamma) = \frac{2\, \gamma^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}-1}}{\Gamma(\mu_{\text{TX}})\, \Gamma(\mu_{\text{RX}})} \left(\frac{\zeta}{\gamma_0}\right)^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}} \times K_{\mu_{\text{TX}}-\mu_{\text{RX}}}\left(2\sqrt{\zeta\, \frac{\gamma}{\gamma_0}}\right), \quad (7)$$

with $K_\nu(\cdot)$ as the ν-th order modified Bessel function of the second kind [5] and the short-hand notation

$$\zeta = m_{\text{TX}}\, m_{\text{RX}}, \quad (8)$$

which has been introduced for brevity.

3. BIT ERROR RATE ANALYSIS

3.1. I-ary Pulse Amplitude Modulation

An exact analytical expression for the average BER of I-PAM modulated signals with Gray mapping has been presented by Cho and Yoon for the case of simple AWGN channels in [7]. According to their results, the bit error probability for a certain SNR γ and constellation size $I = 2^n$ $(n \in \mathbb{N})$ is given as

$$P_{b,\text{AWGN}}(\gamma) = \sum_{r=1}^{\log_2(I)} \sum_{i=0}^{\eta(r;I)} \frac{\xi(i;r;I)}{\log_2(I)}\, \text{erfc}\left(\sqrt{\frac{\chi_i\, \gamma}{I^2-1}}\right), \quad (9)$$

where the short-hand notations

$$\eta(r;I) = (1-2^{-r})\, I - 1 \quad (10)$$

$$\chi_i = 3\,(2i+1)^2 \quad (11)$$

and

$$\xi(i;r;I) = \frac{1}{I}\, (-1)^{\left\lfloor \frac{i\, 2^{r-1}}{I} \right\rfloor} \left(2^{r-1} - \left\lfloor \frac{i\, 2^{r-1}}{I} + \frac{1}{2} \right\rfloor\right) \quad (12)$$

have been introduced for brevity again. Furthermore, $\lfloor \cdot \rfloor$ denotes the floor function, i.e., the largest integer value smaller than or equal to the given argument, and erfc$(\cdot)$ the complementary error function [5]. The respective probability for

MIMO systems employing orthogonal STBC in Nakagami-m keyhole channels hence can be obtained by averaging (9) over the PDF of the instantaneous SNR γ according to (7), i.e., $P_{b,\text{IPAM}} = \int_0^\infty P_{b,\text{AWGN}}(\gamma)\, p_\gamma(\gamma)\, d\gamma$, yielding to

$$P_{b,\text{IPAM}} = \sum_{r=1}^{\log_2(I)} \sum_{i=0}^{\eta(r;I)} \frac{\xi(i;r;I)}{\log_2(I)}\, \Upsilon\left(\sqrt{\frac{\chi_i}{I^2-1}}\right) \quad (13)$$

with

$$\Upsilon(\alpha) = \int_0^\infty \text{erfc}\left(\alpha\sqrt{\gamma}\right)\, p_\gamma(\gamma)\, d\gamma. \quad (14)$$

Plugging (7) in (14), we obtain

$$\Upsilon(\alpha) = \frac{2\,\zeta^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}}}{\Gamma(\mu_{\text{TX}})\,\Gamma(\mu_{\text{RX}})} \int_0^\infty \frac{\gamma^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}-1}}{\gamma_0^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}}} \times K_{\mu_{\text{TX}}-\mu_{\text{RX}}}\left(\sqrt{\frac{4\,\zeta\,\gamma}{\gamma_0}}\right) \text{erfc}(\alpha\sqrt{\gamma})\, d\gamma \quad (15)$$

To the best of our knowledge, the integral in (15) cannot be solved in closed-form using standard tables of integrals or standard integration methods. For that reason, we pursue a somewhat more sophisticated approach in the following by making use Meijer G-functions [6]. From [8] eq. (8.4.14,2), we know that the complementary error function might be written in terms of Meijer G-functions as

$$\text{erfc}(\sqrt{x}) = \frac{1}{\sqrt{\pi}} \cdot G_{1,2}^{2,0}\left[x \,\middle|\, \begin{matrix} 1 \\ 0, \quad \frac{1}{2} \end{matrix}\right]. \quad (16)$$

With this equivalent expression, (15) can be rewritten as

$$\Upsilon(\alpha) = \frac{2}{\sqrt{\pi}\cdot\Gamma(\mu_{\text{TX}})\,\Gamma(\mu_{\text{RX}})} \cdot \left(\frac{\zeta}{\gamma_0}\right)^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}}{2}} \times \int_0^\infty K_{\mu_{\text{TX}}-\mu_{\text{RX}}}\left(2\sqrt{\frac{\zeta\,\gamma}{\gamma_0}}\right) \gamma^{\frac{\mu_{\text{TX}}+\mu_{\text{RX}}-2}{2}} \times G_{1,2}^{2,0}\left[\alpha^2\,\gamma \,\middle|\, \begin{matrix} 1 \\ 0, \quad \frac{1}{2} \end{matrix}\right] d\gamma. \quad (17)$$

Performing the substitution $x = \frac{\zeta\,\gamma}{\gamma_0}$ and making use of [6] eq. (7.821,3), the integral in (17) can be solved analytically in closed-form, yielding to

$$\Upsilon(\alpha) = G_{3,2}^{2,2}\left[\frac{\alpha^2\,\gamma_0}{\zeta} \,\middle|\, \begin{matrix} 1-\mu_{\text{TX}}, & 1-\mu_{\text{RX}}, & 1 \\ 0, & \frac{1}{2} & \end{matrix}\right] \times \frac{1}{\sqrt{\pi}\cdot\Gamma(\mu_{\text{TX}})\,\Gamma(\mu_{\text{RX}})}. \quad (18)$$

By combining this expression with (13), we finally obtain the desired exact result for the average BER of MIMO systems employing orthogonal STBC and I-PAM modulation based on a Gray mapping in Nakagami-m fading channels with keyhole. Please note that the Meijer G-function in (18) might be easily evaluated numerically using standard mathematical software packages—such as Maple or Mathematica—where Meijer G-functions are readily available.

3.2. M-ary Quadrature Amplitude Modulation

M-ary QAM with Gray mapping generally can be considered as two independently amplitude modulated carriers in quadrature. Subsequently, we specifically consider rectangular $I \times J$ signal point constellations with $I \cdot J = M$, where $I = 2^i$ ($i \in \mathbb{N}$) denotes the number of possible amplitude levels for the in-phase component whereas $J = 2^j$ ($j \in \mathbb{N}$) denotes the same parameter for the quadrature component. The average BER of such rectangular M-QAM signals can easily be obtained by simply averaging the individual bit error probabilities of the corresponding in-phase and quadrature components—which are both I-PAM modulated—as has been outlined in [7]. Based on this result, it is hence straightforward to calculate the average BER of orthogonal STBC with M-QAM modulation in Nakagami-m keyhole channels similarly to the previously considered case as

$$P_{b,\text{MQAM}} = \frac{1}{\log_2(M)} \left(\sum_{r=1}^{\log_2(I)} P_I(r) + \sum_{w=1}^{\log_2(J)} P_J(w)\right), \quad (19)$$

where $P_I(r)$ and $P_J(w)$ denote, respectively, the probabilities that the r-th bit of the in-phase and the w-th bit of the quadrature component are in error, which are given by

$$P_I(r) = \sum_{i=0}^{\eta(r;I)} \xi(i;r;I)\,\Upsilon\left(\sqrt{\frac{\chi_i}{I^2+J^2-2}}\right) \quad (20)$$

$$P_J(r) = \sum_{i=0}^{\eta(w;J)} \xi(i;w;J)\,\Upsilon\left(\sqrt{\frac{\chi_i}{I^2+J^2-2}}\right) \quad (21)$$

with $\eta(a;b)$, χ_i, $\xi(a;b;c)$ and $\Upsilon(\alpha)$ according to (10), (11), (12) and (18), respectively.

3.3. M-ary Phase Shift Keying

A tight upper bound on the average BER of Gray-coded M-PSK schemes over AWGN channels, which was found to be in excellent agreement with simulated values, has been presented in [9] and is given by

$$P_{b,\text{AWGN}}(\gamma) = \sum_{i=1}^{\max(\frac{M}{4},1)} \frac{\text{erfc}\left(\sqrt{\gamma}\,\sin\frac{(2i-1)\pi}{M}\right)}{\max(\log_2 M, 2)}. \quad (22)$$

Consequently, it is straightforward to accurately approximate the average BER of MIMO systems employing orthogonal STBC in Nakagami-m keyhole channels with M-PSK modulation similarly to the previously considered cases as

$$P_{b,\text{MPSK}} \cong \sum_{i=1}^{\max\left(\frac{M}{4},1\right)} \frac{\Upsilon\left(\sin\frac{(2i-1)\,\pi}{M}\right)}{\max\left(\log_2(M), 2\right)}, \quad (23)$$

with $\Upsilon(\alpha)$ according to (18). Please note that this expression is exact for $M = 2$ (BPSK) and $M = 4$ (QPSK). In these cases, (23) coincides with the corresponding results for 2-PAM and 4-QAM modulated signals, respectively.

4. HIGH SNR ASYMPTOTICS

Closed-form expressions for the high SNR asymptotics of the average BER curves are of general interest since they are usually easier to evaluate and often more intuitive than the exact formulas. Besides, they might be used to determine the diversity order of the considered system by exploiting that in the high SNR regime the average BER usually can be expressed as $P_b \approx (G_c \cdot \bar{\gamma})^{-G_d}$, where G_c and G_d denote the coding gain and diversity order, respectively [10]. For determining the high SNR asymptotics, we first of all note that for small values of x, the modified Bessel function $K_\nu(x)$ might be reasonably approximated as [5]

$$K_\nu(x) \approx \begin{cases} -\ln\left(\frac{x}{2}\right) & \text{if } \nu = 0 \\ \frac{\Gamma(|\nu|)}{2} \cdot \left(\frac{2}{x}\right)^{|\nu|} & \text{if } \nu \neq 0 \end{cases} . \tag{24}$$

By using this approximation in (7), we obtain an approximation of $p_\gamma(\gamma)$ for large values of $\bar{\gamma}$, which then serves as the basis for determining the high SNR asymptotics of $\Upsilon(\alpha)$, similarly to the method outlined in [10]. However, in this regard we have to distinguish two different cases:

4.1. Case I: $\mathbf{m_{TX}\, N_{TX} \neq m_{RX}\, N_{RX}}$

In this case, we have $\nu = \mu_{\rm TX} - \mu_{\rm RX} \neq 0$. Replacing $K_\nu(x)$ in (7) by its approximation for small x according to (24) and plugging the resulting expression in (14), we obtain a high SNR approximation of $\Upsilon(\alpha)$, which is given as

$$\Upsilon(\alpha) \approx \frac{2\,\alpha\,\zeta^\beta\,\Gamma(|\mu_{\rm TX} - \mu_{\rm RX}|)}{\sqrt{\pi}\,\Gamma(\mu_{\rm TX})\,\Gamma(\mu_{\rm RX})\,\gamma_0^\beta} \times \int_{\gamma=0}^{\infty}\int_{t=1}^{\infty} \gamma^{\beta-\frac{1}{2}}\, e^{-\alpha^2\,\gamma\,t^2}\,dt\,d\gamma, \tag{25}$$

where we introduced for brevity the short-hand notation

$$\beta = \min\left(m_{\rm TX}\,N_{\rm TX}, m_{\rm RX}\,N_{\rm RX}\right) \tag{26}$$

and where we additionally made use of the integral representation $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\,x\,\int_1^\infty e^{-t^2\,x^2}\,dt$ [5]. Changing the order of integration and exploiting [6] eq. (3.381,4), this term can be solved analytically in closed-form, yielding to

$$\Upsilon(\alpha) \approx \frac{\Gamma\left(|\mu_{\rm TX} - \mu_{\rm RX}|\right)\,\Gamma\left(\beta + \frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\mu_{\rm TX})\,\Gamma(\mu_{\rm RX})\,\beta} \cdot \left(\frac{\zeta\,R_C\,N_{\rm TX}}{\alpha^2\,\bar{\gamma}}\right)^\beta. \tag{27}$$

Plugging this expression in the formulas for the average BER derived in the previous sections, we finally obtain the desired high SNR asymptotics of the average BER curves. Based on (27), it can easily be seen that the diversity order is given in this case as $G_d = \beta = \min(m_{\rm TX}\,N_{\rm TX}, m_{\rm RX}\,N_{\rm RX})$, what is in strong contrast to MIMO systems employing orthogonal STBCs in non-keyhole Nakagami-m fading channels, where it would be given as $\tilde{G}_d = m\,N_{\rm TX}N_{\rm RX} \geq G_d$ instead [11] (assuming identical fading levels $m = m_{\rm TX} = m_{\rm RX}$). Consequently, keyholes obviously might lead to a serious performance degradation of the considered system.

4.2. Case II: $\mathbf{m_{TX}\, N_{TX} = m_{RX}\, N_{RX}}$

Likewise, making use of (24) again for approximating $p_\gamma(\gamma)$ for large values of $\bar{\gamma}$, we obtain a high SNR approximation of $\Upsilon(\alpha)$ in case that $\mu_{\rm TX} = \mu_{\rm RX} = \mu$, which is given by

$$\Upsilon(\alpha) \approx \int_0^\infty \frac{\zeta^\mu\,\gamma^{\mu-1}}{(\Gamma(\mu))^2 \cdot \gamma_0^\mu}\,\ln\left(\frac{\gamma_0}{\gamma\,\zeta}\right)\,\text{erfc}(\alpha\,\sqrt{\gamma})\,d\gamma. \tag{28}$$

Replacing the complementary error function by its integral representation again, changing the order of integration and exploiting [6] eq. (4.352,1), we approximately get

$$\Upsilon(\alpha) \approx \frac{\Gamma\left(\mu + \frac{1}{2}\right)}{\left(\Gamma(\mu)\right)^2\,\mu\,\sqrt{\pi}}\,\ln\left(\frac{\alpha^2\,\bar{\gamma}}{R_C\,N_{\rm TX}\,\zeta}\right)\left(\frac{\zeta\,R_C\,N_{\rm TX}}{\alpha^2\,\bar{\gamma}}\right)^\mu, \tag{29}$$

where we considered only the dominant terms for $\bar{\gamma} \to \infty$. Obviously, it is not possible to directly quantify the diversity order in this case since (29) cannot be written as $(G_c\,\bar{\gamma})^{-G_d}$. However, for high values of $\bar{\gamma}$, the steepness of (29) in the logarithmical domain can easily be shown to be given as

$$\theta = -\lim_{\bar{\gamma}\to\infty} \bar{\gamma}\,\left[\frac{\partial}{\partial\bar{\gamma}}\Upsilon(\alpha)\right]/\Upsilon(\alpha) = \mu, \tag{30}$$

independent of α. Therefore, also the steepness of the derived BER expressions and hence to diversity order of our system corresponds to $G_d = \mu$ in this case. In fact, this result is in line with the previously considered case and as a unifying result, we consequently can state that the diversity order is given by $G_d = \min(m_{\rm TX}\,N_{\rm TX}, m_{\rm RX}\,N_{\rm RX})$ in general.

5. NUMERICAL RESULTS

Due to space constraints, we restrict to considering the well-known Alamouti scheme with $N_{\rm TX} = 2$ and $R_C = 1$ here, but the fundamental results hold for any other orthogonal STBC as well. Furthermore, we always assume the same fading level on both sides of the keyhole, i.e., that $m_{\rm TX} = m_{\rm RX} = m$. Fig. 1 depicts the average BERs as well as the corresponding high SNR asymptotics for different modulation schemes in both, keyhole and non-keyhole channels with $m = 1$. For calculating the average BERs of non-keyhole channels, we made use of the results presented in [11]. As can be seen, there is a perfect match between calculated and simulated values, what verifies our theoretical analysis. In addition, it is quite obvious that the existence of keyholes might lead to serious performance degradations due to the usually reduced diversity order compared to non-keyhole channels.

Fig. 2 illustrates the impact of the fading level m on the average BER of QPSK signals in case of one receive antenna. As one would expect, increasing values of m or equivalently less severe fading results in an additional diversity advantage and consequently leads to considerably smaller average BERs, particularly at high SNRs.

Finally, Fig. 3 shows the impact of the number of receive antennas on the average BER in case of QPSK modulation

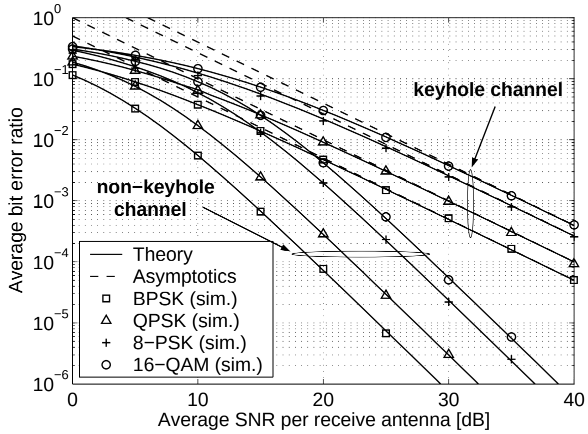

Fig. 1. Average BER versus average SNR per receive antenna for N_{TX} = 2, N_{RX} = 1, $R_C = 1$ and $m = 1$.

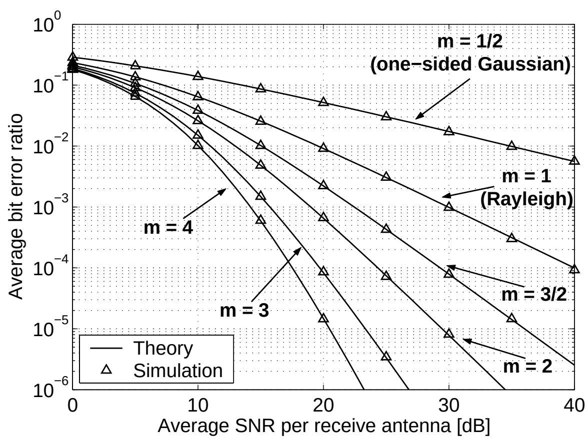

Fig. 2. Impact of the fading parameter m on the average BER of QPSK signals for N_{TX} = 2, N_{RX} = 1 and R_C = 1.

with m = 1. Obviously, using two receive antennas instead of only one leads to a significant performance improvement and especially a higher diversity order as reflected by the steeper slope of the corresponding BER curve in the high SNR regime. However, if the number of receive antennas is further increased, the diversity order remains the same and we obtain only a relatively small coding gain, i.e., a shift of the respective BER curves, which is additionally steadily decreasing with increasing values of N_{RX}. This is because the diversity order in case of keyhole channels is always given by $\min\{m_{\text{TX}}\, N_{\text{TX}}, m_{\text{RX}}\, N_{\text{RX}}\}$, as elucidated before.

6. CONCLUSIONS

We have performed a BER analysis of MIMO systems employing orthogonal STBC in Nakagami-m keyhole channels. In this regard, we have derived exact analytical closed-form expressions for the average BER of I-PAM and M-QAM signals as well as a tight approximation for M-PSK schemes, what has been accomplished by making use of Meijer G-function identities. Besides, we have determined the corresponding high SNR asymptotics, based on which we have proven that the diversity order of the considered system is always given by $G_d = \min\{m_{\text{TX}}\, N_{\text{TX}}, m_{\text{RX}}\, N_{\text{RX}}\}$. Numerical results were shown to be in excellent agreement with analytically calculated values, thus verifying the validity and accuracy of our theoretical analysis.

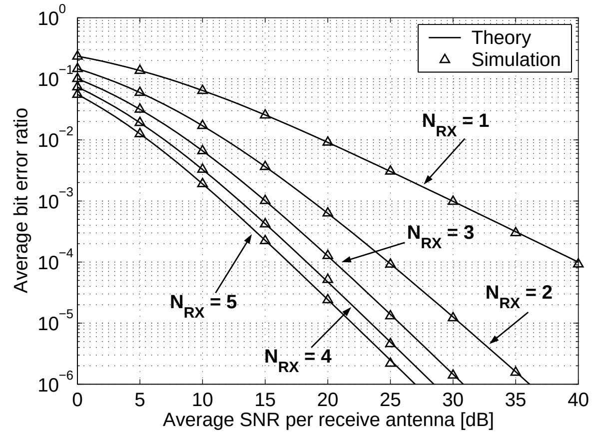

Fig. 3. Impact of the number of receive antennas on the average BER of QPSK signals for m = 1.

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