Capacity of Orthogonal STBC in Spatially Correlated Nakagami-m Fading Channels with Partial Channel Side Information at the Transmitter

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***Abstract*— We analyze the capacity of multiple-input multiple-output (MIMO) systems employing orthogonal space-time block codes (STBC) in spatially correlated Nakagami-m fading channels under different power and rate adaptation policies. In this regard, we assume that partial channel state information (CSI) in form of the instantaneous post-processing signal-to-noise ratio (SNR) at the receiver-side is available to the transmitter, which might be efficiently fed back using a low-rate feedback channel. Numerical results illustrate the impact of fading correlation and the fading severity on the capacity and outline the performance differences between the distinctive adaptation policies.**

I. Introduction

Communication systems employing multiple antennas at both the transmitter and the receiver-side of a wireless link have attracted a lot of research attention during the past few years due to their potential to facilitate significant performance improvements compared to conventional single-input single-output (SISO) systems. In this regard, orthogonal STBC—originally introduced by Alamouti for the special case of two transmit antennas [1] and later generalized by Tarokh *et al.* to an arbitrary number of antennas [2]—provide a powerful means for fully exploiting the spatial diversity inherent to such MIMO systems with only moderate encoding and decoding complexity. Therefore, these codes are expected to find numerous applications in future wireless communication systems.

In this paper, we analyze the capacity of MIMO systems employing orthogonal STBC in spatially correlated Nakagami-m fading channels with partial CSI at the transmitter and full CSI at the receiver for several different power and rate adaptation strategies. Hence, we basically generalize the results presented in [3], where a similar analysis has been performed, but for spatially uncorrelated Rayleigh fading channels only. In particular, we consider exactly the same power and rate adaptation strategies as originally presented by Goldsmith *et al.* in [4] for conventional SISO channels by exploiting a well-known analogy between MIMO systems employing orthogonal STBC and a set of associated SISO systems. The derived results reflect the maximum data rate for which theoretically error-free transmission is possible and consequently might serve as performance benchmarks for any practical implementation.

The remainder of this paper is structured as follows: In Section II, we introduce our system and channel model whereas the SNR statistics at the receiver-side are presented in Section III. In Section IV, we derive the capacity expressions for the considered rate and power adaptation schemes and afterwards some numerical results are given in Section V. Finally, we end up with some concluding remarks in Section VI.

II. System and Channel Model

We consider a frequency-flat MIMO system with N_{TX} and N_{RX} transmit and receive antennas, respectively. The discrete-time equivalent baseband representation of the channel is modeled by the $N_{RX} \times N_{TX}$ matrix $\mathbf{H}$, where the (i,j)-th element $[\mathbf{H}]_{i,j}$ corresponds to the channel coefficient between the j-th transmit and the i-th receive antenna. The phase of each channel coefficient is assumed to be uniformly distributed in $[0; 2\pi)$ whereas the magnitudes are supposed to be Nakagami-m variates with probability density function (PDF)

$$p\left(\zeta = |[\mathbf{H}]_{i,j}|\right) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \zeta^{2m-1} e^{-\frac{m}{\Omega}\zeta^2}, \quad \zeta \geq 0 \tag{1}$$

where $\Gamma(\cdot)$ denotes the Gamma-function [5] and m and Ω the fading parameter and average power gain, respectively. In the following, it is always assumed that all channel coefficients have the same integral fading parameter m and unity average power gain, i.e., $\Omega = 1$. Moreover, the individual elements of $\mathbf{H}$ generally might be correlated and we assume that the channel does not change during the transmission of a block of $L \geq N_{TX}$ symbols, but changes independently from block to block. Consequently, the input-output relationship can be expressed as $\mathbf{Y} = \sqrt{S/N_{TX}} \cdot \mathbf{HC} + \mathbf{N}$, where $\mathbf{C}$ is an $N_{TX} \times L$ codeword matrix with entries having unity average energy, $\mathbf{N}$ an $N_{RX} \times L$ additive white Gaussian noise (AWGN) matrix whose elements are independent and identically distributed (IID) circularly symmetric complex Gaussian random variables with zero mean and variance $2\sigma_n^2$, and $\mathbf{Y}$ is the $N_{RX} \times L$ received signal matrix. S denotes the total (instantaneous) transmit power per symbol time with mean $E[S] = P_T$, which is always equally allocated to the different transmit antennas. The average SNR per receive antenna is defined as $\bar{\gamma} = P_T/(2\sigma_n^2)$.

The codeword matrices $\mathbf{C}$ are created based on $W \leq L$ information symbols, resulting in a code rate equal to $R_C = W/L$ [2]. By design, the row vectors of $\mathbf{C}$ are always pairwise orthogonal. Due to this property, it can be shown that—at the output of the space-time decoder—a MIMO system employing

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orthogonal STBC with transmit power P_T is equivalent to a set of W parallel SISO channels with effective SNR [6]

$$\gamma = \frac{\bar{\gamma}}{R_C\, N_{TX}} \cdot \|\mathbf{H}\|_F^2 = \gamma_0 \cdot \|\mathbf{H}\|_F^2, \tag{2}$$

where $\gamma_0 = \bar{\gamma}/(R_C\, N_{TX})$ and with $\|\cdot\|_F^2$ as the squared Frobenius norm of a matrix. We assume that the receiver has at all times perfect channel knowledge whereas the transmitter has only partial CSI in form of the instantaneous post-processing SNR γ according to (2), which is assumed to be fed back by the receiver using a zero-delay, error-free feedback channel. In contrast to full channel feedback, feeding back only partial CSI requires only a low-rate feedback channel. For that reason, this approach represents an attractive implementation option for practical systems. Based on the partial CSI, the transmitter then can apply different power and rate adaptation strategies, which are always performed on a block-by-block basis, as has already been outlined in [3]. Herein, we specifically consider rate adaptation only (rao), optimal power and rate adaptation (opra) as well as channel inversion (cinv) and truncated channel inversion (tcinv) with fixed rate, respectively, as originally introduced in [4] for the case of conventional SISO channels.

III. Post-Processing SNR Statistics

A Nakagami-m variate with integer fading parameter m and unity average power gain generally can be considered as the square root of the sum of $2m$ squared IID Gaussian random variables with zero mean and variance $1/(2m)$ [7]. In presence of spatial correlation, it is therefore straightforward to model the magnitude of the (i,j)-th element of the channel matrix $\mathbf{H}$ as $|[\mathbf{H}]_{i,j}| = \sqrt{\sum_{l=1}^{m} |[\mathbf{X}_l]_{i,j}|^2}$, where $[\mathbf{X}_l]_{i,j}$ is the (i,j)-th entry of the $N_{RX} \times N_{TX}$ random matrix $\mathbf{X}_l$ $(l = 1, \ldots, m)$. These random matrices are IID circularly symmetric *complex* Gaussian distributed with zero mean, covariance matrix $\mathbf{R}_{\mathbf{X}_l \mathbf{X}_l}$ = $E[\mathrm{vec}(\mathbf{X}_l)\,(\mathrm{vec}(\mathbf{X}_l))^H] = \frac{1}{m}\,\Lambda$ and normalized correlation matrix Λ [7]. Please note that the elements of Λ can be directly related to various physical propagation parameters, such as the antenna spacing, mean angle of arrival, and angular spread, for instance, what makes this model especially suitable for theoretical investigations. Following the approach outlined in [7], it is straightforward to show that the moment-generating function (MGF) of the SNR γ according to (2) is given as

$$M_\gamma(s) = \left[\det\left(\mathbf{I}_{N_{TX}\,N_{RX}} - s \cdot \frac{\gamma_0}{m}\,\Lambda\right)\right]^{-m}, \tag{3}$$

where $\mathbf{I}_N$ denotes the identity matrix of dimension N. Now let us assume that Λ has P distinct non-zero eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_P$ with multiplicities $\alpha_1, \alpha_2, \ldots, \alpha_P$, respectively. Then, exploiting that the determinant of a matrix is given by the product of its eigenvalues, (3) can be rewritten as

$$M_\gamma(-s) = \prod_{i=1}^{P} \frac{1}{\left(1 + s \cdot \frac{\gamma_0}{m}\,\lambda_i\right)^{m\,\alpha_i}}. \tag{4}$$

Making use of partial fraction expansion, we transform (4) to

$$M_\gamma(-s) = \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \frac{\xi_{i,j}}{\left(1 + s \cdot \frac{\gamma_0}{m}\,\lambda_i\right)^{j}}, \tag{5}$$

where the expansion coefficients $\xi_{i,j}$ can be calculated as

$$\xi_{i,j} = \frac{1}{(m\,\alpha_i - j)!\,\left(\frac{\gamma_0}{m}\,\lambda_i\right)^{m\,\alpha_i - j}} \times \frac{\partial^{m\alpha_i - j}}{\partial s^{m\alpha_i - j}} \left[\prod_{\substack{\nu=1\\ \nu\neq i}}^{P} \frac{1}{\left(1 + s\,\frac{\gamma_0}{m}\,\lambda_\nu\right)^{m\alpha_\nu}}\right]\Bigg|_{s=\frac{-m}{\gamma_0\,\lambda_i}} \tag{6}$$

with $\frac{\partial^n}{\partial s^n}(\cdot)$ as the n-th partial derivative with respect to variable s. Performing the inverse Laplace-transform of $M_\gamma(-s)$ finally yields the PDF of γ, given as

$$p(\gamma) = \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j}\, \frac{\gamma^{j-1}}{\Gamma(j)\,\left(\frac{\gamma_0}{m}\,\lambda_i\right)^{j}} \cdot e^{-\frac{\gamma\, m}{\gamma_0\,\lambda_i}}, \tag{7}$$

which serves as the basis for our further analysis.

IV. Capacity Analysis

If partial CSI as given by the instantaneous post-processing SNR γ according to (2) is available at the transmitter-side, we can adjust the instantaneous transmit power S based on γ in order to optimize transmission, i.e., we generally have $S = S(\gamma)$. However, due to the constraint that the average transmit power should be equal to P_T, $S(\gamma)$ has to be chosen such that

$$E\left[S(\gamma)\right] = \int_0^\infty S(\gamma)\, p(\gamma)\, d\gamma = P_T. \tag{8}$$

For a given power allocation strategy $S(\gamma)$, the corresponding capacity in bit/s then can be calculated as [4]

$$C = B\, R_C \int_0^\infty \log_2\left(1 + \frac{S(\gamma)}{P_T} \cdot \gamma\right) p(\gamma)\, d\gamma, \tag{9}$$

where B denotes the system bandwidth in Hertz. Below, we consider now several different approaches for choosing $S(\gamma)$.

A. Optimal Rate Adaptation with Constant Power

In this case, the transmit power is always kept constant, i.e., we have $S(\gamma) \equiv P_T$. Plugging this expression together with (7) in (9) and making use of [8] eq. (7), we obtain

$$\frac{C_{\text{rao}}}{B} = \frac{R_C}{\ln 2} \cdot \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j} \cdot \left[\mathcal{P}_j\left(\frac{-m}{\gamma_0\,\lambda_i}\right) E_1\left(\frac{m}{\gamma_0\,\lambda_i}\right) + \sum_{\eta=1}^{j-1} \frac{1}{\eta} \cdot \mathcal{P}_\eta\left(\frac{m}{\gamma_0\,\lambda_i}\right) \mathcal{P}_{j-\eta}\left(\frac{-m}{\gamma_0\,\lambda_i}\right)\right]. \tag{10}$$

with $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}\, dt$ as the well-known exponential integral and $\mathcal{P}_D(x) = \sum_{\eta=0}^{D-1} \frac{x^\eta}{\eta!}\, e^{-x}$ as the Poisson distribution.

B. Optimal Power and Rate Adaptation

In case of optimal power and rate adaptation, the capacity is given by the solution of the constraint optimization problem

$$C_{\text{opra}} = \max_{S(\gamma)} B\, R_C \cdot \int_0^\infty \log_2\left(1 + \frac{S(\gamma)}{P_T}\,\gamma\right) p(\gamma)\, d\gamma \tag{11}$$

subject to (8) [4]. Since orthogonal STBCs convert a MIMO channel into several parallel SISO channels, we can conclude that the capacity-achieving power allocation strategy with partial CSI at the transmitter is similarly to the conventional SISO case given by temporal waterfilling, i.e., $S(\gamma)$ is given as

$$S(\gamma) = \begin{cases} P_T \cdot \left(\frac{1}{\gamma_{th}} - \frac{1}{\gamma}\right), & \text{for } \gamma \geq \gamma_{th} \\ 0, & \text{for } \gamma < \gamma_{th} \end{cases}, \qquad (12)$$

where transmission is suspended in case that the post-processing SNR γ falls below the fixed threshold value γ_{th}. Plugging (12) into (8), the power constraint hence can be written as

$$\int_{\gamma_{th}}^{\infty} \left(\frac{1}{\gamma_{th}} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma = 1, \qquad (13)$$

and we can finally calculate the actual capacity as

$$C_{\text{opra}} \overset{(9,12)}{=} B\, R_C \cdot \int_{\gamma_{th}}^{\infty} \log_2\left(\frac{\gamma}{\gamma_{th}}\right) p(\gamma)\, d\gamma. \qquad (14)$$

For verifying that for every $\bar{\gamma}$ a unique threshold value γ_{th} satisfying (13) exists, we define a function $g(\gamma_{th})$ as

$$g(\gamma_{th}) = \int_{\gamma_{th}}^{\infty} \left(\frac{1}{\gamma_{th}} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma - 1. \qquad (15)$$

Obviously, (13) has a solution iff $g(\gamma_{th}) = 0$. Differentiating $g(\gamma_{th})$ with respect to γ_{th}, we find

$$\frac{\partial}{\partial \gamma_{th}} g(\gamma_{th}) = -\frac{1}{\gamma_{th}^2} \cdot D_\gamma(\gamma_{th}), \qquad (16)$$

where $D_\gamma(x)$ denotes the complementary cumulative distribution function of γ. We know that generally $D_\gamma(x) \in [0;1]\, \forall\, x$. Hence, it is quite obvious that $\frac{\partial}{\partial \gamma_{th}} g(\gamma_{th}) < 0\, \forall\, \gamma_{th} > 0$, i.e., $g(\gamma_{th})$ is a strictly decreasing function of γ_{th}. Additionally, it is quite evident that $\lim_{\gamma_{th}\to\infty} g(\gamma_{th}) = -1$ and it can easily be shown by means of elementary manipulations and application of l'Hôpital's rule that $\lim_{\gamma_{th}\to 0}(\gamma_{th}) = \infty$. Therefore, since $g(\gamma_{th})$ is a strictly decreasing, continuous function, there is always exactly one value γ_{th} for which $g(\gamma_{th}) = 0$ and consequently the existence and uniqueness of a solution for (13) has been proven. Similarly to [3], it can also be shown that $\gamma_{th} \in]0;1[$ and that it is a strictly increasing function of $\bar{\gamma}$. However, this is omitted here due to space constraints.

Once the optimal threshold value γ_{th} has been determined (what can be done based on the power constraint (13) by means of numerical methods, for example), the actual capacity can be calculated by inserting (7) in (14), yielding to

$$C_{\text{opra}} = \frac{B\, R_C}{\ln 2} \cdot \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \int_{\gamma_{th}}^{\infty} \ln\left(\frac{\gamma}{\gamma_{th}}\right) \cdot \xi_{i,j} \times \frac{\gamma^{j-1}\, e^{-\frac{\gamma\, m}{\gamma_0\, \lambda_i}}}{\Gamma(j) \cdot \left(\frac{\gamma_0}{m}\, \lambda_i\right)^j}\, d\gamma. \qquad (17)$$

Performing the substitution $x = \frac{\gamma}{\gamma_{th}}$ and exploiting [9] eq. (70), we can solve (17) in closed-form as

$$C_{\text{opra}} = \frac{B\, R_C}{\ln 2} \cdot \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j} \cdot \left(E_1\left(\frac{\gamma_{th}\, m}{\gamma_0\, \lambda_i}\right) + \sum_{k=1}^{j-1} \frac{1}{k} \cdot \mathcal{P}_k\left(\frac{\gamma_{th}\, m}{\gamma_0\, \lambda_i}\right)\right). \qquad (18)$$

As already mentioned before, transmission is suspended if the post-processing SNR γ falls below the threshold value γ_{th}. The corresponding outage probability can be calculated as

$$P_{\text{out}} = \int_0^{\gamma_{th}} p(\gamma)\, d\gamma. \qquad (19)$$

Using [10] eq. (3.381,1) and exploiting that $\gamma(n,x) = \Gamma(n) \cdot [1 - \mathcal{P}_n(x)]$ for integer values of n [5], we finally get

$$P_{\text{out}} = \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j} \cdot \left[1 - \mathcal{P}_j\left(\frac{\gamma_{th}\, m}{\gamma_0\, \lambda_i}\right)\right]. \qquad (20)$$

C. Channel Inversion with Fixed Rate

The basic idea of this adaptation scheme is to compensate the channel fading by proper power allocation such that the received power is always constant, i.e., much power is allocated if the channel quality is poor whereas only little power is allocated in good channel conditions. This way, the channel seems to be a time-invariant AWGN channel to the receiver and hence a fixed transmission rate can be used. Clearly, this strategy is only suboptimal, but nevertheless frequently utilized in spread spectrum systems with near-far interference imbalances, for example [3]. In this case, we consequently have [4]

$$S(\gamma) = \frac{\delta}{\gamma} \cdot P_T, \qquad (21)$$

where δ has to be chosen such that the average power constraint (8) is satisfied. Plugging (21) in (8), we obtain

$$P_T \int_0^{\infty} \frac{\delta}{\gamma}\, p(\gamma)\, d\gamma = P_T. \qquad (22)$$

From [10] eqs. (3.381,4) and (2.325,1) we know that

$$\int_0^{\infty} x^{n-2}\, e^{-ax}\, dx = \begin{cases} -\lim_{b\to 0} E_1(ab) & \text{for } n = 1 \\ \frac{1}{a^{n-1}} \cdot \Gamma(n-1) & \text{for } n > 1 \end{cases}. \qquad (23)$$

Hence, we get by combining (23) with (22) and (7)

$$\delta = \lim_{b\to 0} \left(\sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, E_1(\beta_i\, b) + \sum_{i=1}^{P} \sum_{j=2}^{m\,\alpha_i} \frac{\xi_{i,j}\, \beta_i}{j-1}\right)^{-1}, \qquad (24)$$

where we introduced for brevity the short-hand notation

$$\beta_i = \frac{m}{\gamma_0\, \lambda_i}. \qquad (25)$$

Clearly, the capacity is zero iff $\delta = 0$, because in this case $S(\gamma) \equiv 0$. As can be seen from (24), this only occurs if

$$\lim_{b\to 0} \Delta = \lim_{b\to 0} \sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, E_1(\beta_i\, b) = \infty. \qquad (26)$$

Exploiting the identity $E_1(x) = -\zeta - \ln(x) - \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$ with ζ as the Euler-Mascheroni constant [5], we get

$$\lim_{b\to 0} \Delta = -\lim_{b\to 0} \left[\sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, \zeta + \sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, \ln(\beta_i\, b)\right]. \qquad (27)$$

Obviously, the first term is independent of b and finite whereas the second term (denoted as Ψ) can be rewritten as $\Psi =$

$\sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, \ln(\beta_i) + \ln\left(b^{\sum_{i=1}^{P} \xi_{i,1}\, \beta_i}\right)$. This expression converges for $b \to 0$ if and only if

$$\sum_{i=1}^{P} \xi_{i,1}\, \beta_i \overset{(25)}{=} \sum_{i=1}^{P} \xi_{i,1} \cdot \frac{m}{\gamma_0\, \lambda_i} = 0, \tag{28}$$

in which case $\lim_{b\to 0} \Delta = -\sum_{i=1}^{P} \xi_{i,1}\, \beta_i \cdot \ln(\beta_i)$.

For determining under which conditions (28) is satisfied, we rewrite $M_\gamma(-s)$ according to (5) as

$$M_\gamma(-s) = \frac{\sum_{i=1}^{P} \sum_{j=1}^{m\alpha_i} \xi_{i,j} \prod_{l=1}^{P} \left(1+\frac{s}{\beta_l}\right)^{m\alpha_l} \left(1+\frac{s}{\beta_i}\right)^{-j}}{\prod_{\nu=1}^{P} \left(1+\frac{s}{\beta_\nu}\right)^{m\,\alpha_\nu}} \tag{29}$$

It can easily be shown that the sum of all terms in the numerator of (29) with the highest powers of s is given as

$$\chi = \prod_{\eta=1}^{P} \left(\frac{1}{\beta_\eta}\right)^{m\,\alpha_\eta} \cdot \sum_{i=1}^{P} \xi_{i,1}\, \beta_i \left(s^{\sum_{l=1}^{P} m\,\alpha_l - 1}\right). \tag{30}$$

In the following, we distinguish two different cases: On the one hand that the highest power is given as $s^0 = 1$ and on the other hand that the highest power is given as s^n with $n > 0$. From (30), it is quite obvious that the first case is fulfilled iff $\sum_{i=1}^{P} m\,\alpha_i = 1$. In this case, we can conclude by comparing the coefficients of (30) and (4) that $\chi = 1$ and consequently $\sum_{i=1}^{P} \xi_{i,1}\, \beta_i = \prod_{\eta=1}^{P} \beta_\eta^{m\,\alpha_\eta} \neq 0$, i.e., (28) is not fulfilled and therefore we have δ = C_{cinv} = 0. This only occurs if m = P = α_1 = 1, i.e., only in Rayleigh-fading channels with one non-zero eigenvalue having multiplicity one. Such a situation is given for SISO Rayleigh fading channels, but also for fully correlated MIMO channels. In all other cases, the highest power of s in (29) is larger than zero and by comparison of coefficients again, we note that in this case necessarily χ=0. Hence, we can state based on (30) that $\sum_{i=1}^{P} \xi_{i,1}\beta_i$ = 0, what is exactly the necessary condition according to (28) such that $\delta \neq 0$. Consequently, δ and hence also the capacity are zero iff we have a SISO or a fully correlated MIMO Rayleigh fading channel. In all other cases, the capacity is larger than zero and can be calculated by plugging (21) in (9), yielding to

$$C_{\text{cinv}} = B\, R_C \cdot \log_2(1+\delta), \tag{31}$$

with

$$\frac{1}{\delta} = \sum_{i=1}^{P} \sum_{j=2}^{m\,\alpha_i} \frac{\xi_{i,j}\, \beta_i}{j-1} - \sum_{i=1}^{P} \xi_{i,1}\, \beta_i\, \ln(\beta_i)\,. \tag{32}$$

D. Truncated Channel Inversion with Fixed Rate

Truncated channel inversion represents a slightly modified version of the previously considered total channel inversion scheme and generally leads to a higher capacity while avoiding total outage in case of SISO or fully correlated MIMO Rayleigh fading channels at the same time [4]. The basic idea is to invert the channel only if γ is above a certain threshold γ_T whereas otherwise transmission is suspended. Hence, it is straightforward to define the power adaptation policy as

$$S(\gamma) = \begin{cases} \frac{\sigma}{\gamma} \cdot P_T & \text{if } \gamma \geq \gamma_T \\ 0 & \text{otherwise} \end{cases}, \tag{33}$$

where the constant σ has to be chosen such that the power constraint (8) is satisfied. Combining (8) with (33) and (7) and making use of [10] eq. (3.381,3), σ can be calculated as

$$\frac{1}{\sigma} = \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j}\, \frac{\beta_i}{\Gamma(j)}\, \Gamma(j-1, \beta_i\, \gamma_T), \tag{34}$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete Gamma function [10]. Exploiting that $\Gamma(0,x) = E_1(x)$ for $x > 0$ and that $\Gamma(n,x) = \Gamma(n) \cdot \mathcal{P}_n(x)$ for positive integers n, we get

$$\frac{1}{\sigma} = \sum_{i=1}^{P} \left[\xi_{i,1}\, \beta_i\, E_1(\beta_i \gamma_T) + \sum_{j=2}^{m\,\alpha_i} \frac{\xi_{i,j}\beta_i}{j-1} \cdot \mathcal{P}_{j-1}(\beta_i\, \gamma_T)\right]. \tag{35}$$

The actual capacity C_{tcinv} is hence given as

$$C_{\text{tcinv}} = B\, R_C \cdot \max_{\gamma_T} \log_2(1+\sigma) \cdot \int_{\gamma_T}^{\infty} p(\gamma)\, d\gamma, \tag{36}$$

where (using [10] eq. (3.381,3))

$$\int_{\gamma_T}^{\infty} p(\gamma) \cdot d\gamma = \sum_{i=1}^{P} \sum_{j=1}^{m\,\alpha_i} \xi_{i,j} \cdot \mathcal{P}_j\left(\frac{m\,\gamma_T}{\gamma_0\, \lambda_i}\right). \tag{37}$$

Please note that in case of truncated channel inversion also an outage might occur. The corresponding outage probability can be calculated similarly to (20) by replacing γ_{th} with γ_T.

V. Numerical Results

Fig. 1 shows the capacities of the considered power and rate adaptation schemes for three different scenarios. As can be seen, optimal power and rate adaptation generally leads to just a marginal improvement over rate adaptation only. Therefore, we conclude that the additional effort involved with joint power and rate adaptation is in most cases probably not worthwhile. Furthermore, we notice that truncated channel inversion leads at least for small SNRs to a slightly higher capacity than total channel inversion, but the differences are diminishing with increasing SNRs. Finally, we can say that with increasing diversity order (i.e., with more antennas), the differences between the individual adaptation policies become negligible. This is because in that case the equivalent SISO channels approximate more and more AWGN channels, for which all adaptation policies lead to exactly the same capacity.

Fig. 2 illustrates the impact of spatial correlation and the fading level on the capacity, considering optimal power and rate adaptation as well as truncated channel inversion as two examples. For simplicity, we assume that the channel has Kronecker covariance structure, i.e., that Λ = $\Lambda_{\text{TX}} \otimes \Lambda_{\text{RX}}$, where Λ_{TX} and Λ_{RX} are the correlation matrices at the transmitter and receiver, respectively, and where $\otimes$ denotes the matrix Kronecker product. Furthermore, we assume an exponential correlation model, that is the (m,n)-th entry of the correlation matrices $\Lambda_{\text{TX/RX}}$ is given as $\rho_{\text{TX/RX}}^{|m-n|}$, with $\rho_{\text{TX/RX}}$ as a correlation coefficient that can be used for adjusting the degree of correlation. For simplicity, we suppose that ρ_{TX} is always equal to ρ_{RX}, i.e., that we have the same spatial correlation at the transmitter and the receiver-side. As can be

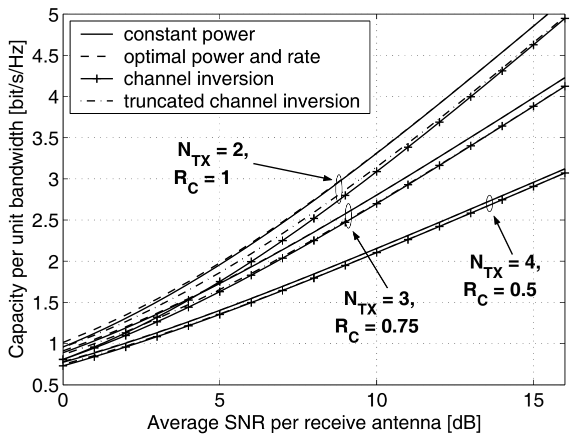

Fig. 1. Capacity per unit bandwidth for the considered power and rate adaptation policies in an uncorrelated channel with $m = 2$ and $N_{RX} = 1$.

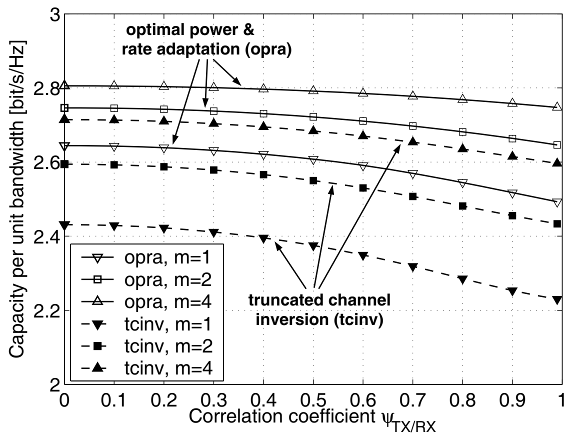

Fig. 2. Impact of spatial correlation and the fading parameter m on the capacity for $N_{RX} = 1$, $N_{TX} = 2$, $R_C = 1$ (Alamouti) and $\bar{\gamma} = 8\,\text{dB}$.

seen from Fig. 2, the impact of spatial correlation is generally relatively small and slightly decreasing with increasing values of m. Furthermore, the differences between both policies are diminishing with increasing m, since this leads similarly to the case that more antennas are used to less severe fading on the equivalent SISO channels. It should also be noted that in case of fully correlated channels (i.e., $\rho_{\mathrm{TX/RX}} = 1$), the achievable capacity is always identical to the capacity of a corresponding SISO channel with the same adaptation policy and fading level, because in this case there is no spatial diversity available.

Fig. 3 depicts the impact of spatial correlation on the outage probability for optimal power and rate adaptation as well as truncated channel inversion. Obviously, spatial correlation has a significant influence on P_{out}, which is generally increasing with increasing values of $\rho_{\mathrm{TX/RX}}$. This is because in presence of spatial correlation the post-processing SNR γ experiences more serious variations, i.e., the probability that γ falls below a given threshold is increasing. It can also be seen that the outage probability of optimal power and rate adaptation is generally much smaller than for truncated channel inversion even though the achievable capacity is always (at least slightly) higher.

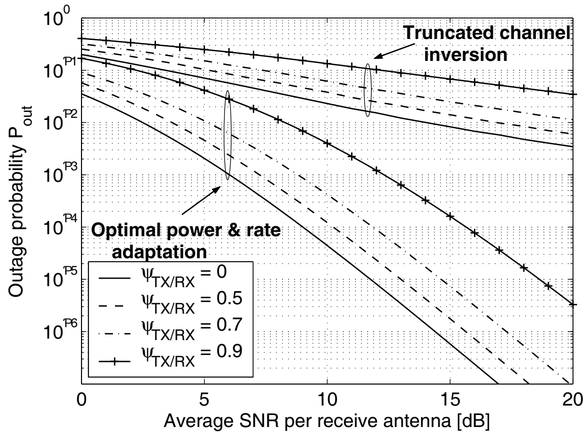

Fig. 3. Impact of spatial correlation on the outage probability for $m = 1$ (Rayleigh fading), $N_{TX} = N_{RX} = 2$ and $R_C = 1$ (Alamouti).

VI. Conclusion

We have analyzed the capacity of MIMO systems employing orthogonal STBC with partial CSI at the transmitter in spatially correlated Nakagami-m fading channels for different power and rate adaptation strategies. It has been shown that the impact of spatial correlation on the capacity is generally relatively small and further decreasing with increasing diversity order of the system. However, we have noticed that spatial correlation might lead to a significant increase of the corresponding outage probabilities. Finally, it has been shown that the capacity differences between the considered adaptation strategies are generally rather small and become almost negligible for systems offering a high diversity order.

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