Adaptive Modulation for MIMO Spatial Multiplexing Systems with Zero-Forcing Receivers in Semi-Correlated Rayleigh Fading Channels

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ABSTRACT

We present and analyze a rate-adaptive multiple-input multiple-output (MIMO) spatial multiplexing system with linear zero-forcing (ZF) receiver in a semi-correlated flat Rayleigh fading channel with spatial correlation at the transmitter-side only. The goal is to maximize the average spectral efficiency (ASE) while satisfying a given bit error ratio (BER) constraint. We consider both continuous and discrete rate adaptation subject to an instantaneous as well as an average BER constraint. Furthermore, we investigate the benefit of using a linear prefilter at the transmitter, which is supposed to optimize transmission by exploiting knowledge of the spatial correlation properties of the channel for mapping a certain number of subchannels to the available transmit antennas. It is shown that the proposed system is capable of realizing very high ASEs and that linear prefiltering might be beneficially exploited for further improving the performance, particularly in highly correlated environments.

Categories and Subject Descriptors: C.2.1 [Computer-Communication Networks]: Network Architecture and Design – *Wireless communication*

Keywords: MIMO systems, adaptive modulation, spatial multiplexing, zero-forcing receiver, spatial correlation, linear transmit prefiltering

1. INTRODUCTION

Multiple antenna systems in conjunction with sophisticated link adaptation techniques, which dynamically adjust various transmission parameters such as the modulation level, code rate or transmission power to the time-varying channel conditions, represent a promising approach for realizing considerably high bit rates for future wireless communications [1]. Ideally, an appropriate link adaptation scheme selects a transmission mode with a low spectral efficiency if the channel conditions are poor whereas high spectral efficiencies can be realized in rather good channel states. This way, the performance compared to non-adaptive systems can be significantly improved since the latter ones are usually designed such that even in the worst possible channel conditions a certain BER target is not exceeded. One of the most prominent link adaptation techniques is adaptive modulation, where depending on the current signal-to-noise ratio (SNR) an appropriate modulation scheme is chosen. Adaptive modulation for conventional single-input single-output channels has been an active field of research for years [2, 3]. Only recently, also research activities related to adaptive transmission methods in MIMO systems have been intensified, which generally provide more degrees of freedom due to the additional spatial dimension. In [4] and [5], for example, adaptive modulation in conjunction with orthogonal space-time block codes has been investigated whereas in [6] it is combined with a two-dimensional beamformer.

In this paper, we present and analyze a rate-adaptive MIMO spatial multiplexing system with ZF receiver in semi-correlated Rayleigh fading channels with spatial correlation at the transmitter-side only. In this regard, the adaptation is performed based on several different M-ary quadrature amplitude modulation (M-QAM) signal point constellations. We consider both, continuous and discrete rate adaptation, where we differentiate in the latter case between discrete rate adaptation subject to an instantaneous and an average BER constraint, respectively. For each case, we calculate the ASEs and BERs and we provide some numerical results, indicating the performance that might be achieved in different scenarios and for various parameter settings.

The remainder of this paper is structured as follows: In Section 2, we outline our system and channel model as well as the structure of the considered linear transmit prefilter. In Section 3, we introduce the proposed rate-adaptation strategies and we analyze their performance in terms of the achievable ASEs and BERs, respectively. Numerical results and a short discussion can be found in Section 4 before we finally give our conclusions in Section 5.

2. SYSTEM AND CHANNEL MODEL

We consider a MIMO system with N_{TX} transmit and N_{RX} receive antennas as illustrated in Fig. 1. At the transmitter, a linear $N_{TX} \times L$ prefilter matrix $\mathbf{F}$ is used for mapping $L \leq N_{TX} \leq N_{RX}$ independent subchannels to the available transmit antennas, via which the resulting signals are subsequently transmitted over a frequency-flat Rayleigh-

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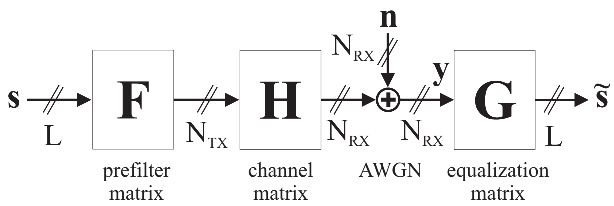

Figure 1: Schematic System Model

fading channel. In this regard, the discrete-time equivalent low-pass representation of the channel is modeled by the $N_{RX} \times N_{TX}$ channel matrix $\mathbf{H}$, where the (m, n)-th entry $[\mathbf{H}]_{m,n}$ denotes the channel coefficient between the n-th transmit and m-th receive antenna, respectively. The input-output relationship of our system is consequently given as

$$\mathbf{y} = \sqrt{\bar{\gamma}/N_{TX}} \cdot \mathbf{HFs} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ denotes the $L \times 1$ data symbol vector with elements having unity average symbol energy, $\mathbf{H}$ the already mentioned channel matrix, $\mathbf{y}$ the $N_{RX} \times 1$ receive symbol vector and $\mathbf{n}$ an additive white Gaussian noise (AWGN) vector, whose elements are i.i.d. circularly symmetric complex Gaussian random variables (CSCGRV) with zero mean and variance one. $\bar{\gamma}$ represents the average SNR per receive antenna. We assume that the MIMO channel is Rayleigh-fading and has Kronecker covariance structure with spatial correlation at the transmitter-side only. In this case, $\mathbf{H}$ can be expressed as $\mathbf{H} = \mathbf{H}_w \, \mathbf{R}_{TX}^{1/2}$, where $\mathbf{H}_w$ is a $N_{RX} \times N_{TX}$ matrix made up of i.i.d. CSCGRVs with zero mean and unity variance and the $N_{TX} \times N_{TX}$ matrix $\mathbf{R}_{TX}^{1/2}$ denotes a matrix root of the transmit correlation matrix $\mathbf{R}_{TX}$. For simplicity, we assume an exponential correlation model, i.e., the (m,n)-th entry of $\mathbf{R}_{TX}$ is given as $[\mathbf{R}_{TX}]_{m,n} = \rho^{|m-n|}$, where ρ with $|\rho| \leq 1$ designates a correlation coefficient, which can be used for adjusting the degree of correlation. Specifically, $\rho = 0$ corresponds to an uncorrelated and $\rho = 1$ to a fully correlated channel. The assumption of spatial correlation at the transmitter-side only is approximately fulfilled in urban scenarios, for example, where a base station (transmitter) is mounted on top of the roofs whereas the mobile terminals (receivers) are located in rich-scattering environments due to surrounding buildings and other obstacles.

Throughout this paper, we consider a statistical transmit prefilter matrix $\mathbf{F}$ as proposed in [7, 8]. This prefilter is based upon the transmit correlation matrix $\mathbf{R}_{TX}$ only—which is subsequently always assumed to be perfectly known to the transmitter—and optimizes transmission for ZF receivers in spatially correlated channels in terms of minimizing the symbol error rate for high SNRs by transmitting data on the long-term eigenmodes of the channel. Generally, using a prefilter that requires only knowledge of the statistical properties of the channel rather than full instantaneous channel knowledge is beneficial in several ways: On the one hand, estimation of the statistical properties can be performed in a relatively wide time window, so that the accuracy of the estimates is usually quite good. On the other hand, the statistical properties usually change at a comparatively slow pace so that only infrequent updates of the prefilter structure are sufficient, what reduces both the feedback load and the computational requirements.

As has been shown in [7] and [8], the structure of the considered prefilter is given as

$$\mathbf{F} = \tilde{\mathbf{V}} \cdot \Phi \cdot \mathbf{D}_L, \tag{2}$$

where $\mathbf{D}_L$ is the $L \times L$ normalized discrete Fourier transform matrix, i.e., the (m, n)-the entry is given as $[\mathbf{D}_L]_{m,n} = \frac{1}{\sqrt{L}} \, e^{-j\frac{2\pi mn}{L}}$, and $\tilde{\mathbf{V}}$ is an $N_{TX} \times L$ matrix containing the eigenvectors corresponding to the L strongest eigenvalues of $\mathbf{R}_{TX}$ as column vectors. These eigenvectors can be obtained from the matrix decomposition

$$\mathbf{R}_{TX} = \begin{bmatrix} \mathbf{V} & \tilde{\mathbf{V}} \end{bmatrix} \cdot \begin{bmatrix} \Lambda & \\ & \tilde{\Lambda} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V} & \tilde{\mathbf{V}} \end{bmatrix}^H, \tag{3}$$

where $\tilde{\Lambda}$ is a diagonal matrix with the L largest eigenvalues of $\mathbf{R}_{TX}$ as diagonal elements and $\mathbf{X}^H$ denotes the conjugate-transpose of an arbitrary matrix $\mathbf{X}$. Moreover, the power allocation matrix Φ is given as

$$\Phi = \sqrt{\frac{\zeta}{\operatorname{tr}\left(\tilde{\Lambda}^{-1/2}\right)}} \cdot \tilde{\Lambda}^{-1/4}, \tag{4}$$

with transmit power constraint ζ, which is always set to $\zeta = N_{TX}$ in the following. In case that no prefilter is used, we simply set $L = N_{TX}$ and $\mathbf{F} = \mathbf{I}_{N_{TX}}$.

At the receiving-end, we subsequently focus exclusively on the case that linear zero-forcing detection is performed. Hence, the received symbol vectors $\mathbf{y}$ are always multiplied by the equalization matrices $\mathbf{G} = \left((\mathbf{HF})^H \, \mathbf{HF}\right)^{-1} (\mathbf{HF})^H$ and in the simplest case—which we consider here—hard decisions are made upon the elements of the post-detection symbol vector $\tilde{\mathbf{s}} = \mathbf{Gy}$. Please note that we assume throughout this paper that the receiver has at all time instants perfect knowledge of the channel and prefilter matrices $\mathbf{H}$ and $\mathbf{F}$, respectively. It has been shown in [9] that the probability density function of the post-detection SNR γ_k on the k-th subchannel at the output of a ZF receiver in presence of transmit correlation only corresponds to a gamma distribution and is given for $\gamma_k \geq 0$ as

$$p(\gamma_k) = \frac{N_{TX}}{\bar{\gamma}\beta_k \cdot \Gamma(D)} \, \exp\left(-\frac{N_{TX}\,\gamma_k}{\bar{\gamma}\,\beta_k}\right) \left(\frac{N_{TX}\,\gamma_k}{\bar{\gamma}\,\beta_k}\right)^{D-1}, \tag{5}$$

where $D = N_{RX} - L + 1$ is the diversity order of the system, $\Gamma(\cdot)$ the well-known Gamma function [10] and

$$\beta_k = \frac{1}{\left[\left(\mathbf{F}^H \, \mathbf{R}_{TX} \, \mathbf{F}\right)^{-1}\right]_{k,k}}. \tag{6}$$

The corresponding cumulative distribution function is

$$F(\gamma_k) = \int_0^{\gamma_k} p(x)\, dx = 1 - \frac{\Gamma\left(D, \frac{N_{TX}\,\gamma_k}{\bar{\gamma}\,\beta_k}\right)}{\Gamma(D)}, \tag{7}$$

with $\Gamma(\cdot, \cdot)$ as the upper incomplete Gamma function [10].

3. ADAPTIVE MODULATION

We propose a rate-adaptive system, where the selection of an appropriate modulation scheme is done on a per-subchannel basis with the goal to maximize the ASE without exceeding a certain target BER δ_0. Only the modulation order M is dynamically adjusted while the total transmit power remains constant. We consider two different cases, namely continuous and discrete rate adaptation, respectively. Continuous rate adaptation means that the number of bits per symbol does not necessarily have to be an integer value. An approach for realizing virtually arbitrary (rational) modulation orders has been presented in [11]. However, such techniques are rarely used in practice due to their complexity and the results presented here serve therefore mainly as a reference, showing what theoretically can be achieved by means of adaptive modulation in general. If discrete rate adaptation is used, on the other hand, only a finite set of modulation orders might be chosen, what is usually the preferred solution in practical systems.

3.1 Continuous Rate Adaptation

Generally, the BER of an AWGN channel with M-QAM modulation, Gray coding and perfect clock and carrier recovery—what is subsequently always assumed—can be reasonably approximated for a given value of $\xi = E_s/N_0$, where E_s denotes the average symbol energy and N_0 the mean noise power, as [6]

$$P_b(M, \xi) \approx \frac{1}{5} \cdot \exp\left(-\frac{3\,\xi}{2\,(M-1)}\right). \tag{8}$$

This approximation represents a tight upper bound for $M \geq 4$ and $P_b \leq 10^{-2}$, which is the BER region we are usually interested in [3]. Since for continuous rate adaptation any positive rational number is a valid value for M, the maximum modulation order that might be used for a given (instantaneous) post-processing SNR γ_k without exceeding the BER constraint δ_0 can easily be determined as

$$M(\gamma_k) = 1 - \frac{3\,\gamma_k}{2 \cdot \ln(5\,\delta_0)}. \tag{9}$$

Assuming ideal Nyquist impulses for the transmitted data symbols, i.e., that the system bandwidth B is given as $B = 1/T_S$ with T_S as the symbol duration, the ASE Υ of our system consequently can be calculated as

$$\Upsilon = \sum_{k=1}^{L} \frac{E[\log_2(M(\gamma_k))]}{T_S \cdot \frac{1}{T_S}} = \sum_{k=1}^{L} I_{2,k}, \tag{10}$$

where

$$I_{2,k} = \frac{1}{\ln 2} \cdot \int_0^\infty \ln\left(M(\gamma_k)\right)\, p(\gamma_k)\, d\gamma_k. \tag{11}$$

Using (5) and (9), this integral can be rewritten as

$$I_{2,k} = \frac{1}{\ln(2)} \cdot \int_0^\infty \ln(1 + b\,\gamma_k)\, \frac{a_k}{\Gamma(D)}\, e^{-a_k \gamma_k}\, \left(a_k \gamma_k\right)^{D-1}\, d\gamma_k, \tag{12}$$

where we introduced for brevity the short-hand notations

$$a_k = \frac{N_{TX}}{\bar{\gamma} \cdot \beta_k} \tag{13}$$

$$b = \frac{-3}{2 \cdot \ln(5\,\delta_0)}. \tag{14}$$

Performing the substitution $x_k = b\,\gamma_k$ and using the result from [12] that

$$\int_0^\infty \ln(1+x)\, \frac{a^n}{\Gamma(n)}\, x^{n-1}\, e^{-ax}\, dx = \mathcal{P}_n(-a)\, E_1(a) + \sum_{k=1}^{n-1} \frac{1}{k}\, \mathcal{P}_k(a)\, \mathcal{P}_{n-k}(-a), \tag{15}$$

where $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}\, dt$ is the exponential integral and $\mathcal{P}_D(x) = \sum_{\eta=0}^{D-1} \frac{x^\eta}{\eta!}\, e^{-x}$ the Poisson distribution, we can directly solve (12) as

$$I_{2,k} = \frac{\mathcal{P}_D\left(-\frac{a_k}{b}\right)\, E_1\left(\frac{a_k}{b}\right)}{\ln(2)} + \sum_{\nu=1}^{D-1} \frac{\mathcal{P}_\nu\left(\frac{a_k}{b}\right)\, \mathcal{P}_{D-\nu}\left(-\frac{a_k}{b}\right)}{\nu \cdot \ln(2)}. \tag{16}$$

Combining (16) with (10) then leads to the desired result for the ASE of our rate-adaptive system in case of continuous rate adaptation. Clearly, both the instantaneous as well as the average BER always correspond to the target BER δ_0 if there are no restrictions on the modulation orders M.

3.2 Discrete Rate Adaptation

In case of discrete rate adaptation, we restrict the available modulation orders to a finite set. For our analysis, we specifically consider modulation orders $M_n = 2^n$ with $n = 1, 2, \ldots N$, where n corresponds to the number of bits per constellation symbol. For selecting an appropriate M_n, we subdivide the SNR range on each subchannel in $N+1$ bins bounded by the switching thresholds $\theta_{k,i}$ ($k = 1, \ldots, L$; $i = 1, \ldots, N+1$), and associate the i-th bin with modulation order M_i, i.e., if $\theta_{k,i} \leq \gamma_k < \theta_{k,i+1}$, we use M_i for transmission on the k-th subchannel. If $\gamma_k < \theta_{k,1}$, data transmission is suspended on the corresponding subchannel since in this case the respective BER constraint cannot be met. Furthermore, please note that for our considerations the thresholds $\theta_{k,N+1}$ are always set to infinity, i.e., $\theta_{k,N+1} = \infty\ \forall\ k$.

Subsequently, we distinguish two different rate-adaptation policies, which are based either upon on an instantaneous or an average BER constraint, respectively. The only difference between both approaches is basically just the way how the switching thresholds $\theta_{k,i}$ are chosen. In the first case, the criterion is to keep the instantaneous BER at *all* time instants below a given target BER δ_0 whereas in the second case only the overall average BER has to be smaller than or equal to δ_0. In the following, we first of all analyze discrete rate adaptation systems for an arbitrary set of switching thresholds $\theta_{k,i}$ in general and only at the end of this section we show how the corresponding thresholds should be calculated under the two considered BER constraints.

Generally, the ASE Υ of our discrete rate-adaptive system is given as (assuming ideal Nyquist impulses again)

$$\Upsilon = \sum_{k=1}^{L} \sum_{i=1}^{N} i \cdot p_{k,i}, \tag{17}$$

where $p_{k,i} = \int_{\theta_{k,i}}^{\theta_{k,i+1}} p(\gamma_k)\, d\gamma_k$ denotes the probability that the instantaneous SNR on the k-th subchannel falls into the i-th bin. This probability can be expressed as

$$p_{k,i} = \mathcal{P}_D\left(a_k\, \theta_{k,i}\right) - \mathcal{P}_D\left(a_k\, \theta_{k,i+1}\right), \tag{18}$$

where we made use of (7) and (13) and the fact that $\Gamma(m, \mu) = \Gamma(m) \cdot \mathcal{P}_m(\mu)$ for positive integer values of m [10].

The average BER $P_{b,avg}$ is generally given as the average number of bits in error divided by the total number of transmitted bits [3]. Hence, it can be calculated as

$$P_{b,avg} = \frac{\sum_{k=1}^{L} \sum_{i=1}^{N} i \cdot P_{b,avg,i,k}}{\sum_{k=1}^{L} \sum_{i=1}^{N} i \cdot p_{k,i}}, \tag{19}$$

where $P_{b,avg,i,k}$ denotes the average BER on the k-th subchannel when the SNR falls into the i-th bin, given as

$$P_{b,avg,i,k} = \int_{\theta_{k,i}}^{\theta_{k,i+1}} P_b(M_i, \gamma_k) \cdot p(\gamma_k)\, d\gamma_k, \qquad (20)$$

with $P_b(M, \xi)$ as the bit error probability of an AWGN channel for modulation order M and SNR ξ. In the following, we restrict to considering square ($M_n = 2^{2i}$) and rectangular ($M_n = 2^{2i+1}$) signal point constellations only, but an extension of our results to other constellations is straightforward. Similar to the case of continuous rate adaptation, we approximate the BER of square M-QAM constellations by (8), but for obtaining more reliable results, we use an even better approximation for the rectangular constellations, which is tight for $P_b \leq 10^{-2}$ and $M \leq 256$ and given as [6]

$$P_{b,\mathrm{rect}}(M, \xi) \approx \frac{1}{5} \cdot \exp\left(-\frac{6\,\xi}{5M-4}\right). \qquad (21)$$

Using (8) and (21), $P_{b,avg,i,k}$ hence can be approximated as

$$\begin{aligned} P_{b,avg,i,k} \;\approx\; & \frac{1}{5} \cdot \left(\frac{a_k}{a_k + c_i}\right)^D \left[\mathcal{P}_D\left((a_k + c_i)\,\theta_{k,i}\right)\right. \\ & \left. -\mathcal{P}_D\left((a_k + c_i)\,\theta_{k,i+1}\right)\right] \end{aligned} \qquad (22)$$

with

$$c_i = \begin{cases} \frac{3}{2\cdot(2^i-1)} & \text{if } i \text{ is even (square } M\text{-QAM)} \\ \frac{6}{5\cdot 2^i - 4} & \text{if } i \text{ is odd (rect. } M\text{-QAM)} \end{cases} \qquad (23)$$

and a_k according to (13). Plugging (22) together with (18) in (19) then yields the desired expression for calculating the average BER in case of discrete rate adaptation for a general set of switching thresholds $\theta_{k,i}$.

3.2.1 Thresholds for Instantaneous BER Constraint

If the adaptation should be done subject to an instantaneous BER constraint, the instantaneous BER always has to be smaller than or equal to the target BER δ_0. Hence, based on (8) and (21), it is straightforward to determine the switching thresholds $\theta_{k,i}$ as

$$\theta_{k,i} = \begin{cases} -\frac{2}{3} \cdot \ln(5\,\delta_0) \cdot (2^i - 1) & \text{for } i \text{ even}, i \leq N \\ -\frac{1}{6} \cdot \ln(5\,\delta_0) \cdot (5 \cdot 2^i - 4) & \text{for } i \text{ odd}, i \leq N \\ \infty & \text{for } i = N+1 \end{cases}. \qquad (24)$$

Please note that in this case the *average* BER is generally smaller than δ_0 because the thresholds are chosen such that the target BER is achieved for all SNRs falling into one specific bin and hence particularly also for the smallest one. However, since in practical systems usually only the average BER is of interest, this leads to an unnecessary loss of spectral efficiency. Therefore, a putatively better approach is to impose an average BER constraint instead.

3.2.2 Thresholds for Average BER Constraint

In case of discrete rate adaptation based on an average BER constraint, we want to maximize the ASE Υ under the constraint that $P_{b,avg} \leq \delta_0$. Defining the set of adjustable thresholds (aside from the fixed thresholds $\theta_{k,N+1}=\infty$) as $\Theta = \{\theta_{k,i} | k=1,\ldots,L; i=1,\ldots,N\}$, our optimization problem consequently can be written as

$$\Theta_{\mathrm{opt}} = \arg\max_{\Theta} \sum_{k=1}^{L} \sum_{i=1}^{N} i \cdot \left(\mathcal{P}_D\left(a_k\,\theta_{k,i}\right) - \mathcal{P}_D\left(a_k\,\theta_{k,i+1}\right)\right) \qquad (25)$$

subject to $P_{b,avg} \leq \delta_0$. A standard approach for solving this problem would be to make use of Lagrange multipliers, but due to the structure of both the objective function and the inequality constraint, an analytical solution is extremely difficult if not impossible to derive. Therefore, it is preferable to rely upon numerical optimization methods. However, since in practical systems the switching thresholds ideally should be computed in almost real-time in order to facilitate a fast adaptation to changing channel conditions, searching the global optimum vector Θ_{opt} is considered to be prohibitively complex. For that reason, we propose a sub-optimum solution in the following, which has proven to lead to almost the same ASE as the numerically determined optimum at a significantly lower complexity while the average BER constraint can still be met.

It can easily be seen from (19) that a sufficient condition for satisfying the average BER constraint δ_0 is $P_{b,avg,i,k} \leq \delta_0 \cdot p_{k,i}\ \forall\ (k,i)$, or equivalently using (18) and (22)

$$\frac{\mathcal{P}_D((a_k + c_i)\,\theta_{k,i}) - \mathcal{P}_D((a_k + c_i)\,\theta_{k,i+1})}{\mathcal{P}_D(a_k\,\theta_{k,i}) - \mathcal{P}_D(a_k\,\theta_{k,i+1})} \leq \frac{\delta_0}{\epsilon_{k,i}}, \qquad (26)$$

where we introduced for brevity the short-hand notation

$$\epsilon_{k,i} = \frac{1}{5} \cdot \left(\frac{a_k}{a_k + c_i}\right)^D. \qquad (27)$$

Since we want to maximize the ASE, we always try to achieve equality in (26) if possible, i.e., we aim to choose the $\theta_{k,i}$ such that $P_{b,avg,i,k} = \delta_0\, p_{k,i}\ \forall\ (k,i)$. Hence, the problem of optimizing the switching thresholds in case of an average BER constraint reduces to a standard nulling problem, for which a wide variety of efficient numerical methods exist. For determining the individual thresholds, we start with $\theta_{k,N}$ and exploit that $\theta_{k,N+1}=\infty$ and $\lim_{x\to\infty} \mathcal{P}_m(x) = 0$. After having determined $\theta_{k,N}$, we use this value for calculating $\theta_{k,N-1}$ and so on. However, please note that achieving equality in (26) is not always feasible with $\theta_{k,i} \geq 0$. If this is the case, the corresponding thresholds are simply set to zero, i.e. the associated modulation levels are completely switched off and at all time instants higher modulation orders are used.

At this point, it should also be noted that in case of adaptation subject to an average BER constraint, we generally require—aside from the higher computational complexity—knowledge of the average SNR $\bar{\gamma}$ and the transmit correlation matrix $\mathbf{R}_{TX}$ at the transmitter for calculating the switching thresholds $\theta_{k,i}$. This implies on the one hand that these parameters must be available at the transmitter (e.g., using a feedback channel), and on the other hand that if one of these parameters changes the thresholds have to be readjusted. In case of adaptation subject to an instantaneous BER constraint, in contrast, the thresholds are fixed and independent of the statistical properties of the channel. Consequently, the required effort under an average BER constraint is generally much higher than for an instantaneous BER constraint.

4. NUMERICAL RESULTS

Fig. 2 shows the achievable ASE for continuous and discrete rate adaptation subject to both an instantaneous and an average BER constraint for $N = \{4, 6, 8\}$, $\delta_0 = 10^{-3}$, $N_{TX} = N_{RX} = L = 2$, and $\rho = 0$, i.e., we do not consider any spatial correlation here. As can be seen, in case of discrete rate adaptation, selecting the switching thresholds subject

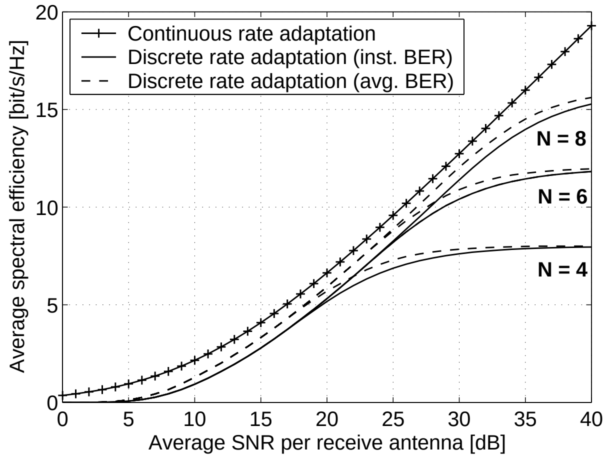

Figure 2: Average spectral efficiency for $\mathbf{N_{TX} = N_{RX} = L = 2}$, $\boldsymbol{\delta_0 = 10^{-3}}$, and $\boldsymbol{\rho = 0}$ with discrete and continuous rate adaptation.

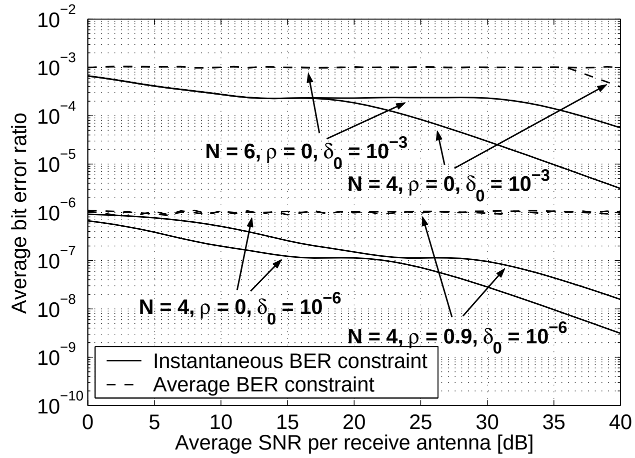

Figure 3: Average BER for $\mathbf{N_{TX} = N_{RX} = L = 2}$, $\mathbf{N = \{4, 6\}}$, $\boldsymbol{\delta_0 = \{10^{-3}, 10^{-6}\}}$, and $\boldsymbol{\rho = \{0, 0.9\}}$ with discrete rate adaptation.

to an average BER constraint leads for moderate SNRs to an improvement of about 1 dB compared to adaptation subject to an instantaneous BER constraint, but for very small or very high SNRs, there is virtually no difference between both approaches. This is quite clear since in these cases transmission is either always suspended (low SNR regime) or always the highest available modulation scheme is used (high SNR regime). However, as the gain from imposing an average instead of an instantaneous BER constraint is generally relatively small, it is questionable whether the additional complexity involved with this approach is worthwhile in practice. We also note that the ASE difference between continuous and discrete rate adaptation is comparatively small, except for very low and very high SNRs, where we get a significant gap. This is because the set of available modulation orders is limited in case of discrete rate adaptation. Most importantly, however, it can be seen that the achievable ASEs are generally rather high, exceeding 10 bits/s/Hz for $N = 6$ and $\bar{\gamma} = 30$ dB, for example.

In Fig. 3, we consider the average BERs in case of discrete rate adaptation, which have been calculated based on the approximation given in (22) for different values of N, ρ, and δ_0. We observe that for adaptation subject to an average BER constraint we basically always get the target BER δ_0, except for very high SNRs, for which the BER is even with the highest available modulation level smaller than δ_0, hence also leading to a smaller average BER. This can be observed for $N = 4$, $\delta_0 = 10^{-3}$, and $\bar{\gamma} > 35$ dB, for example. Furthermore, it can be seen that adaptive modulation based on an instantaneous BER constraint generally leads to smaller average BERs than the target BER δ_0 due to the reasons already mentioned in Section 3. It can also be seen that transmit correlation ($\rho \neq 0$) causes a simple shift of the BER curve that we get without any spatial correlation ($\rho = 0$). This is quite obvious since the average SNR $\bar{\gamma}$ is always multiplied by the constant β_k according to (6), as can be seen from (13), resulting in an SNR penalty equal to $-10\log_{10}(\beta_k)$ on the k-th subchannel. Only for $\rho = 0$ we have $\beta_k = 1 \; \forall \; k$ and consequently no SNR penalty at all.

The influence of the number of subchannels L on the ASE is illustrated in Fig. 4 for discrete rate adaptation subject

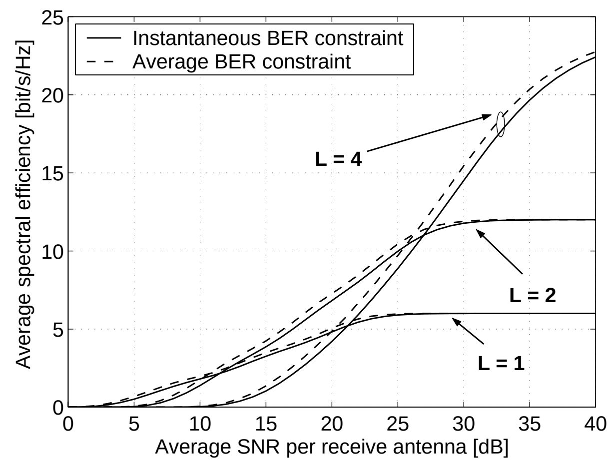

Figure 4: Influence of the number of subchannels L on the ASE for $\mathbf{N_{TX} = N_{RX} = 4}$, $\mathbf{N = 6}$, $\boldsymbol{\delta_0 = 10^{-6}}$, and $\boldsymbol{\rho = 0}$ with discrete rate adaptation.

to both an instantaneous and an average BER constraint, respectively. In this regard, we consider a system with four transmit and four receive antennas, an uncorrelated channel ($\rho = 0$) and a target BER of $\delta_0 = 10^{-6}$. Obviously, at low SNRs, using only one subchannel leads to the highest ASE whereas at high SNRs full spatial multiplexing, i.e. $L = \min(N_{TX}, N_{RX})$, is clearly superior. The performance advantage when using only few subchannels at low SNRs is due to the fact that in this case more transmit power is allocated to the individual subchannels, but also due to the increased diversity order D of the system compared to full spatial multiplexing. This interesting result suggests that aside from the modulation levels ideally also the number of active subchannels L should be dynamically adjusted based on the current channel conditions in order to maximize the overall ASE of our rate-adaptive system.

Finally, Fig. 5 depicts the impact of transmit correlation on the achievable ASE and shows the possible improvement that can be obtained by employing linear prefiltering at the

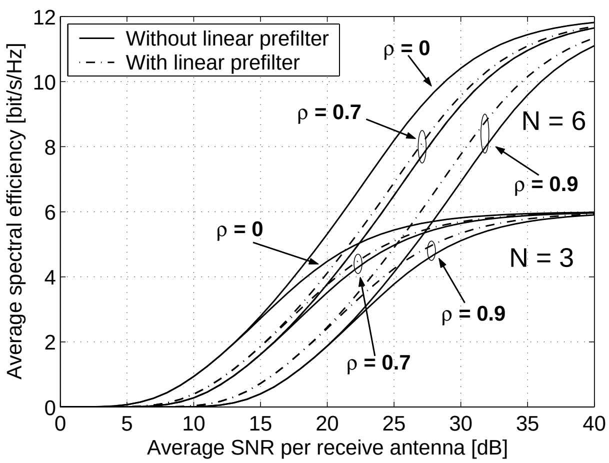

Figure 5: Impact of transmit correlation on the ASE for $N_{TX} = N_{RX} = L = 2$, $N = \{3, 6\}$, and $\delta_0 = 10^{-3}$ with discrete rate adaptation subject to an instantaneous BER constraint.

transmitter as introduced in Section 2. Similarly to the average BERs, transmit correlation leads to a simple shift of the corresponding ASE curves of the uncorrelated channel if the BER constraint is fixed. As can be seen by comparing the average SNRs required for achieving a certain ASE without using a linear prefilter (solid curves), for example, this shift is about 2.5 dB for $\rho = 0.7$ and 6.5 dB for $\rho = 0.9$. Consequently, high transmit correlation clearly degrades the performance of our system. By using the linear long-term prefilter according to (2), this performance degradation can be partially compensated. For $\rho = 0.7$, for example, the prefilter leads to a gain of up to 0.7 dB and for $\rho = 0.9$ to a gain of even up to 1.5 dB compared to the case without linear prefilter. Please note that for $\rho = 0$, i.e., for channels without any spatial correlation, the linear prefilter does not lead to any performance gain at all and therefore the corresponding ASE curves coincide in this case. Furthermore, for very high and very low SNRs, we achieve in all cases, i.e., with and without linear prefilter and for any value of ρ, basically the same ASE for a given set of available modulation schemes since—as already outlined before—in these cases transmission is either always suspended or always the highest available modulation order is used. Finally, it should be noted that the results shown in Fig. 5 correspond to the case that the adaptation is done subject to an instantaneous BER constraint, but the results look quite similar if an average BER constraint is taken into account instead.

5. CONCLUSION

We have presented and analyzed a rate-adaptive MIMO spatial multiplexing system with ZF receiver in Rayleigh fading channels with spatial correlation at the transmitter-side only. In this regard, we have considered both, continuous and discrete rate adaptation subject to either an instantaneous or an average BER constraint, respectively. Our results indicate that our rate-adaptive system can facilitate very high ASEs while guaranteeing a certain BER target even at low SNRs and without any additional coding. It became obvious that discrete rate adaptation subject to an average BER constraint slightly outperforms adaptation based on an instantaneous BER constraint at the expense of a considerably higher complexity. Therefore, it is questionable whether the additional effort is worthwhile in practice. Furthermore, it has been shown that transmit correlation might seriously degrade the performance, particularly at moderate SNRs, but linear prefiltering at the transmitter might be used for partially compensating this effect. Finally, we noted that ideally not only the modulation level but also the number of active subchannels should be dynamically adjusted in order to maximize the overall ASE. A detailed analysis of combined subchannel selection and rate adaptation is left for further studies.

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