

# Pilot-Assisted OFDM Channel Estimation and ICI Cancellation for Double Selective Channels

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**Abstract**—In this paper, we develop a two-stage hybrid channel estimation and ICI cancellation structure for OFDM systems. The double selective channels are approximated by an improved Basis Expansion Model (BEM), which is more accurate than the conventional BEM when the normalized Doppler frequency is smaller than one. Based on this improved model, a new ICI cancellation scheme is proposed to reduce ICI impact. Simulation results show that the framework performs well.

## INTRODUCTION

In mobile communications, the time domain channel impulse response vary with time, thus destroys the orthogonality of OFDM, and give rise to the inter-carrier interference among all the subcarriers.

Different approaches to reduce the inter-carrier interference (ICI) have been investigated widely, mainly time- or frequency-domain equalization [1, 2] and ICI cancellation [3]. However, with all methods time-domain channel information with high reliability must be provided firstly. Basis Expansion Model (BEM) can be applied to approximate time variant channels [4].

Based on the BEM, we basically only need to estimate a limited number of BEM coefficients. A time-domain optimal training structure for block transmissions has been designed in [4]. A frequency domain OFDM pilot assisted channel estimation is proposed in [5]. However, in [4] and [5], the frequency parameter  $\omega_q$  is set to a multiple of  $2\pi/K$ , here  $K$  is the BEM observation window length. This will reduce the accuracy of BEM channel model when  $f_{\max}KT_s < 1$ , where  $f_{\max}$  is the maximum Doppler spread and  $T_s$  is the sampling interval.

In this paper, we present a modified BEM to increase estimation accuracy. Furthermore, we introduce a new block-type OFDM channel estimation method, which overcomes the non-full rank problem in [5] while calculating the BEM coefficients. To this end, we adopt a new ICI cancellation scheme which will use the information from the channel estimation. According to the simulation results, the combined receiver can provide a good performance in both channel estimation and interference cancellation of the OFDM systems.

*Notation:* We use upper (lower) bold face letters to denote matrices (column vector). Superscripts  $(\cdot)^H$ ,  $(\cdot)^*$  and  $(\cdot)^+$  represent Hermitian, conjugate and pseudo inverse, respectively. We reserve  $E_x\{\cdot\}$  for expectation value with respect to the random variable  $x$  within the brackets.  $\otimes$  stands for Kronecker product,  $\lceil \cdot \rceil$  for integer ceiling,  $\lfloor \cdot \rfloor$  for integer

floor,  $\angle$  for the argument of a complex number.  $\mathbf{I}_N$  denotes the identity matrix of size  $N$  and  $\text{diag}\{\mathbf{x}\}$  is a diagonal matrix with vector  $\mathbf{x}$  on its diagonal. The discrete Fourier transform (DFT) matrix of size  $N \times N$  is given by  $[\mathbf{F}]_{m,k} = e^{(-j2\pi mk/N)/\sqrt{N}}$  ( $m, k \in 0, \dots, N-1$ ).

## I. SYSTEM MODEL

In this section, we first present our improvement on BEM. Then we introduce the OFDM system based on this modified double-selective channel models.

### A. Double-selective channel models

First we give the BEM to approximate the double-selective channels [4]. We define the maximum channel delay-spread as  $\tau_{\max}$  and the maximum Doppler spread as  $f_{\max}$ . Assuming the sampling interval  $T_s$  and the BEM observation window length  $K$ , we can accurately model the discrete time-varying impulse response of tap  $l$  in parsimonious finite parameters as follows:

$$h^{BEM}(n;l) = \sum_{q=-Q/2}^{Q/2} h_{q,l}^{BEM} e^{j\omega_q n}, n \in [0, N-1], l \in [0, L] \quad (1)$$

$$\omega_q := 2\pi q/K; L := \lceil \tau_{\max}/T_s \rceil; Q = 2\lceil f_{\max}KT_s \rceil$$

Here, we assume  $2f_{\max}\tau_{\max} < 1$ . Furthermore, we define  $\xi = f_{\max}KT_s$  as the channel varying speed. If  $\xi$  is smaller than one, the varying velocity of the original BEM can not match the velocity of the real channel. This will reduce the BEM accuracy. We propose a modified BEM in (2), in which  $\omega_q$  is replaced by  $\omega'_q$  for  $\xi < 1$ .

$$h(n;l) = \begin{cases} \sum_{q=-Q/2}^{Q/2} h_{q,l} e^{j\omega_q n}, 0 \leq n \leq K-1, \text{ if } f_{\max}KT_s \geq 1 \\ \text{where } Q = 2\lceil f_{\max}KT_s \rceil, \xi = 1, \omega_q = 2\pi\xi q/K \\ \sum_{q=-Q/2}^{Q/2} h_{q,l} e^{j\omega'_q n}, 0 \leq n \leq K-1, \text{ if } f_{\max}KT_s < 1 \\ \text{where } Q = 2\lceil f_{\max}KT_s \rceil, \xi = f_{\max}KT_s, \omega'_q = 2\pi\xi q/K \end{cases} \quad (2)$$

### B. Block Transmission for OFDM Systems

From [3], we know that the average channel impulse estimate of every multipath in time domain can be obtained by the IFFT of the channel frequency response on the pilot subcarriers. In order to get different  $Q+1$  estimates of all  $L+1$  multipaths in time-domain, we construct a block-form OFDM



$$\hat{\mathbf{h}}_{LS} = \Xi^T \mathbf{y}^{(p)} \quad (8)$$

$$\begin{aligned} \hat{\mathbf{h}}_{LMMSE} &= \mathbf{R}_h \Xi^H (\Xi \mathbf{R}_h \Xi^H + \mathbf{R}_\Psi + \mathbf{R}_n^{(p)})^{-1} \mathbf{y}^{(p)} \\ \mathbf{R}_\Psi &= E_{\mathbf{h}_s(d)} \{ \Psi \Psi^H \}, \mathbf{R}_n^{(p)} = E_{\mathbf{n}^{(p)}} \{ \mathbf{n}^{(p)} \mathbf{n}^{(p)H} \} \end{aligned} \quad (9)$$

Since we adopt the block transmission to obtain enough time domain channel estimates, the matrix  $\Xi$  has full rank except for poor channel situations.

### B. ICI Cancellation Receiver Design

In the conventional receivers for the consistent carrier frequency offset (CFO), we can simply multiply the conjugate of the estimated exponential frequency offset to overcome the ICI impact. Every basis function of the modified BEM is similar to a fixed channel amplitude multiplied by an exponent of the frequency offset. Hence, we combine the conventional consistent CFO cancellation method with the demodulation, and then propose a new receiver based on the modified BEM.

The ICI cancellation unit is separated into four stages. The detailed structure of the receiver is shown in Fig. 2. Since the processing of every sub-OFDM symbol is the same, we define  $\hat{\mathbf{y}}$  as the time domain received data and omit the subscript “ $i$ ” in the subsequent. The first stage is to calculate the time-domain phase compensation factor  $\hat{\xi}_{n,CP}$  and  $\hat{\xi}_{n,CH}$ . Here  $n$  indicates the time instant and the subscripts “ $CP$ ” and “ $CH$ ” the estimated source. The second stage is to build the compensation Matrix  $\tilde{\mathbf{D}}_{CP}$  and  $\tilde{\mathbf{D}}_{CH}$  using the information from stage 1. In stage 3 we use the windowed DFT-based method [7] to obtain the rotary demodulation coefficients. In the end, all the modules of the rotary channel coefficients are compared and the largest one is selected to fulfil the demodulation.

Assume the maximal Doppler frequency in one sub-OFDM symbol to be a consistent carrier frequency offset, a raw Doppler frequency index  $\hat{\xi}_{n,CP}$  can be estimated by the cyclic prefix (CP) [8].

$$\hat{\xi}_{n,CP} = \frac{n}{N} \angle \left[ \frac{1}{P-L} \sum_{a=L}^{P-L} \left( \frac{1}{Q+1} \sum_{i=0}^Q y_i^*(a) y_i(a+N) \right) \right] \quad (10)$$

Here,  $N$  is the subcarrier number in a sub-OFDM symbol and  $P$  is the length of the CP. We set the  $\hat{\xi}_{n,CP}$  to be equidistant at every time instant, so the Matrix  $\tilde{\mathbf{D}}_{CP}$  can be described as

$$\tilde{\mathbf{D}}_{CP} = \text{diag} \left[ (1, e^{j2\pi k_{CP} \hat{\xi}_{n,CP} / N}, \dots, e^{j2\pi k_{CP} \hat{\xi}_{N-1,CP} / N})^T \right] \quad (11)$$

$k_{CP}$  is a factor like the  $q$  in BEM and its value can take on  $\{-2, -1, -0.5, 0.5, 1, 2\}$ . So we have six different  $\tilde{\mathbf{D}}_{CP}$  to multiply the received data with, which are numbered from  $\tilde{\mathbf{D}}_0$  to  $\tilde{\mathbf{D}}_5$ .

The calculation methods for the  $\hat{\xi}_{n,CH}$  and  $\tilde{\mathbf{D}}_{CH}$  are as follows. The  $\hat{\xi}_{n,CH}$  is the accumulated phase discrimination between every two adjacent time-domain channel impulses. To reduce the number of matrices  $\tilde{\mathbf{D}}_{CH}$  and the complexity of the

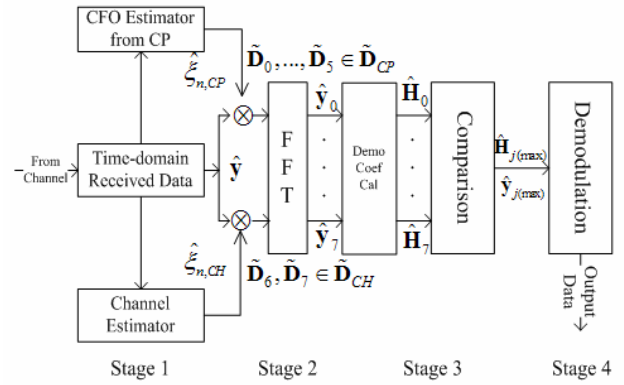


Figure 2. ICI Cancellation Receiver receiver, we only choose the two most powerful multipaths to calculate the  $\hat{\xi}_{n,CH}$  and the  $\tilde{\mathbf{D}}_{CH}$  (numbered from  $\tilde{\mathbf{D}}_6$  to  $\tilde{\mathbf{D}}_7$ ).

$$\hat{\xi}_{n,CH} = \sum_{n=0}^n \angle (\hat{h}_{n+1,l} / \hat{h}_{n,l}) \quad (12)$$

$$\tilde{\mathbf{D}}_{CH} = \text{diag} [ (1, e^{j2\pi \hat{\xi}_{n,CH} / N}, \dots, e^{j2\pi \hat{\xi}_{n,CH} / N})^T ] \quad (13)$$

Here  $\hat{h}_{n,l}$  is the estimated channel impulse response at time instant  $n$  from the two most powerful multipaths  $l$ .

After stage 3, eight optional received data ( $\hat{\mathbf{y}}_0$  to  $\hat{\mathbf{y}}_7$ ) can be obtained after multiplying compensation matrices  $\tilde{\mathbf{D}}_j$  ( $\tilde{\mathbf{D}}_0, \dots, \tilde{\mathbf{D}}_5$  from  $\tilde{\mathbf{D}}_{CP}$  and  $\tilde{\mathbf{D}}_6, \tilde{\mathbf{D}}_7$  from  $\tilde{\mathbf{D}}_{CH}$ ) and FFT. We consider the  $\hat{\mathbf{y}}_j (= \mathbf{F} \tilde{\mathbf{D}}_j \mathbf{y})$  as the new received frequency data (including the pilot and information) and use the simple DFT-based window [7] to obtain the rotary frequency domain channel coefficient. The DFT-based window frequency channel coefficients can be obtained by the following algorithm:

1) Use the Least Square (LS) method to get the rotary channel frequency response  $\tilde{\mathbf{H}}_{LS,j}^p$  of the pilot subcarriers. The superscript “ $p$ ” indicates the pilot position in every pilot cluster. Here  $k \in [0, \dots, M-1]$  and  $p \in [1, 2]$ .

$$\tilde{H}_{LS,j}^p(k) = \frac{y_j(L_p k + 2p - 1)}{s(L_p k + 2p - 1)}; k \in [0, \dots, M-1], p \in [1, 2] \quad (14)$$

$$\tilde{\mathbf{H}}_{LS,j}^p = [\tilde{H}_{LS,j}^p(0), \dots, \tilde{H}_{LS,j}^p(M-1)]^T$$

2) An IFFT of size  $M$  is performed on  $\tilde{\mathbf{H}}_{LS,j}^p$  and we calculate the average value  $\tilde{h}_j(n)$  of two groups of channel time responses.

$$\tilde{h}_{LS,j}^p(n) = \frac{1}{M} \sum_{m=0}^{M-1} \tilde{H}_{LS,j}^p(L_p m + 2p - 1) \exp(j2\pi \frac{mn}{M}) \quad (15)$$

$$\tilde{h}_j(n) = (\tilde{h}_{LS,j}^1(n) + \tilde{h}_{LS,j}^2(n)) / 2; n \in [0, 1, \dots, M-1]$$

3) Use FFT of size  $N$  to obtain the final channel frequency response  $\hat{H}_j(k)$  and select the information subcarriers based on  $\hat{H}_j(k)$ .

$$\hat{H}_j(k) = \sum_{l=0}^{L-1} \tilde{h}_j(l) \exp(-j2\pi kn/N); k \in [0, \dots, N-1] \quad (16)$$

Now from eight groups of optional  $\hat{\mathbf{y}}_j$  and  $\hat{\mathbf{H}}_j$ , we need to find the one which is influenced by the ICI at least. Since the mean power of the channel impulse response is fixed in one sub-OFDM symbol, the effect of the time variation on the channel frequency matrix is to spread the power on the non-diagonal positions. If the  $|\hat{H}_j(k)|$  is the maximal of all eight optional objects after the steps above, the rotary process has congregated more power on the diagonal elements of the channel frequency matrix. Then, for all the information subcarriers, the related  $\hat{\mathbf{y}}_{j(\max)}(k)$  from  $\hat{\mathbf{y}}_j$  and  $\hat{H}_{j(\max)}(k)$  are picked out to do demodulation.

$$\hat{H}_{j(\max)}(k) = \arg \left\{ \left| \hat{H}_j(k) \right| \right\}, 0 \leq k \leq N-1 \quad (17)$$

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we show some simulation results for the proposed BEM based channel estimation and ICI mitigation. Here the parameters of the OFDM system are as follows: the total number of the subcarriers is  $N = 256$  and cyclic prefix is  $P = 16$  in one sub-OFDM symbol. The length of cyclic prefix is larger than the maximum path delay. We have  $M = 16$  equidistant pilot clusters, each containing  $L_P = 6$  pilot subcarriers. In every pilot cluster, we set the second and fifth subcarrier to be the pilot data and the other to be the guard subcarriers. A six-path double selective channel generated by Jake's model is selected. The carrier frequency  $f_c$  is 5GHz and the sampling interval is  $T_s = 0.8\mu s$  (QPSK modulation). We normalize the maximum Doppler frequency  $f_{\max}$  to one sub-OFDM spacing as  $f_d = f_{\max} NT_s$ . Here we select five channel types, for which  $f_d$  equals to 0.0948, 0.142, 0.1896, 0.237 and 0.284. In all these situations,  $Q = 2$  is enough to build a double selective channel by the modified BEM model as in (2) in respect that  $f_{\max} KT_s < 1$ .

The first simulation addresses the analysis of the proposed modified BEM channel estimation based on block transmission. We compared our block type channel estimation algorithm with the algorithm proposed in [5] by simulations. To be fair, the channel model used in [5] has been replaced by the modified BEM. Two types of double selective channels are selected: (i)  $f_{d1} = 0.0948$ , (ii)  $f_{d2} = 0.1896$ . We use mean square error (MSE) in (18) to evaluate the performance. Here  $h_j(m, l)$  is the simulated channel generated by Jake's model and  $\bar{h}(m, l)$  is the channel estimation result based on modified BEM described in Part A of Section III.

$$MSE = \sum_{m=0}^{N-1} \sum_{l=0}^{L-1} E \left\{ [h_j(m, l) - \bar{h}(m, l)]^2 / N \right\} \quad (18)$$

Fig. 3 shows the MSE performance of the proposed channel estimation for different Doppler effects. Here we use  $Q + 1 = 3$

basis to rebuild the channel with the modified BEM (2). The results show that we can improve the performance of the original algorithm in [5] both in the high and low SNR regime. There the matrix  $\Xi$  in (8) and (9) is full rank. With the support of  $Q + 1$  sub-OFDM symbols as well as the modified BEM model, the method can rebuild the double-selective channel more accurately when  $f_{\max} KT_s < 1$ . The performance is also compared with the CRLB. It is clear that the performance of the Block-LMMSE is close to the CRLB.

The effect of the ICI cancellation based on the two types of phase compensation for the block transmission OFDM system is examined next. Fig. 4 shows the BER performances, for which  $f_d$  equals to 0.0948, 0.1896 and 0.284. It can be easily observed that the combined phase compensation algorithm significantly improves the system performance. Fig. 5 shows the bit error ratio (BER) for "CP" and "CH" based cancellation with  $\tilde{\mathbf{D}}_{CP}$  and  $\tilde{\mathbf{D}}_{CH}$ , individually. As can be seen, both the  $\tilde{\mathbf{D}}_{CP}$  from consistent carrier frequency offset hypothesis and the  $\tilde{\mathbf{D}}_{CH}$  from BEM channel estimation can improve the system performance. Obviously, the combined structure can provide further improvement over the single type of the phase compensation matrix.

### CONCLUSION

In this paper we have proposed a modified Basis Expansion Model (BEM), which is more accurate than the original BEM for double-selective channels when the normalized maximum Doppler frequency is below one. Based on this model, a block type OFDM channel estimation and an ICI cancellation algorithm are presented. By applying the BEM based channel estimation, we can obtain more accurate channel information, reduce the ICI caused by time-variant channels and improve the BER performance of the systems effectively.

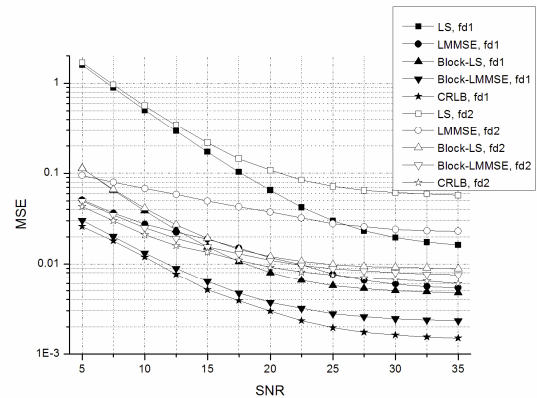


Figure 3. The channel estimation performances

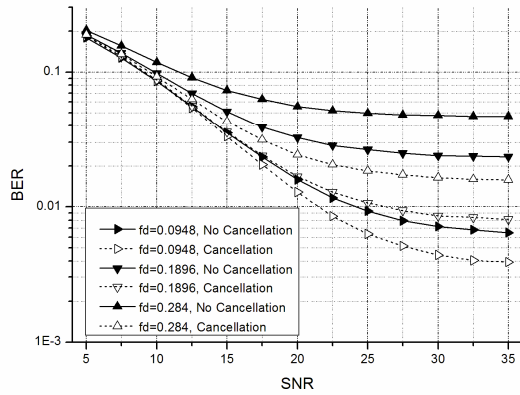


Figure 4. Performances of combined ICI cancellation

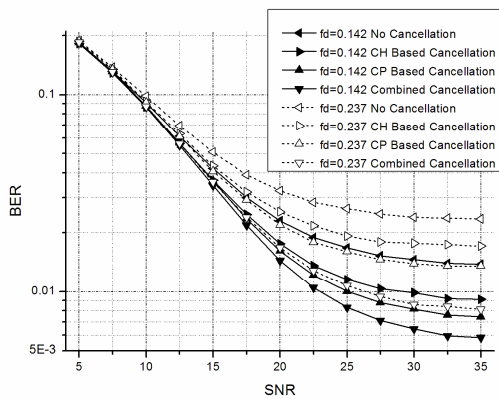


Figure 5. Performance of different compensation matrices

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