# Linear and Nonlinear Electronic Feed-Forward Equalizers for DQPSK

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***Abstract*— We investigate and compare linear and nonlinear electronic feed-forward equalizers for DQPSK transmission with direct detection with respect to their ability to increase chromatic and first order polarization mode dispersion tolerance.**

## I. Introduction

In recent years differential quadrature phase-shift keying (DQPSK) [1] has attracted quite some attention due to its higher spectral efficiency and reduced symbol rate compared to binary modulation formats such as conventional on-off keying (OOK) or differential binary phase-shift keying (DBPSK). In this respect, DQPSK is currently also considered as a potential candidate for future 100 Gbit/s Ethernet using only 50 Gbit/s equipment [2].

In this paper we investigate the application of linear and nonlinear electronic feed-forward equalizers (FFE) to increase the tolerance of DQPSK against chromatic (CD) and polarization mode dispersion (PMD). We show that a significant increase in dispersion tolerance can only be achieved by fractionally-spaced nonlinear FFE (NL-FFE).

The application of a nonlinear equalizer is motivated by the fact, that dispersion leads to nonlinear distortions in the electrical domain after photo detection (square law detection), which can not be completely equalized by a linear equalizer.

## II. Nonlinear Electronic Feed-Forward Equalization

The considered NL-FFE, which is based on the Volterra theory, has already been suggested in [3] for OOK, DBPSK and duobinary modulation formats. In principle the NL-FFE may equivalently be implemented as analog or digital filter.

Fig. 1 shows an example of an NL-FFE of order $N = 2$ (i.e., $N-1 = 1$ delay element) and nonlinear order $n = 2$. The difference compared to a linear equalizer is the nonlinear combination of the delayed samples $y_l$ . Thereby, it is possible to combat nonlinear distortions up to a certain degree depending on the filter order $N$ and the order of nonlinearity $n$. The nonlinear part of the equalizer presented in Fig. 1 is indicated by the dashed box. We introduce the term NL[$n$]-FFE[$N$] as a short-hand notation for the NL-FFE.

The complexity of an NL-FFE, i.e. the number of coefficients $K$ and multiplications $M$, is given by

$$K = \binom{N+n}{n} - 1 \quad \text{and} \quad M = 2K - N. \tag{1}$$

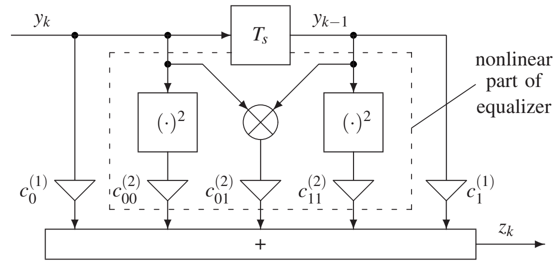


Fig. 1. Example of a nonlinear feed-forward equalizer of order $n = 2$

Defining the state vector

$$\mathbf{y}_k = [y_k, y_{k-1}, \ldots, y_{k-(N-1)}, y_k^2, y_k y_{k-1}, \ldots, y_{k-(N-1)}^2, \ldots, y_k^n, y_k^{(n-1)} y_{k-1}, \ldots, y_{k-(N-1)}^n]^T \tag{2}$$

and the coefficient vector

$$\mathbf{c} = [c_0^{(1)}, c_1^{(1)}, \ldots, c_{N-1}^{(1)}, c_{00}^{(2)}, c_{01}^{(2)}, \ldots, c_{(N-1)(N-1)}^{(2)}, \ldots, c_{0\ldots0}^{(n)}, c_{0\ldots01}^{(n)}, \ldots, c_{(N-1)\ldots(N-1)}^{(n)}]^T \tag{3}$$

it is easily shown that the output $z_k$ of the equalizer may be written as

$$z_k = \mathbf{c}^T \cdot \mathbf{y}_k. \tag{4}$$

From (4) one can conclude, that the output of the NL-FFE is linearly depending on its coefficients. As a consequence the determination of the coefficients according to the minimum mean squared error (MMSE) criterion may be done in the same way as for a linear FFE. Therfore, the condition

$$\mathbf{c}_{\text{opt}} = \arg\min_{\mathbf{c}} \mathrm{E}\left[e_k^2\right] = \arg\min_{\mathbf{c}} \mathrm{E}\left[(z_k - a_k)\right], \tag{5}$$

where $a_k$ denotes the reference data, results in the well known Wiener solution [4]

$$\mathbf{c}_{\text{opt}} = \mathbf{R}^{-1} \cdot \mathbf{p}. \tag{6}$$

Compared to the case of a linear FFE the autocorrelation matrix $\mathbf{R} = \mathrm{E}[\mathbf{y}_k \mathbf{y}_k^T]$ and the cross correlation vector $\mathbf{p} = \mathrm{E}[a_k \mathbf{y}_k]$ are now composed of moments up to the order of $2n$ instead of second order moments only. Moreover, $\mathbf{R}$ is no longer a Toeplitz matrix and might therefore in some cases not be invertible. However, the invertibility is usually assured if noise is taken into account.

From the fact, that the coefficient optimization according to the MMSE criterion results in the Wiener solution, it follows that

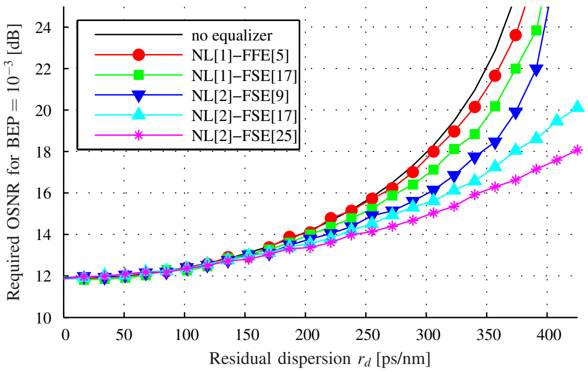


Fig. 2. Required OSNR for BEP $= 10^{-3}$ vs. residual dispersion $r_d$

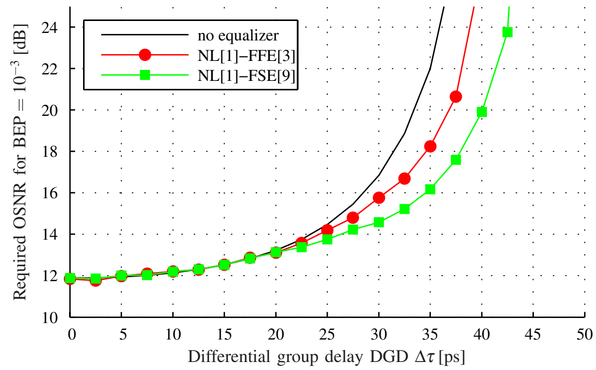


Fig. 3. Required OSNR for BEP $= 10^{-3}$ vs. differential group delay $\Delta\tau$.

adaptation algorithms like e.g. the least mean square algorithm (LMS) can be applied. Nevertheless, all results presented here were obtained with equalizer coefficients calculated according to (6). Please note that the considered NL-FFE is a generalization of the linear FFE, which is determined by $n = 1$.

## III. Simulation Results

The investigated DQPSK system uses a transmitter with two parallel Mach-Zehnder modulators and time-domain raised-cosine impulse shapers with a roll-off factor of 0.5 and a receiver comprising two Mach-Zehnder interferometers and two balanced detectors as in [1]. The considered bit rate is $R_b = 42.7$ Gbit/s, including 6.8% FEC-overhead. Each DQPSK tributary is processed independently by a separate equalizer. We consider a linear channel model with CD and first order PMD, thus neglecting fiber nonlinearities. Therefore, all results presented here may be scaled to any other data rate $R_b'$ by scaling the axes in Figs. 2 and 3 according to $r_d' = r_d \cdot \gamma^2$, $\Delta\tau' = \Delta\tau \cdot \gamma$ and $\mathrm{OSNR}' = \mathrm{OSNR} - 10\log_{10}\gamma$, where $\gamma = R_b/R_b'$. The residual dispersion $r_d$ quantifies the effect of CD, $\Delta\tau$ is the differential group delay (DGD), quantifying the effect of first order PMD, $R_s = 1/T_s = R_b/2$ is the symbol rate and $T_s$ is the symbol interval.

The optical receiver filter is a second order Gaussian band-pass with a 3-dB bandwidth of $1.2R_s$ and the electrical receiver filter is a third order Bessel lowpass with a 3-dB cut-off frequency of $1.0R_s$ . The filter bandwidths were optimized to achieve the lowest optical signal-to-noise ratio (OSNR) required for zero residual dispersion.

Fig. 2 shows the required OSNR to achieve a bit error ratio (BER) of $10^{-3}$ versus $r_d$. It can be observed, that the linear FFE is not specifically effective in compensating distortions caused by CD. This has already been observed for DBPSK [3]. The reason is that CD leads to nonlinear distortions in the electrical domain after square law detection, which may not be fully compensated by a linear equalizer. A linear fractionally-spaced feed-forward equalizer (FSE) with a tap-spacing of $T_s/2$ or even $T_s/4$ (where the latter is assumed for all FSE in this paper) does not improve the CD-tolerance considerably. Finally, it is shown in Fig. 2 that a fractionally-spaced NL-FFE with nonlinearities of the second order performs much better than the linear FFE; the 3dB-CD-tolerance is improved by about 30% by an NL[2]-FSE[25]. Further simulations have shown that an increase of the order of nonlinearity of $n > 2$ does not result in a significant performance improvement and does therefore not justify the considerably higher complexity according to (1). This is plausible considering that the photodiode is a second order nonlinearity.

Fig. 3 shows the required OSNR for BER $= 10^{-3}$ versus $\Delta\tau$. As the inter-symbol interference caused by first order PMD does only affect the two neighboring symbols as long as $\Delta\tau \le 2T_s$, only FFE with a total memory length of $3T_s$ are considered. Moreover, it is easily shown that first order PMD results in linear distortions in the electrical domain after square law detection. If nonlinear distortions introduced by the modulator and optical receiver filter are negligible, the application of an NL-FFE does not considerably improve the performance, which was also verified by numerical simulations. Consequently, Fig. 3 presents results for linear FFE only. One can observe that the 3dB-PMD-tolerance is increased by 10% for an FSE, whereas the improvement is less for an FFE.

## IV. Conclusions

We investigated the performance of linear and nonlinear electronic feed-forward equalizers for DQPSK with direct detection. It turned out that a linear FFE is not effective to compensate for neither CD nor PMD even if the filter taps are fractionally spaced. Opposed to that an NL-FFE increases the CD-tolerance by about 30%, whereas it does not increase the PMD-tolerance.

## Acknowledgement

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