Performance Analysis of Switch-and-Stay Transmit Diversity with Feedback Errors

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***Abstract*— We analyze the performance of switch-and-stay transmit diversity systems in presence of an erroneous feedback link. First of all, the analysis is conducted for arbitrary fading distributions and a rather general class of performance functions. Assuming a certain feedback error probability P_e, we determine the distribution of the signal-to-noise ratio (SNR) at the receiver-side and we provide a generic expression for the optimum switching threshold, which turns out to be independent of P_e. Furthermore, it is shown that the performance degradation due to feedback errors is always a linear function of the feedback error rate. Finally, we consider a concrete system as an example and we demonstrate how our generic results can be used to easily investigate the performance of this system in terms of the average SNR, the system capacity as well as the average bit error rate.**

I. Introduction

Switched diversity schemes are known to offer a good trade-off between complexity, hardware costs, and the capability to substantially mitigate the detrimental effects of multipath fading on the performance of wireless communication systems [1]–[5]. In case that only one out of several antenna elements may be used, it is clearly optimal to select always the antenna for which the signal-to-noise ratio (SNR) at the receiver-side is maximized, but this requires permanent estimation of the SNR that could be achieved with any antenna element and therefore might significantly increase the complexity compared to non-diversity schemes [1]. With switched diversity systems, in contrast, a certain antenna element is generally used as long as the induced signal quality is acceptable, i.e., as long as the corresponding SNR exceeds a certain threshold value. If the SNR falls below this threshold, different strategies might be realized, such as to simply switch to the next antenna element (switch-and-stay) or to switch to the next antenna element for which the induced SNR is acceptable (switch-and-examine).

The performance analysis and optimization of switched diversity schemes have attracted a lot of research attention during the past decade, see for example [2]–[6], but mostly these schemes have only been applied at the receiver-side. The main difference in case that we perform switched diversity at the transmitter-side is that we usually require an efficient feedback link for that purpose, via which the receiver can signal the transmitter when it should switch to another antenna element since the SNR becomes unacceptable [6]. However, in practice such a feedback link might induce feedback errors as well as a certain feedback delay, wherefore the performance analysis of switched diversity reception systems generally cannot be directly applied to switched transmit diversity schemes as well.

In this paper, we analyze—to the best of our knowledge for the first time—the performance of switch-and-stay transmit diversity systems in presence of an erroneous feedback link. In a first step, we conduct a rather general performance analysis by considering arbitrary fading distributions as well as a broad class of performance functions, for which we derive generic expressions for the average system performance, the optimum switching threshold and the performance degradation due to feedback errors. In a second step, we consider a system with maximum ratio combining (MRC) at the receiver-side and Nakagami-m fading as a concrete example and we show how the generic results presented before can be used to easily derive analytical closed-form expressions for the average SNR, the capacity, and the average BER of this system. Please note that even though we focus on switched transmit diversity herein, our results can readily be applied to switch-and-stay receive combining as well by exploiting the duality between both approaches in the absence of feedback errors.

The remainder of this paper is structured as follows: In Section II, we shortly introduce our system model. The statistics of the SNR at the receiver-side are derived in Section III, followed by the generic performance analysis in Section IV. In Section V, we consider a concrete system as an example before finally the conclusions are given in Section VI.

II. System Model

We consider a block-fading MIMO system with two transmit antennas and N_{RX} receive antennas as depicted in Fig. 1. The basic mode of operation is basically similar to the one already considered in [6]. The transmitter uses always only one antenna for data transmission while the receiver generally combines the received signals first of all in an appropriate way (using MRC, equal gain combining, or selection combining, for example) before performing the actual data detection. The selection of an appropriate transmit antenna is done based upon feedback information from the receiver. For that purpose, the receiver periodically measures the instantaneous SNR γ_{eff} at the output of the signal combiner and compares it to a given threshold value γ_T. If γ_{eff} is smaller than γ_T, a '1' is fed back to the transmitter, indicating that it should switch to the other antenna whereas otherwise a '0' is fed back, indicating that no switching is required. Please note that for proper operation the feedback interval should be in the order of the coherence time of the channel, so that the channel is approximately time-invariant during each interval, what is always assumed in the

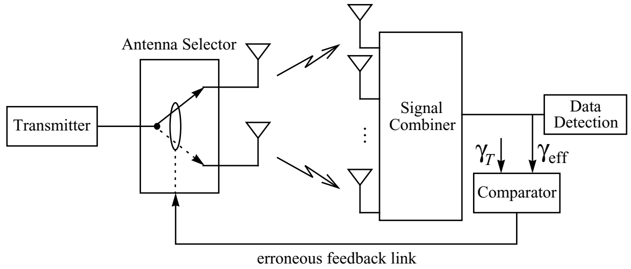

Fig. 1. Considered switch-and-stay transmit diversity system.

following. Besides, the feedback bit might be erroneously detected by the transmitter, what is taken into account by assuming a certain feedback error probability P_e. The receiver itself is supposed to have always perfect channel knowledge.

III. SNR Statistics

First of all, we determine the distribution of the SNR γ_{eff} as indicated in Fig. 1. Let us denote the (instantaneous) combined SNR at the receiver-side in case that always the same transmit antenna is used (i.e., for a system without transmit diversity) by γ_0 in the following and the corresponding expected value by $\mathbb{E}[\gamma_0] = \bar{\gamma}$. For simplicity, we assume that the distribution of γ_0 does not depend on which of the two antenna elements is active. A generalization of our results to the case of unequal SNR distributions induced by the two transmit antennas is straightforward, but would significantly complicate notation. If we denote the cumulative distribution function (cdf) of γ_0 by $F_{\gamma_0}(\gamma)$, the cdf of γ_{eff} generally can be expressed in terms of $F_{\gamma_0}(\gamma)$, the feedback error probability P_e as well as the switching threshold γ_T as given by the following theorem:

Theorem 1: The cdf of γ_{eff} is generally given by

$$F_{\gamma_{\text{eff}}}(\gamma) = \begin{cases} F_{\gamma_0}(\gamma)\ [\rho(\gamma_T, P_e) + 2\,P_e], & \gamma < \gamma_T \\ F_{\gamma_0}(\gamma) + \rho(\gamma_T, P_e)\ [F_{\gamma_0}(\gamma) - 1], & \gamma \geq \gamma_T \end{cases}, \tag{1}$$

where we introduced for brevity the short-hand notation

$$\rho(\gamma_T, P_e) = (1 - 2\,P_e)\, F_{\gamma_0}(\gamma_T). \tag{2}$$

Proof: It can easily be seen that in general

$$\begin{aligned} F_{\gamma_{\text{eff}}}(\gamma) \;=\; & (1-P_e)\,\text{Prob}[\gamma_{\text{eff}} \leq \gamma|\text{no error}] \\ & + \text{Prob}[E_1]\,\text{Prob}[\gamma_{\text{eff}} \leq \gamma|E_1] \\ & + \text{Prob}[E_2]\,\text{Prob}[\gamma_{\text{eff}} \leq \gamma|E_2], \end{aligned} \tag{3}$$

where E_1 refers to the event that we switch even if it is not necessary, i.e., that a '0' is fed back to the transmitter which misinterprets it as a '1', and E_2 refers to the event that we do not switch even if it was requested, what is the case if a '1' is fed back and we misinterpret it as a '0'. It can easily be shown that $\text{Prob}[E_1] = P_e\,(1 - F_{\gamma_0}(\gamma_T))$ and $\text{Prob}[E_2] = P_e\, F_{\gamma_0}(\gamma_T)$. Furthermore, due to the duality of our system with a conventional switch-and-stay receive combining scheme in the error-free case, it is straightforward to show that [3]

$$\begin{aligned} &\text{Prob}[\gamma_{\text{eff}} \leq \gamma|\text{no error}] \\ &\quad = \begin{cases} F_{\gamma_0}(\gamma_T)\, F_{\gamma_0}(\gamma), & \gamma < \gamma_T \\ F_{\gamma_0}(\gamma)\ [1 + F_{\gamma_0}(\gamma_T)] - F_{\gamma_0}(\gamma_T), & \gamma \geq \gamma_T \end{cases}. \end{aligned} \tag{4}$$

Besides, whenever we switch to the other antenna element, the distribution of γ_{eff} corresponds to the distribution of the SNR without transmit diversity since we do not have any additional information about the SNR induced by that antenna. Therefore, we clearly can say that $\text{Prob}[\gamma_{\text{eff}} \leq \gamma|E_1] = F_{\gamma_0}(\gamma)$. If we do not switch even though we are supposed to, γ_{eff} surely is always smaller than the threshold value γ_T, so that we get $\text{Prob}[\gamma_{\text{eff}} \leq \gamma|E_2] = \min\left\{\frac{F_{\gamma_0}(\gamma)}{F_{\gamma_0}(\gamma_T)}, 1\right\}$. Putting everything together then yields after some rearrangements to (1). ■

It can easily be seen that in the error-free case (P_e = 0), (1) reduces to the well-known result from [3], [4] whereas in the worst case ($P_e = 0.5$), we have $F_{\gamma_{\text{eff}}} = F_{\gamma_0}(\gamma)$, i.e., in this case our system has exactly the same performance as a system with only one transmit antenna, what is quite obvious since in this case the feedback bits are totally unreliable and hence useless.

Corollary 1: The probability density function (pdf) of the SNR γ_{eff} is given by

$$p_{\gamma_{\text{eff}}}(\gamma) \;=\; \begin{cases} p_{\gamma_0}(\gamma)\ [\rho(\gamma_T, P_e) + 2\,P_e], & \gamma < \gamma_T \\ p_{\gamma_0}(\gamma)\ [1 + \rho(\gamma_T, P_e)], & \gamma \geq \gamma_T \end{cases} \tag{5}$$

with $\rho(\gamma_T, P_e)$ according to (2) and $p_{\gamma_0}(\gamma) = \frac{\partial}{\partial\gamma} F_{\gamma_0}(\gamma)$.

Proof: It can easily be seen that $p_{\gamma_{\text{eff}}}(\gamma) = \frac{\partial}{\partial\gamma} F_{\gamma_{\text{eff}}}(\gamma)$ with $F_{\gamma_{\text{eff}}}(\gamma)$ according to (1). ■

IV. Generic Performance Analysis

Many important measures characterizing the performance of communication systems in fading environments, such as the average symbol error rate (SER), the average bit error rate (BER), or the ergodic system capacity, can be determined by averaging appropriate performance functions, which reflect the corresponding performance in an additive white Gaussian noise channel, over the distribution of the effective SNR at the receiver-side. If we denote such a performance function by $\xi(\gamma)$, the average performance of our switch-and-stay transmit diversity system generally can be calculated as

$$\Xi = \mathbb{E}_{\gamma}[\xi(\gamma)] = \int_0^{\infty} \xi(\gamma)\, p_{\gamma_{\text{eff}}}(\gamma)\, d\gamma, \tag{6}$$

with $p_{\gamma_{\text{eff}}}(\gamma)$ according to (5). For determining the average (combined) SNR, for example, we set $\xi(\gamma) = \gamma$ whereas for obtaining the average BER in case of coherent binary phase shift keying, we set $\xi(\gamma) = Q(\sqrt{2\,\gamma})$, where $Q(\cdot)$ denotes the Gaussian Q-function. Inserting (5) in (6), we get

$$\Xi = \Xi_0\ [1 + \rho(\gamma_T, P_e)] - (1 - 2\,P_e) \int_0^{\gamma_T} \xi(\gamma)\, p_{\gamma_0}(\gamma)\, d\gamma, \tag{7}$$

where

$$\Xi_0 = \int_0^{\infty} \xi(\gamma)\, p_{\gamma_0}(\gamma)\, d\gamma \tag{8}$$

denotes the corresponding (average) performance in a system without transmit diversity, i.e., with a single transmit antenna only. Closed-form expressions for Ξ_0 are readily available from literature for a wide variety of performance functions, so that the calculation of the average performance of our system reduces in most cases basically to the solution of the single integral with finite integration limits given in (7).

In order to keep our further analysis as general as possible, we consider a broad class of performance functions $\xi(\gamma)$ in the following, which we require to satisfy the following properties:

- P1) $\xi(\gamma)$ is a real-valued, strictly monotonic continuous function with domain $\mathbb{D} = \mathbb{R}^+$.
- P2) $\xi(\gamma) \geq 0 \;\forall\; \gamma \in \mathbb{D}$.
- P3) The performance improves with increasing SNR γ, i.e., if $\xi(\gamma)$ is strictly monotonic increasing, higher values reflect a better performance whereas in the case that it is strictly monotonic decreasing, smaller values are better.

It can easily be checked that most performance functions of practical interest actually satisfy these properties, wherefore this restriction does not represent a very strong limitation.

The performance of switch-and-stay diversity schemes generally is strongly dependent on the choice of the switching threshold γ_T, of course. In this regard, we can characterize the optimum threshold leading to the best performance by deriving a generic expression for it as given by the following theorem:

Theorem 2: The optimal switching threshold is independent of the feedback error probability P_e and generally given by

$$\gamma_{T,opt} = \xi^{-1}(\Xi_0), \tag{9}$$

where $\xi^{-1}(\gamma)$ denotes the inverse function of $\xi(\gamma)$.

Proof: A necessary requirement for a certain switching threshold γ_T to be optimal is that it is a stationary point of Ξ. In this regard, we find based on (7)

$$\frac{\partial}{\partial \gamma_T} \Xi = (1 - 2\,P_e) p_{\gamma_0}(\gamma_T) \left[\Xi_0 - \xi(\gamma_T)\right], \tag{10}$$

which equals only zero if we choose γ_T according to (9). In order to verify that we really get the optimum this way, we consider the behavior of (10) for thresholds slightly smaller or larger than $\gamma_{T,opt}$. Since the maximum feedback error probability is $P_e = 0.5$, in which case we would make random decisions on the feedback bits, it is obvious that $1 - 2\,P_e > 0$ in practice. Besides, $p_{\gamma_0}(\gamma)$ is always non-negative, Ξ_0 is independent of γ_T and $\xi'(\gamma) = \frac{\partial}{\partial \gamma}\xi(\gamma)$ is always positive if $\xi(\gamma)$ is strictly monotonic increasing and always negative if $\xi(\gamma)$ is strictly monotonic decreasing. Hence, it can easily be seen that we have a change of sign of (10) at $\gamma_T = \gamma_{T,opt}$ from plus to minus in the first case (maximum) and from minus to plus in the second case (minimum). Furthermore, due to P1), (9) has always a unique solution and it can easily be checked that $\Xi(\gamma_T = 0) = \Xi(\gamma_T \to \infty) = \Xi_0$. Since Ξ is under the afore made assumptions always a continuous function of γ_T, we finally can conclude together with P3) that $\gamma_{T,opt}$ according to (9) is really the globally optimal switching threshold. ■

Please note that the generic expression for $\gamma_{T,opt}$ according to (9) is also valid for conventional switch-and-stay receive combining schemes and hence basically unifies the results already presented in [3], [4]. Furthermore, since Ξ_0 is readily available in literature for a wide variety of performance functions and fading distributions, we often can directly determine $\gamma_{T,opt}$ based on (9), without the need for any further calculations.

A general expression for the actual average performance that we obtain by using $\gamma_{T,opt}$ is given by the following corollary:

Corollary 2: The best possible average performance of our system generally can be calculated as

$$\Xi_{opt} = \Xi_0 + (1 - 2\,P_e) \int_0^{\gamma_{T,opt}} F_{\gamma_0}(\gamma)\, \xi'(\gamma)\, d\gamma. \tag{11}$$

Proof: Inserting $\gamma_{T,opt}$ according to (9) in (7) and making use of partial integration yields the given expression. ■

This general but concise expression reveals that there is a linear relationship between the average system performance and the feedback error probability P_e. Hence, it is possible to directly quantify the system performance for a given value of P_e without the need for lengthy calculations if we know the performance for the error-free case ($P_e = 0$) as well as Ξ_0, i.e., the performance for the case that we have a single transmit antenna only, which obviously leads to the same performance as our switched transmit diversity system with $P_e = 0.5$. Clearly, the actual performance degradation due to feedback errors is generally given by $\Delta_\xi = \Xi_{opt}\big|_{P_e=0} - \Xi_{opt}\big|_{P_e>0} = 2\,P_e \int_0^{\gamma_{T,opt}} F_{\gamma_0}(\gamma)\, \xi'(\gamma)\, d\gamma$, which can be upper-bounded as stated in the following corollary:

Corollary 3: For any fading distribution and arbitrary performance functions $\xi(\gamma)$, the performance degradation Δ_ξ due to feedback errors can be upper-bounded by

$$|\Delta_\xi| \leq 2\,P_e\,\Xi_0. \tag{12}$$

Proof: Rewriting the general expression for Δ_ξ given above by means of partial integration as $\Delta_\xi = 2P_e \left[F_{\gamma_0}(\gamma_{T,opt})\,\Xi_0 - \int_0^{\gamma_{T,opt}} p_{\gamma_0}(\gamma)\,\xi(\gamma)\,d\gamma\right]$ and further noting that $F_{\gamma_0}(\gamma_{T,opt})\,\Xi_0 \leq \Xi_0$ and $\int_0^{\gamma_{T,opt}} \xi(\gamma)\,p_{\gamma_0}(\gamma)\,d\gamma \geq 0$, it directly follows that $\Delta_\xi \leq 2\,P_e\,\Xi_0$. Likewise, it can easily be seen that $F_{\gamma_0}(\gamma_{T,opt})\,\Xi_0 \geq 0$ and $\int_0^{\gamma_{T,opt}} p_{\gamma_0}(\gamma)\,\xi(\gamma)\,d\gamma \leq \Xi_0$ and consequently $\Delta_\xi \geq -2\,P_e\,\Xi_0$. Putting both results together, we finally obtain (12). ■

Before concluding our generic analysis, we formally prove the intuitive idea that the optimal switching threshold according to (9) is always a strictly increasing function of the average SNR $\bar{\gamma}$, as stated by the following corollary:

Corollary 4: The optimal switching threshold $\gamma_{T,opt}$ according to (9) is unique and a strictly increasing function of the average (combined) SNR $\bar{\gamma}$.

Proof: Due to the strict monotonicity of the considered performance functions, the only thing that has to be shown for that purpose is that if $\xi(\gamma)$ is monotonically increasing, we have $\Xi_0|_{\bar{\gamma}_1} < \Xi_0|_{\bar{\gamma}_2 > \bar{\gamma}_1}$ and likewise if $\xi(\gamma)$ is monotonically decreasing, we have $\Xi_0|_{\bar{\gamma}_1} > \Xi_0|_{\bar{\gamma}_2 > \bar{\gamma}_1}$. In this regard, we note that for a given fading distribution, the distribution of the SNR γ_{eff} at the output of the signal combiner at the receiver-side generally represents a scale-family with the average SNR as scale parameter. This implies that the pdf $p_{\gamma'}(\gamma)$ with average SNR $\bar{\gamma}'$ might be expressed by means of a reference pdf $p_{\gamma_r}(\gamma)$ with average SNR one as $p_{\gamma'}(\gamma) = \frac{1}{\bar{\gamma}'}\, p_{\gamma_r}\left(\frac{\gamma}{\bar{\gamma}'}\right)$. For strictly monotonic increasing performance functions $\xi(\gamma)$, we consequently have $\bar{\Xi}_0\big|_{\bar{\gamma}_2 > \bar{\gamma}_1} = \frac{1}{\bar{\gamma}_2} \int_0^\infty \xi(\gamma)\, p_{\gamma_r}\left(\frac{\gamma}{\bar{\gamma}_2}\right) d\gamma = \int_0^\infty \xi\left(\bar{\gamma}_2\, x\right)\, p_{\gamma_r}(x)\, dx > \int_0^\infty \xi\left(\bar{\gamma}_1\, x\right)\, p_{\gamma_r}(x)\, dx = \bar{\Xi}_0\big|_{\bar{\gamma}_1}$ whereas for strictly monotonic decreasing $\xi(\gamma)$ the last inequality simply has to be reversed. ■

V. Concrete Example

In the following, we consider a switch-and-stay transmit diversity system with MRC combining at the receiver-side and IID Nakagami-m fading with integer fading parameter m as a concrete example and we show how our generic results derived in the previous section can be used to easily investigate the performance of this system in terms of the average SNR, the ergodic capacity, as well as the average BER for binary modulation. It can easily be shown that for such a scenario

$$p_{\gamma_0}(\gamma) = \frac{\gamma^{mN_{RX}-1}}{\Gamma(mN_{RX})} \left(\frac{m}{\bar{\eta}}\right)^{mN_{RX}} e^{-\gamma \frac{m}{\bar{\eta}}}, \quad \gamma \geq 0 \tag{13}$$

where $\Gamma(\cdot)$ denotes the well-known gamma function and with $\bar{\eta}$ as the average SNR per receive antenna. Hence, the average SNR at the output of the MRC combiner is simply given by $\bar{\gamma} = N_{RX}\,\bar{\eta}$ and the corresponding cdf can be determined as

$$F_{\gamma_0}(\gamma) = 1 - e^{-\frac{m}{\bar{\eta}}\gamma} \sum_{k=0}^{mN_{RX}-1} \frac{1}{k!} \left(\frac{m}{\bar{\eta}}\gamma\right)^k, \quad \gamma \geq 0 \tag{14}$$

where we made use of [7] eqs. (3.381,1) and (8.352,1).

A. Average SNR

Theorem 3: For arbitrary switching thresholds γ_T, the average SNR at the output of the signal combiner is given by

$$\bar{\gamma}_{\text{eff}} = \bar{\gamma} \left[1 + \frac{(1-2\,P_e)\,e^{-\frac{m\,N_{RX}\,\gamma_T}{\bar{\gamma}}}}{\Gamma(m\,N_{RX}+1)} \left(\frac{m\,N_{RX}\,\gamma_T}{\bar{\gamma}}\right)^{m\,N_{RX}}\right] \tag{15}$$

whereas for the optimum threshold $\gamma_{T,opt} = \bar{\gamma}$, we obtain

$$\bar{\gamma}_{\text{eff,opt}} = \bar{\gamma} \left[1 + \frac{1-2\,P_e}{\Gamma(m\,N_{RX})} (m\,N_{RX})^{m\,N_{RX}-1} e^{-m\,N_{RX}}\right]. \tag{16}$$

Proof: If we consider the average SNR as performance measure, we set $\xi(\gamma) = \gamma$, which obviously satisfies all properties P1)-P3). Clearly, we have $\Xi_0 = \bar{\gamma}$ and consequently it directly follows from (9) that $\gamma_{T,opt} = \bar{\gamma}$. Besides, (15) and (16) can be determined by inserting (13) and (14) in (7) and (8) and by making use of [7] eqs. (8.356,2) and (3.381,1). ■

It can easily be checked that for $P_e = 0$ and $N_{RX} = 1$, (16) reduces to the well-known result already presented in [4]. A graphical illustration of the impact of the switching threshold and the feedback error probability on $\bar{\gamma}_{\text{eff}}$ is depicted in Fig. 2. Obviously, we really get the best performance for $\gamma_T = \bar{\gamma}$, independently of P_e, and increasing values of P_e lead to a linear decrease of $\bar{\gamma}_{\text{eff}}$. Furthermore, it can be seen that there is basically perfect match between our analytical results and results obtained from Monte Carlo simulations, what verifies the validity of our theoretical analysis.

B. System Capacity

Theorem 4: The optimum switching threshold which maximizes the capacity of our system is given by $\gamma_{T,opt} = 2^{C_0} - 1$ with

$$C_0 = \frac{e^{\frac{m}{\bar{\eta}}}}{\ln 2} \sum_{k=0}^{m\,N_{RX}-1} \Gamma\left(-k, \frac{m}{\bar{\eta}}\right) \left(\frac{m}{\bar{\eta}}\right)^k, \tag{17}$$

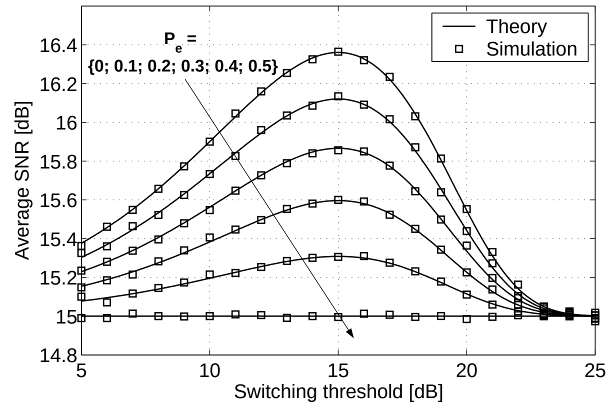

Fig. 2. Impact of the switching threshold on the average SNR for $N_{RX} = 1$, $m = 1$, $\bar{\gamma} = 15\,$dB, and different feedback error probabilities P_e.

for which we get (in bits per channel use)

$$\begin{aligned} C_{opt} = C_0 + (1-2\,P_e) \Bigg[C_0 - \frac{1}{\ln 2} \sum_{k=0}^{mN_{RX}-1} \frac{1}{k!} e^{\frac{m}{\bar{\eta}}} \\ \times \left(\frac{m}{\bar{\eta}}\right)^k \Bigg[(-1)^k \left[E_1\left(\frac{m}{\bar{\eta}}\right) - E_1\left(\frac{m}{\bar{\eta}} 2^{C_0}\right)\right] \\ + \sum_{s=1}^{k} \binom{k}{s} (-1)^{k-s} \left(\frac{\bar{\eta}}{m}\right)^s \\ \times \left[\Gamma\left(s, \frac{m}{\bar{\eta}}\right) - \Gamma\left(s, \frac{m}{\bar{\eta}} 2^{C_0}\right)\right]\Bigg]\Bigg], \end{aligned} \tag{18}$$

where $E_1(\cdot)$ denotes the exponential integral function and $\Gamma(\cdot,\cdot)$ the upper incomplete gamma function [7].

Proof: Now, we have $\xi(\gamma) = \log_2(1+\gamma)$, which clearly satisfies all properties P1)-P3) again. C_0 corresponds to the capacity of a system with one transmit antenna only, which is known from [8], and based on this result we can directly determine $\gamma_{T,opt}$ by capitalizing on (9) again. For calculating C_{opt}, we then plug (14) and $\xi'(\gamma) = \frac{(1+\gamma)^{-1}}{\ln 2}$ in (11), yielding to

$$C_{opt} = C_0 + (1-2\,P_e) \left[C_0 - \frac{1}{\ln 2} \mathcal{I}_1\right], \tag{19}$$

where we have introduced for brevity the short-hand notation

$$\mathcal{I}_1 = \sum_{k=0}^{mN_{RX}-1} \frac{1}{k!} \left(\frac{mN_{RX}}{\bar{\gamma}}\right)^k \int_0^{2^{C_0}-1} e^{-\frac{mN_{RX}}{\bar{\gamma}}\gamma} \frac{\gamma^k}{\gamma+1}\, d\gamma. \tag{20}$$

Performing the substitution $z = \gamma + 1$ and making use of the binomial theorem, $\mathcal{I}_1$ can be reformulated as

$$\mathcal{I}_1 = e^{\frac{mN_{RX}}{\bar{\gamma}}} \left[E_1\left(\frac{mN_{RX}}{\bar{\gamma}}\right) - E_1\left(\frac{mN_{RX}}{\bar{\gamma}} 2^{C_0}\right)\right] - \mathcal{I}_2, \tag{21}$$

with

$$\begin{aligned} \mathcal{I}_2 \;=\; & \sum_{k=1}^{mN_{RX}-1} \frac{1}{k!} \left(\frac{mN_{RX}}{\bar{\gamma}}\right)^k e^{\frac{mN_{RX}}{\bar{\gamma}}} \\ & \times \sum_{s=0}^{k} \binom{k}{s} \int_1^{2^{C_0}} z^{s-1} (-1)^{k-s} e^{\frac{mN_{RX}\,z}{-\bar{\gamma}}}\, dz, \end{aligned} \tag{22}$$

which after some basic mathematical manipulations can be solved in closed-form by making use of [7] eq. (3.381,1). ■

Please note that to the best of our knowledge, the problem of finding an analytical closed-form expression for the capacity of switch-and-stay diversity schemes as given by Theorem 4 has only been considered in [9] before. However, in contrast to (18), the result in [9] corresponds to an infinite series and therefore has to be truncated for numerical evaluation, thus always inducing computational errors.

The impact of the feedback error probability on the maximum capacity for a system with optimum switching threshold is illustrated in Fig. 3. Since it is expected that generally $P_e \ll 0.1$ in practical systems, the capacity reduction due to feedback error obviously can usually be reasonably neglected.

C. Average Bit Error Rate

Theorem 5: The optimum switching threshold which minimizes the average BER for coherent binary phase shift keying is given by $\gamma_{T,opt} = \frac{1}{2}\left[Q^{-1}(\bar{P}_0)\right]^2$, with $Q^{-1}(\cdot)$ as the inverse Gaussian Q-function and where

$$\bar{P}_0 = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\eta}}{m+\bar{\eta}}} \sum_{k=0}^{m N_{RX}-1} \frac{\binom{2k}{k}}{\left[4\left(1+\frac{\bar{\eta}}{m}\right)\right]^k}\right]. \quad (23)$$

denotes the average BER for a similar system with one transmit antenna only.

Proof: For investigating the average BER performance, we set $\xi(\gamma) = Q(\sqrt{2\,\gamma})$. The average BER for a system without transmit diversity according to (23) is well-known from literature and can be obtained from the results provided in [10], for example. Based on this result, we then can directly determine $\gamma_{T,opt}$ by means of (9) again. ■

Theorem 6: The minimum average BER of coherent binary phase shift keying, which can be obtained by using the optimum switching threshold, is given by

$$\bar{P}_{\text{opt}} = \bar{P}_0 - (1-2P_e)\left[\frac{1}{2} - \bar{P}_0 - \sum_{k=0}^{mN_{RX}-1} \frac{1}{2\sqrt{\pi}\,k!}\left(\frac{m}{\bar{\eta}}\right)^k \right.$$
$$\left. \times \left(\frac{1}{\frac{m}{\bar{\eta}}+1}\right)^{k+\frac{1}{2}} \Gamma_L\left(k+\frac{1}{2}, \gamma_{T,opt}\left(\frac{m}{\bar{\eta}}+1\right)\right)\right] \quad (24)$$

with $\gamma_{T,opt}$ and $\bar{P}_0$ according to Theorem 5 and $\Gamma_L(\cdot,\cdot)$ as the lower incomplete gamma function [7].

Proof: First of all, it can easily be shown that $\xi'(\zeta) = \frac{-1}{2\sqrt{\pi\,\gamma}} e^{-\gamma}$. Inserting this relationship together with (14) in (11), the calculation of the average BER can traced back to the calculation of two different types of integrals, which can both be solved in closed-form by making use of [7] eqs. (3.361,1) and (3.381,1). If we further exploit that $\frac{1}{2}\text{erf}\left(\sqrt{\gamma_{T,opt}}\right) = \frac{1}{2} - \bar{P}_0$, we finally end up with the given formula. ■

Please note that a similar analysis and particularly the derivation of the optimum switching threshold can be done for a wide variety of different coherent and noncoherent modulation schemes in a straightforward manner, what, however, is not explicitly presented here due to space constraints.

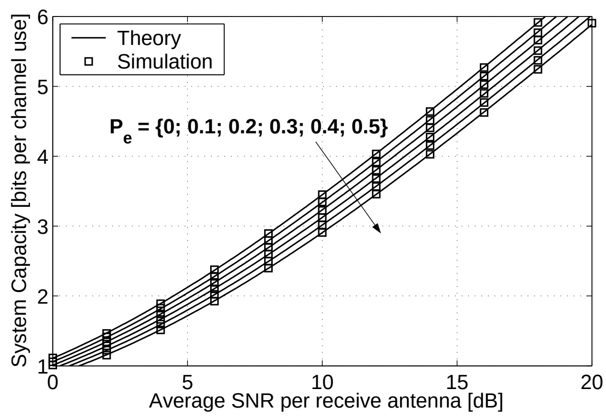

Fig. 3. Impact of the feedback error probability P_e on the system capacity for N_{RX} = 1, m = 1 (Rayleigh-fading), and $\gamma_T = \gamma_{T,opt}$.

VI. Conclusion

We have analyzed the performance of switch-and-stay transmit diversity systems in presence of an erroneous feedback link. First, we have determined the distribution of the induced SNR at the receiver-side as a function of the feedback error probability for arbitrary fading distributions and we have determined generic but simple expressions for the average performance and optimal switching threshold for a broad class of performance functions, thus unifying and generalizing existing results in literature. In a second step, we have considered a system with MRC combining at the receiver-side and Nakagami-m fading as a concrete example and we have shown how the previously derived generic results can be used to easily investigate the performance of this system in terms of the average SNR, the capacity, and the average BER.

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