Orthogonal Space-Time Block Codes with Receive Antenna Selection: Capacity and SER Analysis

Andreas Müller and Joachim Speidel
Institute of Telecommunications, University of Stuttgart, Germany
E-mail: {andreas.mueller, joachim.speidel}@inue.uni-stuttgart.de

***Abstract*— We analyze the performance of multiple-input multiple-output (MIMO) systems employing orthogonal space-time block codes (STBC) combined with receive antenna selection in Nakagami-m fading channels with possible spatial correlation at the transmitter-side. We derive exact analytical closed-form expressions for the system capacity as well as the average symbol error rate (SER) in case of M-PSK and M-QAM modulation, respectively. Furthermore, we determine the corresponding high SNR asymptotes, which are significantly simpler than the exact results while providing important insights into the SER performance at high SNRs. Finally, we compare the performance of our system with the performance of a full-complexity system without receive antenna selection as well as a single-input multiple-output system with selection combining and we verify the validity of our analytical results by means of Monte-Carlo simulations.**

I. Introduction

Multiple-input multiple-output (MIMO) systems offer numerous benefits over conventional single-input single-output (SISO) systems, such as the potential to facilitate considerably higher data rates or to significantly improve the reliability of a wireless link. However, this superior performance generally comes along with an increased signal processing complexity as well as substantially higher hardware requirements, what hence represents a limiting factor for the production and wide-spread deployment of low-cost MIMO devices. A promising approach for partially alleviating these drawbacks is to make use of some form of antenna selection, whereby many of the advantages offered by MIMO systems can be retained while the number of required radio frequency (RF) chains and hence the costs and the complexity can be significantly reduced [1].

In this paper, we analyze the performance of MIMO systems employing orthogonal space-time block codes (STBC) combined with antenna selection at the receiver-side. This might be an attractive solution for the downlink of cellular networks, for example, where the complexity and the costs of the mobile user equipment generally should be kept as low as possible whereas this is usually only of secondary importance for the base stations. Previous studies dealing with the combination of antenna selection and orthogonal STBCs have mainly focused on performing antenna subset selection at the transmitter-side (see for example [2], [3] and references therein), but only very few results have been reported concerning the performance of orthogonal STBC with receive antenna selection so far. In [4], for instance, the authors present upper bounds on the average bit error rates for binary modulation in uncorrelated Rayleigh fading as well as an exact expression for the special case that the well-known Alamouti scheme is employed. Besides, the effects of channel estimation errors on the performance of orthogonal STBCs combined with receive antenna selection recently have been investigated in [5]. However, to the best of our knowledge, a comprehensive performance analysis of orthogonal STBCs with receive antenna selection has not been presented yet, what we therefore will do herein. In particular, we derive exact analytical closed-form expressions for the capacity of such a system as well as the average symbol error rates (SER) in case of M-ary quadrature amplitude modulation (M-QAM) and M-ary phase shift keying (M-PSK), considering Nakagami-m fading channels with possible spatial correlation at the transmitter-side. In addition, we compare the performance of our system with the performance of a full-complexity system as well as a single-input multiple-output (SIMO) system with selection combining.

The remainder of this paper is structured as follows: In Section II we outline our system and channel model whereas the statistics of the effective SNR at the receiver-side are determined in Section III. The actual capacity and SER analysis is presented in Section IV, followed by numerical results in Section V as well as some concluding remarks in Section VI.

II. System and Channel Model

We consider a frequency-flat block-fading MIMO system with N_{TX} transmit antennas and N_{RX} receive antennas. In the discrete-time equivalent baseband domain, the channel can be modeled by the matrix $\mathbf{H} \in \mathbb{C}^{N_{RX} \times N_{TX}}$, where the (i, j)-th element $h_{i,j}$ corresponds to the channel coefficient between the j-th transmit antenna and the i-th receive antenna. The phase of each channel coefficient is assumed to be uniformly distributed in $[0; 2\,\pi)$ whereas the magnitudes are supposed to be Nakagami-m variates with unity average power gain and integer fading parameter m. Furthermore, the channel might exhibit spatial correlation at the transmitter-side, i.e., entries standing in the same row of $\mathbf{H}$ might be correlated. Please note that the assumption of spatial correlation at the transmitter-side only is valid in urban scenarios, for example, where a base station (transmitter) is mounted on top of the roofs whereas the mobile terminals (receivers) are located in rich-scattering environments due to surrounding buildings, cars, and so on.

The transmitter encodes the data to be sent by means of an orthogonal STBC with code rate R_C while the receiver—which is assumed to have perfect knowledge of the channel—selects always only the best out of the available N_{RX} receive antenna elements for processing the faded received signals, which are additionally perturbed by additive white Gaussian noise. It is well-known that after appropriate signal combining, orthogonal STBCs transform a MIMO system into a set of equivalent SISO channels [6] and in case that the i-th receive antenna is selected, it can easily be shown that these equivalent SISO channels have the effective signal-to-noise ratio (SNR)

$$\gamma_i = \frac{\bar{\gamma}}{R_C\, N_{TX}} \sum_{j=1}^{N_{TX}} |h_{i,j}|^2 = \gamma_0 \sum_{j=1}^{N_{TX}} |h_{i,j}|^2, \qquad (1)$$

with the short-hand notation $\gamma_0 = \bar{\gamma}/(R_C\, N_{TX})$, where $\bar{\gamma}$ denotes the average SNR per receive antenna. Clearly, selecting the best antenna hence corresponds to selecting the antenna for which this effective SNR is maximized.

III. SNR Statistics

A Nakagami-m variate with integer fading parameter m and unity average power gain generally can be considered as the square root of the sum of $2\,m$ squared i.i.d. Gaussian random variables with zero mean and variance $1/(2m)$ [7]. In presence of spatial correlation at the transmitter-side only, it is therefore straightforward to model the magnitude of the coefficient $h_{i,j}$ as $|h_{i,j}| = \sqrt{\sum_{l=1}^{m} |x_{i,j}^l|^2}$, where $x_{i,j}^l$ is the (i,j)-th entry of the $N_{RX} \times N_{TX}$ random matrix $\mathbf{X}_l$ $(l = 1, \ldots, m)$. These random matrices are i.i.d. circularly symmetric complex Gaussian distributed with zero mean and covariance matrix $\frac{1}{m}(\Lambda_{TX} \otimes \mathbf{I}_{N_{RX}})$, where Λ_{TX} represents the normalized transmit correlation matrix whose diagonal elements are all equal to one, $\mathbf{I}_n$ is the identity matrix of dimension n and $\otimes$ denotes the matrix Kronecker product. Please note that the elements of Λ_{TX} can be directly related to various physical propagation parameters such as the antenna spacing, mean angle of arrival, and angular spread, for instance, what makes this model especially suitable for theoretical investigations.

Following the approach outlined in [7], it can be shown that the moment-generating function (mgf) of γ_i is given by

$$M_{\gamma_i}(s) = \left[\det\left(\mathbf{I}_{N_{TX}} - s\,\frac{\gamma_0}{m}\,\Lambda_{TX}\right)\right]^{-m}, \qquad (2)$$

where $\det(\cdot)$ denotes the determinant operator. In the following, we assume without loss of generality that Λ_{TX} has P distinct non-zero eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_P$ with multiplicities $\alpha_1, \alpha_2, \ldots, \alpha_P$, respectively. Consequently, it can easily be seen that (2) can be rewritten as

$$M_{\gamma_i}(-s) = \prod_{j=1}^{P} \frac{1}{(1 + s\,\epsilon_j)^{\mu_j}}, \qquad (3)$$

where we have introduced for brevity the short-hand notations

$$\mu_j = m\,\alpha_j \qquad (4)$$

$$\epsilon_j = \frac{\gamma_0}{m}\,\lambda_j. \qquad (5)$$

Expanding this term into partial fractions, we obtain

$$M_{\gamma_i}(-s) = \sum_{j=1}^{P}\sum_{k=1}^{\mu_j} \frac{\xi_{j,k}}{(1 + s\,\epsilon_j)^k}, \qquad (6)$$

where the expansion coefficients $\xi_{j,k}$ can be calculated as

$$\xi_{j,k} = \frac{\epsilon_j^{k-\mu_j}}{(\mu_j - k)!}\,\frac{\partial^{\mu_j - k}}{\partial s^{\mu_j - k}} \left[\prod_{\substack{\nu=1\\ \nu\neq j}}^{P} \frac{1}{(1 + s\,\epsilon_\nu)^{\mu_\nu}}\right]\Bigg|_{s=\frac{-1}{\epsilon_j}}. \qquad (7)$$

The probability density function (pdf) of γ_i then corresponds to the inverse Laplace transform of $M_{\gamma_i}(-s)$ and is given by

$$p_{\gamma_i}(\zeta) = \sum_{j=1}^{P}\sum_{k=1}^{\mu_j} \frac{\xi_{j,k}\,\zeta^{k-1}}{\Gamma(k)\,\epsilon_j^k}\, e^{-\frac{\zeta}{\epsilon_j}}, \quad \zeta \geq 0 \qquad (8)$$

where $\Gamma(\cdot)$ denotes the well-known gamma function [8]. The corresponding cumulative distribution function (cdf) $F_{\gamma_i}(\zeta) = \int_0^\zeta p_{\gamma_i}(t)\,dt$ consequently can be obtained by capitalizing on [8] eqs. (3.381,1) and (8.352,1) as well as the fact that $\sum_{j=1}^{P}\sum_{k=1}^{\mu_j} \xi_{j,k} = 1$ (what can readily be checked by comparing (6) with (3)) as

$$F_{\gamma_i}(\zeta) = 1 - \sum_{j=1}^{P}\sum_{k=1}^{\mu_j}\sum_{t=0}^{k-1} \frac{\xi_{j,k}}{\Gamma(t+1)} \left(\frac{\zeta}{\epsilon_j}\right)^t e^{-\frac{\zeta}{\epsilon_j}}, \quad \zeta \geq 0. \qquad (9)$$

As already mentioned before, the receiver always selects the antenna for which the effective SNR according to (1) is maximized, i.e., the actual SNR γ_{S} of the selected branch is given by $\gamma_{\mathrm{S}} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_{N_{RX}}\}$ Due to the statistical independence of all γ_i, the cdf of γ_{S} is then simply given by $F_{\gamma_{\mathrm{S}}}(\zeta) = \prod_{i=1}^{N_{RX}} F_{\gamma_i}(\zeta) = (F_{\gamma_i}(\zeta))^{N_{RX}}$ and the corresponding pdf hence can be expressed as

$$p_{\gamma_{\mathrm{S}}}(\zeta) = \frac{\partial}{\partial \zeta} F_{\gamma_{\mathrm{SC}}}(\zeta) = N_{RX}\, p_{\gamma_i}(\zeta)\, \left(F_{\gamma_i}(\zeta)\right)^{N_{RX}-1}, \ \zeta \geq 0. \qquad (10)$$

Inserting (8) and (9) in (10) and using [8] eq. (1.111), we get

$$p_{\gamma_{\mathrm{S}}}(\zeta) = N_{RX} \sum_{\nu=1}^{P}\sum_{\tau=1}^{\mu_\nu}\sum_{n=0}^{N_{RX}-1} \binom{N_{RX}-1}{n} \frac{\xi_{\nu,\tau}\,\zeta^{\tau-1}}{\Gamma(\tau)\,\epsilon_\nu^\tau} \times e^{-\frac{\zeta}{\epsilon_\nu}} \left(-\sum_{j=1}^{P}\sum_{k=1}^{\mu_j}\sum_{t=0}^{k-1} \frac{\xi_{j,k}\left(\frac{\zeta}{\epsilon_j}\right)^t}{\Gamma(t+1)}\, e^{-\frac{\zeta}{\epsilon_j}}\right)^n \qquad (11)$$

In order to transform this expression into a more convenient form, we make use of the multinomial theorem and obtain after some basic mathematical manipulations (a detailed derivation is omitted here due to space constraints)

$$p_{\gamma_{\mathrm{S}}}(\zeta) = \sum_{\nu=1}^{P}\sum_{\tau=1}^{\mu_\nu}\sum_{n=0}^{N_{RX}-1} \sum_{\{\beta_{j,k,t}\}\in\mathbb{U}_n} \binom{N_{RX}}{\{\beta_{j,k,t}\}} \frac{\xi_{\nu,\tau}}{\Gamma(\tau)\,\epsilon_\nu^\tau} \times \frac{\zeta^{\tau+f(\{\beta_{j,k,t}\})-1}\, e^{-\zeta\left(g(\{\beta_{j,k,t}\})+\frac{1}{\epsilon_\nu}\right)}\, h(\{\beta_{j,k,t}\})}{(-1)^n\,\Gamma(N_{RX}-n)}, \qquad (12)$$

where the inner summation has to be taken over all possible index tuples $\{\beta_{j,k,t}\} \in \mathbb{U}_n$ $(1 \leq j \leq P$, $1 \leq k \leq \mu_j$, $0 \leq t \leq$

$k-1)$ with

$$\mathbb{U}_n = \left\{ \{\beta_{j,k,t}\} \middle| \{\beta_{j,k,t}\} \in \mathbb{N}_0^{\Phi} \wedge \sum_{j=1}^{P} \sum_{k=1}^{\mu_j} \sum_{t=0}^{k-1} \beta_{j,k,t} = n \right\}, \tag{13}$$

where $\Phi = \sum_{j=1}^{P} \frac{\mu_j\,(\mu_j+1)}{2}$ denotes the number of elements belonging to one such tuple. Furthermore, we have introduced for brevity the short-hand notations

$$f(\{\beta_{j,k,t}\}) = \sum_{j=1}^{P} \sum_{k=1}^{\mu_j} \sum_{t=0}^{k-1} t\, \beta_{j,k,t} \tag{14}$$

$$g(\{\beta_{j,k,t}\}) = \sum_{j=1}^{P} \sum_{k=1}^{\mu_j} \sum_{t=0}^{k-1} \frac{\beta_{j,k,t}}{\epsilon_j} \tag{15}$$

$$h(\{\beta_{j,k,t}\}) = \prod_{j=1}^{P} \prod_{k=1}^{\mu_j} \prod_{t=0}^{k-1} \left(\epsilon_j^{-t} \frac{\xi_{j,k}}{\Gamma(t+1)} \right)^{\beta_{j,k,t}} \tag{16}$$

and $\binom{N_{RX}}{\{\beta_{j,k,t}\}}$ denotes a multinomial coefficient, which might be calculated as

$$\binom{N_{RX}}{\{\beta_{j,k,t}\}} = \frac{N_{RX}!}{\prod_{j=1}^{P} \prod_{k=1}^{\mu_j} \prod_{t=0}^{k-1} \beta_{j,k,t}!}. \tag{17}$$

Based on (12), it is now straightforward to determine the mgf of γ_{S} by means of the Laplace transform as

$$M_{\gamma_{\mathrm{S}}}(s) = \sum_{\nu=1}^{P} \sum_{\tau=1}^{\mu_\nu} \sum_{n=0}^{N_{RX}-1} \sum_{\{\beta_{j,k,t}\} \in \mathbb{U}_n} \binom{N_{RX}}{\{\beta_{j,k,t}\}} \times \frac{(-1)^n\, \xi_{\nu,\tau}\, \Gamma(\tau + f(\{\beta_{j,k,t}\}))}{\Gamma(\tau)\, \epsilon_\nu^{\tau}\, \Gamma(N_{RX}-n)} \times \frac{h(\{\beta_{j,k,t}\})}{\left(g(\{\beta_{j,k,t}\}) + \frac{1}{\epsilon_\nu} - s \right)^{\tau + f(\{\beta_{j,k,t}\})}}. \tag{18}$$

Finally, it should be noted that also the cdf of γ_{S} can easily be obtained from (12) by using [8] eq. (3.381,1), but due to space restrictions this result is not explicitly given here.

IV. Performance Analysis

A. System Capacity

The system capacity in the Shannon sense generally reflects the maximum average spectral efficiency that can be achieved without any delay or complexity constraints. Similarly to [6], [9], it can be calculated in our case as (in bits per channel use) $C = R_C \int_0^\infty \log_2(1+\zeta)\, p_{\gamma_{\mathrm{S}}}(\zeta)\, d\zeta$. Inserting the pdf of γ_{S} according to (12) and using the well-known integration result from [10] appendix B, we obtain

$$C = R_C \sum_{\nu=1}^{P} \sum_{\tau=1}^{\mu_\nu} \sum_{n=0}^{N_{RX}-1} \sum_{\{\beta_{j,k,t}\} \in \mathbb{U}_n} \binom{N_{RX}}{\{\beta_{j,k,t}\}} \frac{\xi_{\nu,\tau}}{\Gamma(\tau)\, \epsilon_\nu^{\tau}} \times \frac{h(\{\beta_{j,k,t}\})\, \Gamma(\tau + f(\{\beta_{j,k,t}\}))}{\Gamma(N_{RX}-n)\, e^{-g(\{\beta_{j,k,t}\}) - \frac{1}{\epsilon_\nu}}} \sum_{i=1}^{\tau + f(\{\beta_{j,k,t}\})} \times \frac{\Gamma(i - \tau - f(\{\beta_{j,k,t}\}), g(\{\beta_{j,k,t}\}) + \frac{1}{\epsilon_\nu})}{(-1)^n\, (g(\{\beta_{j,k,t}\}) + \frac{1}{\epsilon_\nu})^i}, \tag{19}$$

with $\Gamma(\cdot,\cdot)$ as the upper incomplete gamma function, which might be easily evaluated numerically by exploiting that [8]

$$\Gamma(-n, x) = \frac{(-1)^n}{\Gamma(n+1)} \left[E_1(x) - e^{-x} \sum_{j=0}^{n-1} \frac{(-1)^j\, \Gamma(j+1)}{x^{j+1}} \right]. \tag{20}$$

for $n \in \mathbb{N}_0$ and with $E_1(x)$ as the exponential integral.

B. Average Symbol Error Rate

1) Exact Analysis: It is well-known that the exact average SER in case of regular M-PSK can be calculated as [11]

$$P_s = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} M_{\gamma_{\mathrm{S}}} \left(-\frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2 \phi} \right) d\phi \tag{21}$$

and the one in case of M-QAM as

$$P_s = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/2} M_{\gamma_{\mathrm{S}}} \left(-\frac{\frac{3}{2\,(M-1)}}{\sin^2 \phi} \right) d\phi - \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right)^2 \int_0^{\pi/4} M_{\gamma_{\mathrm{S}}} \left(-\frac{\frac{3}{2\,(M-1)}}{\sin^2 \phi} \right) d\phi. \tag{22}$$

Inserting the corresponding expression for the mgf according to (18), it can be seen that the calculation of the average SERs can in both cases be traced back to the solution of integrals of the form $\mathcal{I}_1 = \int_0^a \frac{\sin^2 \phi}{\sin^2 \phi + c}\, d\phi$. Such a solution is given in [11] appendix 5A, for example, thus yielding the desired expressions, which are not explicitly provided here due to space constraints. However, since these exact expressions are relatively complex and not very intuitive, we also derive the corresponding high SNR asymptotes in the following.

2) High SNR Asymptotics: For determining the high SNR asymptotes, we pursue an approach similar to the one presented in [12]. For $\bar{\gamma} \to \infty$, (3) may be written as

$$M_{\gamma_i}(-s) = \left(\frac{m}{\gamma_0}\right)^{\alpha} \frac{1}{\prod_{\nu=1}^{P} \lambda_\nu^{\mu_\nu}} \frac{1}{s^\alpha} + o\left(\frac{1}{s^\alpha}\right), \tag{23}$$

with the short-hand notation

$$\alpha = m\, \mathrm{rank}(\Lambda_{TX}), \tag{24}$$

and where $o(x)$ represents an (arbitrary) function for which $\lim_{x\to 0} o(x)/x = 0$. Hence, it is quite obvious that the pdf and cdf of γ_i can be expressed in this case as

$$p_{\gamma_i}(\zeta) = \left(\frac{m}{\gamma_0}\right)^{\alpha} \frac{1}{\prod_{\nu=1}^{P} \lambda_\nu^{\mu_\nu}} \frac{\zeta^{\alpha-1}}{\Gamma(\alpha)} + o\left(\zeta^{\alpha-1}\right) \tag{25}$$

as well as

$$F_{\gamma_i}(\zeta) = \left(\frac{m}{\gamma_0}\right)^{\alpha} \frac{1}{\prod_{\nu=1}^{P} \lambda_\nu^{\mu_\nu}} \frac{\zeta^{\alpha}}{\Gamma(\alpha+1)} + o\left(\zeta^{\alpha}\right). \tag{26}$$

The pdf of the combined SNR γ_{S} is then given by

$$p_{\gamma_{\mathrm{S}}}(\zeta) = \frac{N_{RX}}{\Gamma(\alpha)} \left(\frac{m}{\gamma_0}\right)^{\alpha N_{RX}} \left(\prod_{\nu=1}^{P} \lambda_\nu^{\mu_\nu} \right)^{-N_{RX}} \frac{\zeta^{\alpha N_{RX}-1}}{(\Gamma(\alpha+1))^{N_{RX}-1}} + o\left(\zeta^{\alpha N_{RX}-1}\right) \tag{27}$$

and the corresponding mgf consequently can be determined by means of the Laplace transform of (27) as

$$M_{\gamma_S}(s) = \frac{N_{RX}}{\Gamma(\alpha)} \left(\frac{m}{(-s)\,\gamma_0}\right)^{\alpha\, N_{RX}} \left(\prod_{\nu=1}^{P} \lambda_\nu^{\mu_\nu}\right)^{-N_{RX}} \frac{\Gamma(\alpha\, N_{RX})}{(\Gamma(\alpha+1))^{N_{RX}-1}} + o\left(\frac{1}{s^{\alpha\, N_{RX}}}\right). \qquad (28)$$

The actual high SNR asymptotes can then be obtained by neglecting the higher-order terms $o\left(\frac{1}{s^{\alpha N_{RX}}}\right)$ in (28) and inserting the resulting expression in (21) and (22), respectively [12]. This requires in both cases the solution of integrals of the form

$$\mathcal{I}_2(\theta, n) = \int_0^\theta \sin^{2n}\phi\, d\phi, \quad n \in \mathbb{N}_0. \qquad (29)$$

In this regard, we find by capitalizing on [8] eq. (2.513,1)

$$\mathcal{I}_2(\theta, n) = \frac{\theta}{2^{2n}}\binom{2n}{n} + \frac{(-1)^n}{2^{2n-1}} \times \sum_{k=0}^{n-1}(-1)^k \binom{2n}{k} \frac{\sin(2\,\theta\,(n-k))}{2\,(n-k)} \qquad (30)$$

and hence the high SNR asymptotes for M-PSK are given by

$$P_s \approx \frac{\vartheta}{\pi\,\sin^{2\alpha N_{RX}}\left(\frac{\pi}{M}\right)} \frac{1}{\bar{\gamma}^{\alpha\, N_{RX}}} \mathcal{I}_2\left(\tfrac{M-1}{M}\,\pi, \alpha\, N_{RX}\right) \qquad (31)$$

whereas in case of M-QAM we find

$$P_s \approx \frac{4}{\pi} \frac{\vartheta}{\bar{\gamma}^{\alpha\, N_{RX}}} \left(1 - \tfrac{1}{\sqrt{M}}\right) \left(\frac{2\,(M-1)}{3}\right)^{\alpha\, N_{RX}} \times \left[\mathcal{I}_2\left(\tfrac{\pi}{2}, \alpha\, N_{RX}\right) - \left(1 - \tfrac{1}{\sqrt{M}}\right) \mathcal{I}_2\left(\tfrac{\pi}{4}, \alpha\, N_{RX}\right)\right], \qquad (32)$$

where we have introduced for brevity the short-hand notation

$$\vartheta = \frac{N_{RX}\,\left(m\, R_c\, N_{TX}\right)^{\alpha\, N_{RX}}\,\Gamma(\alpha\, N_{RX})}{\Gamma(\alpha)\,\left(\Gamma(\alpha+1)\right)^{N_{RX}-1}\left(\prod_{\nu=1}^{P}\lambda_\nu^{\mu_\nu}\right)^{N_{RX}}}. \qquad (33)$$

Based on these high SNR asymptotes, it can easily be seen that in the high SNR regime $P_S \sim \bar{\gamma}^{-\alpha\, N_{RX}}$, i.e., the diversity order with receive antenna selection is exactly the same as for a full-complexity system without antenna selection.

V. Numerical Results

Fig. 1 shows the average SER of our system versus the average SNR in case that the well-known Alamouti scheme is employed in a spatially correlated Rayleigh-fading channel and for $N_{RX} \in \{1; 2\}$. In this regard, we assume an exponential correlation model, i.e., the (i, j)-th entry of Λ_{TX} is given by $[\Lambda_{TX}]_{i,j} = \rho^{|i-j|}$, where ρ represents a correlation coefficient for adjusting the degree of correlation. As can be seen, there is basically a perfect match between calculated and simulated values, what verifies the accuracy of our theoretical analysis. Furthermore, it can be seen that the high SNR asymptotes (which are only given for QPSK for clarity) are really tight in the high SNR regime and that spending an additional receive antenna element leads to considerably smaller SERs, particularly at high SNRs, where the additional diversity gain that can be obtained this way is most effective.

Fig. 2 depicts the system capacity for two different orthogonal STBCs and $N_{RX} \in \{1; 2; 4\}$. Obviously, the capacity can be significantly increased with more receive antennas, but this increase is clearly smaller for the STBC with four transmit antennas and rate $1/2$ than for the Alamouti code with N_{TX} = 2 and R_C = 1. This is because with more transmit antennas the variations of the individual SNRs γ_i according to (1) are less severe due to the higher diversity order that we get this way, so that the probability that the SNR γ_S of the selected branch is very high is decreasing as well.

In Figs. 3 and 4, we compare the performance of our system with that of a full-complexity system where always all antennas are used as well as a SIMO system with selection combining. Please note that the SIMO system basically represents a special case of the system considered herein with N_{TX} = 1 and R_C = 1 whereas the capacity and average SER for the full complexity system can be taken from [9], for instance.

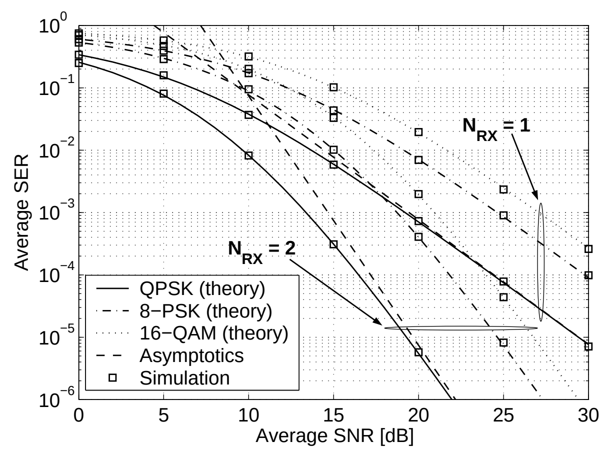

Fig. 1. Average SER for several different modulation schemes in a Rayleigh fading channel (m = 1) with ρ = 0.5, N_{TX} = 2, R_C = 1, and $N_{RX} \in \{1; 2\}$.

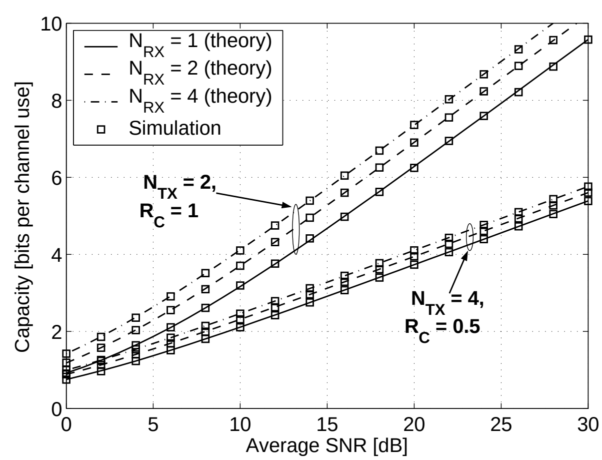

Fig. 2. Capacity versus average SNR for different numbers of receive antenna elements in uncorrelated Rayleigh-fading channels (m = 1, ρ = 0).

Fig. 3 shows the capacity difference between the aforementioned schemes and the considered system for various different degrees of spatial correlation. Obviously, with two receive antennas the SIMO system with selection combining leads to a slightly lower capacity than our system, but if more receive antenna elements are available, the capacity surprisingly is higher, despite the reduced training and signal processing complexity required in that case in practice. This can be explained in the same way as before, namely that the variations of the individual SNRs γ_i are reduced if an orthogonal STBC is used, thus reducing the probability that γ_{S} is very high as well. This is an important result, of course, which reveals that simultaneously using diversity techniques at both the transmitter and the receiver-side might actually lead to a performance degradation—at least from a capacity point of view. However, with increasing spatial correlation (i.e., for $\rho \to 1$), the capacity difference between the two schemes is diminishing since in this case the diversity advantage of orthogonal STBC is diminishing as well.

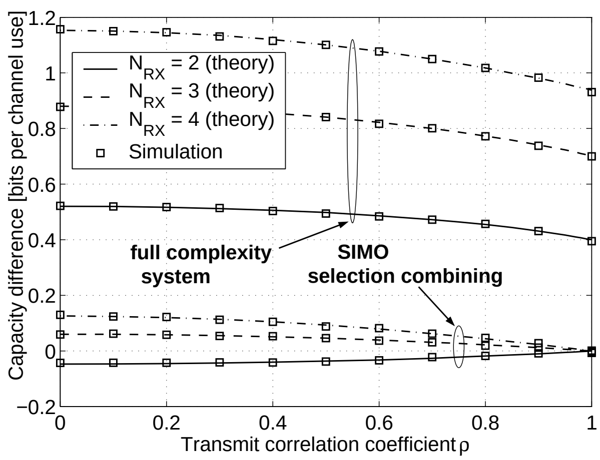

Fig. 3. Capacity difference between a full-complexity system as well as a SIMO system with selection combining and the considered MIMO system with antenna selection for N_{TX} = 2, R_C = 1, m = 1, and $\bar{\gamma}$ = 10 dB.

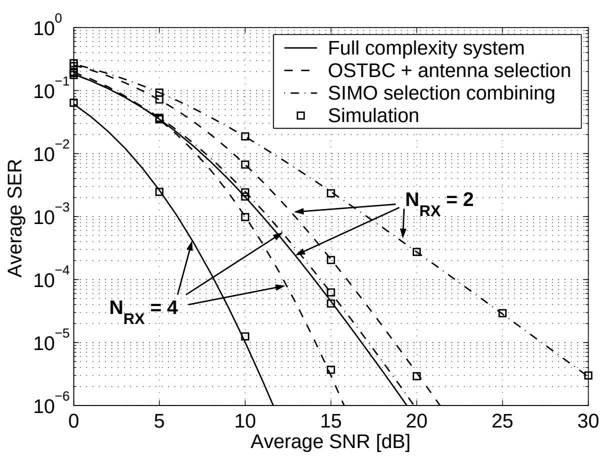

Fig. 4. Comparison of the average SERs of the considered systems for N_{TX} = 2, R_C = 1, m = 1, ρ = 0, and QPSK modulation.

In case of the average SER performance, we have a different situation, as can be seen from Fig. 4. Here, our system with antenna selection clearly outperforms the SIMO system with selection combining due to the higher diversity order that we have in this case. This is because the average SER performance is mainly governed by deep fades, which, however, can be drastically reduced with higher diversity. Another thing that can be observed from Fig. 4 is that there is a significant SNR gap between the full-complexity system and our system with antenna selection, which can be explained by the fact that more power can be extracted with the full-complexity system since always all receive antenna elements are used.

VI. Conclusion

We have conducted a comprehensive performance analysis of MIMO systems employing orthogonal STBC combined with receive antenna selection in semi-correlated Nakagami-m fading channels. We have derived exact analytical closed-form expressions for the capacity as well as the average SER in case of M-QAM and M-PSK modulation and we have determined the corresponding high SNR asymptotes, which are somewhat more intuitive than the exact expressions. Furthermore, we have shown that from a capacity point of view using only one transmit antenna might lead to a superior performance than employing orthogonal STBCs, despite the lower implementation complexity required in that case. Numerical results were shown to be in perfect agreement with simulation results, thus verifying the validity of our theoretical analysis.

References

[1] A. Molisch and M. Z. Win, “MIMO systems with antenna selection,” *IEEE Microwave Magazine*, vol. 5, no. 1, pp. 46 – 56, Mar. 2004.

[2] D. J. Love, “On the probability of error of antenna-subset selection with space-time block codes,” *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1799 – 1803, Nov. 2005.

[3] S. Kaviani and C. Tellambura, “Closed-form BER analysis for antenna selection using orthogonal space-time block codes,” *IEEE Commun. Letters*, vol. 10, no. 10, pp. 704 – 706, Oct. 2006.

[4] X. N. Zeng and A. Ghrayeb, “Performance bounds for space-time block codes with receive antenna selection,” *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 2130 – 2137, Sept. 2004.

[5] W. Li and N. C. Beaulieu, “Effects of channel-estimation errors on receiver selection-combining schemes for Alamouti MIMO systems with BPSK,” *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 169 – 178, Jan. 2006.

[6] H. Shin and J. H. Lee, “Performance analysis of space-time block codes over keyhole Nakagami-m fading channels,” *IEEE Trans. Veh. Technol.*, vol. 53, no. 2, pp. 351 – 362, Mar. 2004.

[7] J. Luo, J. R. Zeidler, and S. McLaughlin, “Performance analysis of compact antenna arrays with MRC in correlated Nakagami fading channels,” *IEEE Trans. Veh. Technol.*, vol. 50, no. 1, pp. 267 – 277, Jan. 2001.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 6th ed. New York: Academic Press, 2000.

[9] A. Maaref and S. Aissa, “Performance analysis of orthogonal space-time block codes in spatially correlated MIMO Nakagami fading channels,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 4, pp. 807 – 817, Apr. 2006.

[10] M.-S. Alouini and A. Goldsmith, “Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques,” *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165 – 1181, July 1999.

[11] M. K. Simon and M.-S. Alouini, *Digital communications over fading channels*. New York: John Wiley & Sons, 2000.

[12] Z. Wang and G. B. Giannakis, “A simple and general parameterization quantifying performance in fading channels,” *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389 – 1398, Aug. 2003.