Capacity of Multiple-Input Multiple-Output Keyhole Channels with Antenna Selection

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***Abstract*— This paper presents a comprehensive capacity analysis of multiple-input multiple-output (MIMO) keyhole channels with antenna selection. In this regard, two different scenarios are considered, with either receive antenna selection only or with joint antenna selection at both the transmitter and the receiver-side, respectively. For both cases, we derive exact analytical closed-form expressions for the ergodic capacity and we present somewhat simpler upper and lower bounds, which are shown to be asymptotically tight for high signal-to-noise ratios. Furthermore, we present concise mathematical expressions for the corresponding information outage probabilities, thus completely characterizing the statistical properties of the mutual information. Numerical results are shown to be in perfect agreement with results obtained from Monte-Carlo simulations and illustrate the impact of several different parameters on the capacity.**

I. Introduction

Multiple-input multiple-output (MIMO) systems are known to offer tremendous capacity gains over conventional single antenna arrangements and are therefore expected to find numerous applications in future wireless communication systems. In fact, it has been shown in [1] that in case of independent and identically distributed Rayleigh-fading, the ergodic capacity scales linearly with the minimum out of the number of transmit and receive antenna elements, respectively. However, in real-world propagation scenarios the actual capacity gains are often much smaller than those reported in [1] due to the detrimental effects of spatial fading correlation as well as possible degenerate channel phenomena caused by so-called keyholes. Such keyhole channels characterize rank-deficient MIMO channels, which may have sufficient scattering around the transmitter and the receiver to obtain uncorrelated or at least only weakly correlated signals, but due to other propagation effects, such as certain diffraction or waveguiding phenomena, the channel matrix might nevertheless exhibit only low rank [2].

Naturally, using multiple antennas on both sides of a wireless link generally increases the signal processing complexity and leads to significantly higher hardware costs, particularly due to the additional radio-frequency (RF) chains required for amplifying, up- and down-converting as well as digital-to-analog and analog-to-digital converting the corresponding transmit and receive signals, respectively. Since the costs for the additional antenna elements themselves are usually more or less negligible, an attractive approach for partially alleviating these drawbacks is to perform antenna selection, whereby the number of required RF chains can be reduced while most of the benefits offered by MIMO systems can be retained [3].

In this paper, we perform a comprehensive capacity analysis of MIMO keyhole channels with antenna selection, where we consider two different cases: On the one hand receive antenna selection only with uninformed transmitter and on the other hand joint transmit and receive antenna selection, where the selection at the transmitter-side is based upon partial channel state information (CSI) fed back from the receiver. For both cases, we derive exact as well as approximate closed-form expressions for the ergodic capacity and we determine the corresponding information outage probabilities, thus completely characterizing the statistical properties of the mutual information. Please note that a first analysis of the capacity of keyhole channels with antenna selection has already been presented in [4], but there the authors basically only show that the diversity order of a keyhole channel with antenna selection is the same as the diversity order of the same channel without antenna selection whereas the actual capacity is only investigated by means of Monte-Carlo simulations. The capacity of keyhole channels without antenna selection, in contrast, has already been extensively studied in literature before, see for example [5], [6] and [7].

The remainder of this paper is structured as follows: In Section II, we shortly introduce our system and channel model. The actual capacity analysis is done in Section III whereas some numerical results are presented in Section IV. Finally, our main conclusions are given in Section V.

II. System and Channel Model

We consider a frequency-flat MIMO system with N_{TX} transmit antennas and N_{RX} receive antennas. The discrete-time equivalent baseband representation of the channel is modeled by the matrix $\mathbf{H} \in \mathbb{C}^{N_{RX} \times N_{TX}}$ and we assume to have a perfect keyhole channel, i.e., the only way for the radio waves to propagate from the transmitter to the receiver is to pass through a keyhole, what could be a hallway acting as a single-mode waveguide, for example [2]. In this case, the channel can be considered as a concatenation of a multiple-input single-output (MISO) channel from the individual transmit antennas to the keyhole and a (statistically independent) single-input multiple-output (SIMO) channel from the keyhole to the various receive antennas [2], i.e., we have $\mathbf{H} = \mathbf{h}_{\text{SIMO}}\, \mathbf{h}_{\text{MISO}}^H$, where $(\cdot)^H$ refers to the conjugate-transpose of a vector or matrix and $\mathbf{h}_{\text{SIMO}} \in \mathbb{C}^{N_{RX}}$ as well as $\mathbf{h}_{\text{MISO}} \in \mathbb{C}^{N_{TX}}$ denote the aforementioned SIMO and MISO channels, respectively. All elements of these vectors are modeled as i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance one, thus corresponding to a Rayleigh-fading scenario. Generally, we select M out of the available N_{TX} transmit antennas and N out of the available N_{RX} receive antennas. Due to the special structure of the considered keyhole channel, it can easily be seen that the joint transmit and receive antenna selection problem can be decomposed into two independent antenna selection problems at the transmitter and the receiver-side, respectively [4]. The effective channel matrix hence can be written as $\mathbf{H}_{\text{eff}} = \mathbf{x}\,\mathbf{y}^H$, where $\mathbf{x} \in \mathbb{C}^N$ and $\mathbf{y} \in \mathbb{C}^M$ denote the effective SIMO- and MISO channels from and to the keyhole, which result from selecting the M and N strongest elements of $\mathbf{h}_{\text{SIMO}}$ and $\mathbf{h}_{\text{MISO}}^H$, respectively. The input-output relationship of our system is then given by

$$\mathbf{r} = \sqrt{\bar{\gamma}/M}\,\mathbf{H}_{\text{eff}}\,\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{r} \in \mathbb{C}^N$ denotes the received signal, $\mathbf{s} \in \mathbb{C}^M$ the transmitted signal, and $\mathbf{n} \in \mathbb{C}^N$ an additive white Gaussian noise vector, whose elements are i.i.d. circularly symmetric complex Gaussian distributed with zero mean and variance one. Furthermore, $\bar{\gamma}$ denotes the average signal-to-noise ratio (SNR) per receive antenna and the transmit signal $\mathbf{s}$ is assumed to be composed of M statistically independent unity average power components, each being circularly symmetric complex Gaussian distributed with zero mean. The channel is always assumed to be perfectly known by the receiver whereas it is either unknown to the transmitter or where the transmitter just knows the indices of the antenna elements to be selected. This partial CSI might be fed back by the receiver using a low-rate feedback channel, what makes this approach an attractive solution for practical implementations.

III. Capacity Analysis

Since we assume that the transmitter has either no or just partial knowledge of the channel, we always perform uniform power allocation among the different transmit antenna elements. For a fixed realization of $\mathbf{H}_{\text{eff}}$, the mutual information between the transmit signal $\mathbf{s}$ and the received signal $\mathbf{y}$ is hence given by (in nats per channel use) [1]

$$I(\mathbf{s};\mathbf{r}) = \ln\det\left(\mathbf{I}_{N_{RX}} + \frac{\bar{\gamma}}{M}\,\mathbf{H}_{\text{eff}}\,\mathbf{H}_{\text{eff}}^H\right), \tag{2}$$

where $\mathbf{I}_n$ denotes the identity matrix of dimension n. Exploiting the special structure of $\mathbf{H}_{\text{eff}}$ for the considered keyhole channels and further making use of the determinant identity $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$ for arbitrary matrices $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{AB}$ is square, (2) can be shown to be equivalent to [7]

$$I(\mathbf{s};\mathbf{r}) = \ln\left(1 + \frac{\bar{\gamma}}{M}\,\Xi\right), \tag{3}$$

where $\Xi = X\,Y$ with $X = ||\mathbf{x}||^2$, $Y = ||\mathbf{y}||^2$, and $||\cdot||^2$ as the squared Euclidian norm of a vector. The ergodic capacity can then be obtained by averaging (3) over the distribution of Ξ, i.e., $C_{\text{erg}} = \mathbb{E}_\Xi\left[\ln(1+\bar{\gamma}/M\,\Xi)\right]$ with $\mathbb{E}[\cdot]$ as the expectation operator. Clearly, the distributions of X and Y and hence also the distribution of Ξ depend on the realized antenna selection strategy and for that reason we will separately analyze the two different strategies that we consider herein in the following.

A. Receive Antenna Selection Only

If the transmitter does not have any CSI, it always uses all available transmit antennas, i.e., $M = N_{TX}$, and consequently it generally requires N_{TX} different RF chains. The receiver, in contrast, always selects the $N \le N_{RX}$ best receive antennas for which $X = ||\mathbf{x}||^2$ is maximized. The ergodic capacity for this case is given by the following theorem:

Theorem 1: The ergodic capacity of keyhole channels with receive antenna selection only and no CSI at the transmitter is given by (in nats per channel use)

$$\begin{aligned} C_I \;=\; & \frac{\binom{N_{RX}}{N}}{\Gamma(N_{TX})}\left[\frac{1}{\Gamma(N)}\,G_{2,4}^{4,1}\left[\frac{N_{TX}}{\bar{\gamma}}\,\middle|\,\begin{matrix}0, & 1\\ N, & N_{TX},0,0\end{matrix}\right]\right.\\ & + \sum_{i=1}^{N_{RX}-N}\beta_i\left[\frac{N}{N+i}\,G_{1,3}^{3,1}\left[\frac{N_{TX}}{\bar{\gamma}\,\frac{N}{N+i}}\,\middle|\,\begin{matrix}0\\ N_{TX},0,0\end{matrix}\right]\right.\\ & \left.\left. - \sum_{j=1}^{N-1}\eta_{i,j}\,G_{2,4}^{4,1}\left[\frac{N_{TX}}{\bar{\gamma}}\,\middle|\,\begin{matrix}0, & 1\\ j, & N_{TX},0,0\end{matrix}\right]\right]\right], \end{aligned} \tag{4}$$

where $G_{p,q}^{m,n}\left[\cdot\,|\cdot\right]$ denotes the Meijer G-function and $\Gamma(\cdot)$ the well-known gamma function [8]. Furthermore, for the sake of brevity, we have introduced the short-hand notations

$$\beta_i = (-1)^i\binom{N_{RX}-N}{i}\left(\frac{-N}{i}\right)^{N-1} \tag{5}$$

$$\eta_{i,j} = \frac{1}{\Gamma(j)}\left(\frac{-i}{N}\right)^{j-1}. \tag{6}$$

Proof: Following the approach outlined in [9], for example, the probability density function (pdf) of the random variable X can be determined as

$$\begin{aligned} p_X(x) \;=\; & \binom{N_{RX}}{N}\left[x^{N-1}\,\frac{e^{-x}}{\Gamma(N)} + \sum_{i=1}^{N_{RX}-N}\beta_i\,e^{-x}\right.\\ & \left.\times\left(e^{-\frac{i\,x}{N}} - \sum_{j=1}^{N-1}\eta_{i,j}\,x^{j-1}\right)\right], \quad x \ge 0 \end{aligned} \tag{7}$$

with β_i and $\eta_{i,j}$ according to (5) and (6), respectively. Furthermore, since always all transmit antennas are used and the available transmit power is equally distributed among the various antenna elements, Y is gamma distributed with pdf

$$p_Y(y) = \frac{1}{\Gamma(N_{TX})}\,y^{N_{TX}-1}\,e^{-y}, \quad y \ge 0. \tag{8}$$

The pdf of the product $\Xi = X\,Y$ can then be determined as

$$p_\Xi(\xi) = \int_0^\infty p_X(x)\,p_Y\left(\frac{\xi}{x}\right)\frac{1}{x}\,dx, \quad \xi \ge 0. \tag{9}$$

With the expressions for $p_X(x)$ and $p_Y(y)$ according to (7) and (8), this integral can be solved analytically in closed-form by making use of [8] eq. (3.471,9), yielding to (10), which is given at the top of the next page, and where $K_\nu(\cdot)$ denotes the ν-th order modified Bessel function of the second kind [8]. The ergodic capacity then can be calculated as $C_I = \int_0^\infty \ln\left(1+\xi\,\bar{\gamma}/N_{TX}\right)\,p_\Xi(\xi)\,d\xi$. By inserting (10) in this formula, it turns out that the problem of calculating the ergodic

$$p_\Xi(\xi) = \frac{2}{\Gamma(N_{TX})}\binom{N_{RX}}{N}\left[\frac{1}{\Gamma(N)}\,\xi^{\frac{N_{TX}+N}{2}-1}\,K_{N-N_{TX}}\left(2\sqrt{\xi}\right) + \sum_{j=1}^{N_{RX}-N}\beta_i\left[\left(\frac{N+i}{N}\,\xi\right)^{\frac{N_{TX}-1}{2}} K_{N_{TX}-1}\left(2\sqrt{\xi\,\frac{N+i}{N}}\right) - \sum_{j=1}^{N-1}\eta_{i,j}\,\xi^{\frac{N_{TX}+j}{2}-1}\,K_{j-N_{TX}}\left(2\sqrt{\xi}\right)\right]\right]. \quad (10)$$

capacity can be traced back to the problem of solving integrals of the form

$$\mathcal{I} = \int_0^\infty \ln(1+a\,x)\,x^p\,K_n\left(2\sqrt{b\,x}\right)\,dx \quad (11)$$

with positive constants a and b and non-negative n and p. Due to the complicated structure of the integrand, it is to the best of our knowledge not possible to find an analytical closed-form solution for this integral for general values of a, b, n, and p by means of conventional integration methods. For that reason, we resort to an approach based upon Meijer G-functions instead, which has already been used in [5]–[7]. Meijer G-functions are very general functions, which contain virtually all known elementary functions as special cases [8]. In particular, it is well-known that the logarithm in (11) might be expressed in terms of a Meijer G-function as (cf. [10] eq. (8.4.6,5))

$$\ln(1+a\,x) = G_{2,2}^{1,2}\left[a\,x\,\middle|\,\begin{matrix}1, & 1\\ 1, & 0\end{matrix}\right]. \quad (12)$$

With this identity, (11) can be expressed in a form for which we can invoke the relationship given in [8] eq. (7.821,3) and hence solve the integral (11) in closed-form as

$$\mathcal{I} = \frac{1}{2\,b^{p+1}}\,G_{4,2}^{1,4}\left[\frac{a}{b}\,\middle|\,\begin{matrix}-p-\frac{n}{2}, & -p+\frac{n}{2}, & 1, & 1\\ 1, & 0 & & \end{matrix}\right]. \quad (13)$$

After some simple mathematical manipulations and exploitation of various basic properties of Meijer G-functions (see for example [8] eqs. (9.31,1) and (9.31,2)), we finally get the result given in (4), what eventually concludes the proof. ■

As a special case of the more general result provided by Theorem 1, we can formulate the following corollary:

Corollary 1: The ergodic capacity of keyhole channels without antenna selection is given by (in nats per channel use)

$$C_I' = \frac{1}{\Gamma(N_{TX})\,\Gamma(N_{RX})}\,G_{2,4}^{4,1}\left[\frac{N_{TX}}{\bar{\gamma}}\,\middle|\,\begin{matrix}0,1\\ N_{RX},N_{TX},0,0\end{matrix}\right]. \quad (14)$$

Proof: By setting $N = N_{RX}$ in (4) we directly obtain the given expression. ■

Please note that this result is perfectly in line with [5] and [7], where this special case has already been covered before.

Even though the exact capacity expression according to (4) might be easily evaluated numerically since Meijer G-functions are readily available in standard mathematical software packages, these functions are not very well-known and particularly not very intuitive, i.e., it is hard to analyze the impact of various different parameters on the capacity solely based on this expression. For that reason, we present somewhat simpler upper and lower bounds in the following, which are quite close to the exact ergodic capacity and which can be expressed by means of elementary functions only.

Theorem 2: A lower bound on the exact ergodic capacity C_I according to (4) is given by

$$C_{\text{I,low}} = \ln\left(1+\frac{\bar{\gamma}}{N_{TX}}\,e^{\psi(N_{TX})+\zeta}\right), \quad (15)$$

with

$$\zeta = \binom{N_{RX}}{N}\left[\psi(N) - \sum_{i=1}^{N_{RX}-N}\beta_i\left[\sum_{j=1}^{N-1}\left(\frac{-i}{N}\right)^{j-1}\psi(j) + \frac{N}{N+i}\left(\epsilon+\ln\frac{N+i}{N}\right)\right]\right] \quad (16)$$

and $\psi(\cdot)$ as Euler's Psi-function, which is given for integer arguments n as $\psi(n) = -\epsilon + \sum_{k=1}^{n-1}\frac{1}{k}$, where $\epsilon = 0.577215\ldots$ denotes the Euler-Mascheroni constant [8].

Proof: The derivation of the lower bound can be done similarly to the derivation of the lower bound on the capacity of spatially correlated keyhole channels without antenna selection presented in [11]. Exploiting that $\ln(1+a\,e^x)$ is a convex function in x for any $a>0$, rewriting the general capacity formula as $C_I = \mathbb{E}\left[\ln\left(1+\frac{\bar{\gamma}}{N_{TX}}\exp(\ln X+\ln Y)\right)\right]$ and making use of Jensen's inequality, we obtain

$$C_I \geq C_{\text{I,low}} = \ln\left(1+\frac{\bar{\gamma}}{N_{TX}}\,e^{\mathbb{E}_X[\ln X]+\mathbb{E}_Y[\ln Y]}\right). \quad (17)$$

Capitalizing on [8] eq. (4.352,1) and the known pdfs of X and Y, we find $\mathbb{E}_X[\ln X] = \zeta$ as well as $\mathbb{E}_Y[\ln Y] = \psi(N_{TX})$. ■

Theorem 3: An upper bound on the exact ergodic capacity C_I according to (4) is given by

$$C_{\text{I,up}} = \ln\left(\frac{\bar{\gamma}}{N_{TX}}\right)+\zeta+\psi(N_{TX})+\sqrt{\frac{N_{TX}}{\bar{\gamma}}}\,\frac{\Gamma\left(N_{TX}-\frac{1}{2}\right)}{\Gamma(N_{TX})}\,\chi, \quad (18)$$

with ζ according to (16) and the short-hand notation

$$\chi = \binom{N_{RX}}{N}\left[\frac{\Gamma\left(N-\frac{1}{2}\right)}{\Gamma(N)} + \sum_{i=1}^{N_{RX}-N}\beta_i \times\left[\sqrt{\frac{\pi\,N}{N+i}} - \sum_{j=1}^{N-1}\eta_{i,j}\,\Gamma\left(j-\frac{1}{2}\right)\right]\right], \quad (19)$$

which has been introduced for brevity again.

Proof: It can easily be checked that $\ln(1+x) \leq \ln(x) + x^{-1/2} \;\;\forall\;\; x \geq 0$. Hence, $C_I = \mathbb{E}\left[\ln\left(1+\bar{\gamma}/N_{TX}\,X\,Y\right)\right]$ is upper-bounded by

$$C_I \leq \mathbb{E}\left[\ln\left(\frac{\bar{\gamma}\,X\,Y}{N_{TX}}\right)\right] + \sqrt{\frac{N_{TX}}{\bar{\gamma}}}\,\mathbb{E}\left[(X\,Y)^{-1/2}\right]. \quad (20)$$

Exploiting the statistical independence of X and Y, we obtain

$$\mathbb{E}\left[\ln\left(\frac{\bar{\gamma}\,X\,Y}{N_{TX}}\right)\right] = \ln\left(\frac{\bar{\gamma}}{N_{TX}}\right) + \mathbb{E}_X[\ln(X)] + \mathbb{E}_Y[\ln(Y)] \tag{21}$$

as well as $\mathbb{E}\left[(X\,Y)^{-1/2}\right] = \mathbb{E}_X\left[X^{-1/2}\right]\,\mathbb{E}_Y\left[Y^{-1/2}\right]$. The expectations of $\ln(X)$ and $\ln(Y)$ have already been determined before and are given by ζ according to (16) and $\psi(N_{TX})$, respectively. The expectations of $1/\sqrt{X}$ and $1/\sqrt{Y}$, on the other hand, can directly be calculated based on (7) and (8) by making use of [8] eq. (3.381,4) as well as the well-known relationship that $\Gamma(1/2) = \sqrt{\pi}$ [12], what finally leads to the expression given in (18). ■

Corollary 2: Both, the upper bound $C_{\text{I,up}}$ as well as the lower bound $C_{\text{I,low}}$ are asymptotically tight for high SNRs and the actual high SNR asymptotics are given by

$$C_{\text{I,asymp}} = \ln\left(\frac{\bar{\gamma}}{N_{TX}}\right) + \zeta + \psi(N_{TX}), \tag{22}$$

with ζ according to (16).

Proof: It can easily be checked that for $\bar{\gamma} \to \infty$ both $C_{\text{I,up}}$ and $C_{\text{I,low}}$ approach $C_{\text{I,asymp}}$. Since $C_{\text{I,low}} \leq C_I \leq C_{\text{I,up}}$ holds in general, this automatically implies that $C_I \to C_{\text{I,asymp}}$ for $\bar{\gamma} \to \infty$ as well. ■

An interesting question that arises in this context is what capacity gain we get if we spend more transmit or receive antennas. In this regard, we particularly consider the high SNR regime again, for which we can quantify this capacity gain by means of elementary functions only. The corresponding results are given by the following two corollaries.

Corollary 3: In the high SNR regime, the additional capacity gain that can be obtained by increasing the number of transmit antennas from N_{TX} to $N_{TX}+L$ is given by

$$\Delta_{C_I}(N_{TX}, N_{TX}+L) = \sum_{k=N_{TX}}^{N_{TX}+L-1} \frac{1}{k} + \ln\,\frac{N_{TX}}{N_{TX}+L}. \tag{23}$$

Proof: It can immediately be seen from (22) that $\Delta_{C_I}(N_{TX}, N_{TX}+L) = \psi(N_{TX}+L) - \psi(N_{TX}) + \ln\frac{N_{TX}}{N_{TX}+L}$, what directly corresponds to the expression given in (23). ■

Corollary 4: In the high SNR regime, the capacity gain that we get by increasing the number of receive antennas from N_{RX} to $N_{RX}+L$ (while keeping N constant) is given by

$$\Delta_{C_I}(N_{RX}, N_{RX}+L) = \sum_{i=N_{RX}}^{N_{RX}+L-1} \Delta_{C_R}(i), \tag{24}$$

with

$$\begin{aligned}\Delta_{C_{I,R}}(i) \;=\; & \binom{i}{N-1}\psi(N) - \sum_{k=1}^{i-N+1}(-1)^k\binom{i}{N+k-1} \\ & \times\binom{N+k-1}{k}\left(\frac{-N}{k}\right)^{N-1}\left[\epsilon + \ln\frac{N+k}{N}\right. \\ & \left. + \left(\frac{N+k}{N}\right)\sum_{j=1}^{N-1}\left(\frac{-k}{N}\right)^{j-1}\psi(j)\right].\end{aligned} \tag{25}$$

Proof: It can easily be seen from (22) that the capacity gain that we get in the high SNR regime by increasing the number of receive antennas from i to $i+1$ is given by

$$\Delta_{C_{I,R}}(i) = \zeta|_{N_{RX}=i+1} - \zeta|_{N_{RX}=i}\,, \tag{26}$$

with ζ according to (16). Plugging this expression in (26), we find after some basic mathematics and exploitation of the fact that $\binom{i}{k} = \binom{i-1}{k} + \binom{i-1}{k-1}$ the result given in (24). ■

While the ergodic capacity that has been considered so far is generally suitable for characterizing ergodic fading channels, it is usually more expedient to consider the information outage probability in case of non-ergodic fading channels instead, which we therefore will determine in the following. In fact, the information outage probability represents the probability that a certain transmission rate R cannot be supported by the channel and corresponds to the cumulative distribution function (cdf) of the mutual information according to (3). The corresponding result is given by the following theorem:

Theorem 4: The information outage probability of keyhole channels with receive antenna selection is given by

$$\begin{aligned}P_I \;=\; & \binom{N_{RX}}{N}\left[1 - \frac{\Phi(N,0)}{\Gamma(N)} - \sum_{i=1}^{N_{RX}-N}\beta_i\left[\Phi_1\left(1, \frac{i}{N}\right)\right.\right. \\ & \left.\left. -\frac{N}{N+i} + \sum_{j=1}^{N-1}\eta_{i,j}\,\left[\Gamma(j) - \Phi_1(j,0)\right]\right]\right],\end{aligned} \tag{27}$$

with the short-hand notations

$$\begin{aligned}\Phi_1(a,b) \;=\; & 2\sum_{k=0}^{N_{TX}-1}\frac{\Lambda(R)^{\frac{a+k}{2}}}{k!}\,(1+b)^{\frac{k-1}{2}} \\ & \times K_{a-k}\left(2\sqrt{\Lambda(R)\,(1+b)}\right)\end{aligned} \tag{28}$$

and

$$\Lambda(R) = \frac{N_{TX}}{\bar{\gamma}}\,\left(e^R - 1\right). \tag{29}$$

Proof: It can easily be seen that P_I = Prob$[I(\mathbf{s};\mathbf{r}) \leq R]$ = Prob$[\Xi \leq \Lambda(R)]$, with $\Lambda(R)$ as defined in (29). Hence, P_I generally can be calculated as $P_I = \int_0^{\Lambda(R)} p_\Xi(\xi)\,d\xi$. Even though a closed-form expression for $p_\Xi(\xi)$ is already available with (10), directly solving this integral seems to be mathematically intractable. For that reason, we resort to the more generic expression of $p_\Xi(\xi)$ according to (9) and obtain after changing the order of integration

$$P_I = \int_0^\infty \frac{1}{x}\,p_X(x)\int_0^{\Lambda(R)} p_Y\left(\frac{\xi}{x}\right)\,d\xi\,dx. \tag{30}$$

Inserting the corresponding expressions for $p_X(x)$ and $p_Y(y)$ according to (7) and (8), respectively, the inner integral can be solved in closed-form by making use of [8] eq. (3.381,1). If we then exploit that the lower incomplete gamma function $\gamma(n,x)$ can be calculated for integer arguments n as $\gamma(n,x) = \Gamma(n)\left[1 - e^{-x}\sum_{k=0}^{n-1}\frac{x^k}{k!}\right]$ [12] and capitalize on [8] eqs. (3.381,4) as well as (3.471,9), the outer integral can be solved in closed-form as well, thus leading after some rearrangements to the expression provided in (27). ■

B. Joint Transmit and Receive Antenna Selection

If antenna selection is performed at both the transmitter- and the receiver-side, we assume that the transmitter knows only the indices of the antennas to be selected—which might be fed back by the receiver using a low-rate feedback channel—and the total available transmit power is then equally distributed among the selected antenna elements. However, it has been shown in [4] and it can readily be verified that if only this partial CSI is available at the transmitter-side, it is optimal to select always only the best transmit antenna. Hence, we have $M = 1$ in this case and we solely require $\lceil \log_2 N_{TX} \rceil$ feedback bits for informing the transmitter which antenna to select.

Theorem 5: The ergodic capacity of keyhole channels with antenna selection at both the transmitter- and the receiver-side is given by the expression according to (31) at the top of the next page (in nats per channel use), where the coefficients α_k can be calculated as

$$\alpha_k = N_{TX} \binom{N_{TX}-1}{k} (-1)^k \tag{33}$$

and with β_i and $\eta_{i,j}$ according to (5) and (6), respectively.

Proof: In case that antenna selection is performed on both sides of the channel, the pdf of X is the same as in (7) whereas the cdf of Y can easily be shown to be given by

$$F_Y(y) = \left(1 - e^{-y}\right)^{N_{TX}}. \tag{34}$$

Hence, the corresponding pdf can be determined as

$$p_Y(y) = \frac{\partial}{\partial y} F_Y(y) = \sum_{k=0}^{N_{TX}-1} \alpha_k \, e^{-y\,(1+k)}, \tag{35}$$

where we made use of the binomial theorem and with α_k according to (33). In order to get the pdf of $\Xi = XY$, we then make use of (9) again, what results in (32), which is given at the top of the next page. The actual ergodic capacity can then be determined as $C_{II} = \int_0^\infty \ln(1+\bar{\gamma}\,\Xi)\, p_\Xi(\xi)\, d\xi$. Similarly to the previously considered case, this integral can be solved analytically in closed-form by making use of Meijer G-functions, what eventually leads to the expression given in (31) and hence concludes the proof. ■

As before, the exact capacity expression according to (31) corresponds to a finite sum of weighted Meijer G-functions and is hence not very intuitive. For that reason, we subsequently present a lower as well as an upper bound again, which both can be expressed by means of elementary functions only.

Corollary 5: A lower bound on the exact ergodic capacity C_{II} according to (31) is given by

$$C_{\text{II,low}} = \ln\left(1 + \bar{\gamma}\, e^{\zeta+\theta}\right), \tag{36}$$

where

$$\theta = \sum_{k=0}^{N_{TX}-1} \frac{\alpha_k}{1+k} \left(\epsilon + \ln(1+k)\right) \tag{37}$$

and with ζ as already defined in (16).

Proof: The proof can be done in a straightforward manner similarly to the proof of Theorem 2 by exploiting that $\mathbb{E}_X[\ln X] = \zeta$ as well as $\mathbb{E}_Y[\ln Y] = \theta$, where the latter result can be calculated by capitalizing on the pdf of Y according to (35) as well as [8] eq. (4.331,1). ■

Corollary 6: An upper bound on the exact ergodic capacity C_{II} according to (31) is given by

$$C_{\text{II,up}} = \ln(\bar{\gamma}) + \zeta + \theta + \frac{1}{\sqrt{\bar{\gamma}}}\, \chi\, \phi, \tag{38}$$

with ζ, θ, and χ according to (16), (37), and (19), respectively, as well as

$$\phi = \sum_{k=0}^{N_{TX}-1} \alpha_k \sqrt{\frac{\pi}{1+k}}. \tag{39}$$

Proof: The basic idea of the proof is the same as the one that has been used in Theorem 3, but now we have $\mathbb{E}_Y[\ln Y] = \theta$ as well as $\mathbb{E}_Y[1/\sqrt{Y}] = \phi$ according to (39). ■

Corollary 7: Both, the upper bound $C_{\text{II,up}}$ as well as the lower bound $C_{\text{II,low}}$ are asymptotically tight for high SNRs and the actual high SNR asymptotics are given by

$$C_{\text{II,asymp}} = \ln\left(\frac{\bar{\gamma}}{N_{TX}}\right) + \zeta + \theta, \tag{40}$$

with ζ and θ according to (16) and (37), respectively.

Proof: The proof is basically identical to the proof of corollary 2. ■

Below, we quantify the capacity gain that can be obtained in the high SNR regime by spending more transmit/receive antenna elements again. The calculation of this gain can be done analogously to the previously considered case and therefore we provide here only the final result without proof.

Corollary 8: In the high SNR regime, the capacity gain that can be obtained by increasing the number of transmit antennas from N_{TX} to $N_{TX} + L$ can be calculated as

$$\Delta_{C_{II}}(N_{TX}, N_{TX}+L) = \sum_{i=N_{TX}}^{N_{TX}+L-1} \Delta_{C_{II,T}}(i), \tag{41}$$

with

$$\Delta_{C_{II,T}}(i) = \sum_{k=0}^{i} \binom{i}{k} (-1)^{k+1} \left(\epsilon + \ln(1+k)\right). \tag{42}$$

The capacity gain that we get by increasing the number of receive antennas from N_{RX} to $N_{RX} + L$, in contrast, is the same as already given in Corollary 4.

Finally, we determine the information outage probability again, which is provided by the following theorem:

Theorem 6: The outage probability of keyhole channels with antenna selection on both sides of the channel is given by

$$\begin{aligned} P_{II} \;=\; & \sum_{k=1}^{N_{TX}} \frac{\alpha_{k-1}}{k} \left[1 - \binom{N_{RX}}{N} \left[\frac{\Phi_2(N,0,k)}{\Gamma(N)} \right.\right. \\ & + \sum_{i=1}^{N_{RX}-N} \beta_i \left[\Phi_2\left(1, \frac{i}{N}, k\right) \right. \\ & \left.\left.\left. - \sum_{j=1}^{N-1} \eta_{i,j}\, \Phi_2(j,0,k)\right]\right]\right], \end{aligned} \tag{43}$$

$$C_{II} = \binom{N_{RX}}{N} \sum_{k=0}^{N_{TX}-1} \frac{\alpha_k}{1+k} \left[\frac{1}{\Gamma(N)} G_{3,1}^{1,3}\left[\frac{\bar{\gamma}}{1+k} \middle| \begin{array}{l} 1-N,1,1 \\ 1 \end{array} \right] \right.$$
$$\left. + \sum_{i=1}^{N_{RX}-N} \beta_i \left[\frac{N}{N+i} G_{3,1}^{1,3}\left[\frac{\bar{\gamma} N}{(1+k)(N+i)} \middle| \begin{array}{l} 0,1,1 \\ 1 \end{array} \right] - \sum_{j=1}^{N-1} \eta_{i,j} G_{3,1}^{1,3}\left[\frac{\bar{\gamma}}{1+k} \middle| \begin{array}{l} 1-j,1,1 \\ 1 \end{array} \right] \right] \right] \quad (31)$$

$$p_{\Xi}(\xi) = 2\binom{N_{RX}}{N} \sum_{k=0}^{N_{TX}-1} \alpha_k \left[\frac{(\xi(1+k))^{\frac{N-1}{2}}}{\Gamma(N)} K_{N-1}\left(2\sqrt{\xi(1+k)}\right) \right.$$
$$\left. + \sum_{i=1}^{N_{RX}-N} \beta_i \left[K_0\left(2\sqrt{\frac{x(1+k)(N+i)}{N}}\right) - \sum_{j=1}^{N-1} \eta_{i,j} \left(\xi(1+k)\right)^{\frac{j-1}{2}} K_{j-1}\left(2\sqrt{\xi(1+k)}\right) \right] \right] \quad (32)$$

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with the short-hand notation

$$\Phi_2(a,b,k) = 2\left(\frac{\Lambda(R)\,k}{N_{TX}(1+b)}\right)^{\frac{a}{2}} K_a\left(2\sqrt{\frac{\Lambda(R)(1+b)\,k}{N_{TX}}}\right) \quad (44)$$

and $\Lambda(R)$ according to (29).

Proof: The proof is quite similar to the proof of Theorem 4. Again, we can express the outage probability P_{II} as

$$P_{II} = \int_0^\infty \frac{1}{x} p_X(x) \int_0^{\frac{\Lambda(R)}{N_{TX}}} p_Y\left(\frac{\xi}{x}\right) d\xi\, dx, \quad (45)$$

where the inner integral can easily be shown to be given by

$$\int_0^{\frac{\Lambda(R)}{N_{TX}}} p_Y\left(\frac{\xi}{x}\right) d\xi = \sum_{k=1}^{N_{TX}} \frac{\alpha_{k-1}}{k}\, x \left[1 - e^{-\frac{\Lambda(R)}{N_{TX}\, x} k}\right]. \quad (46)$$

Hence, we obtain for P_{II} by combining (45) with (46)

$$P_{II} = \sum_{k=1}^{N_{TX}} \frac{\alpha_{k-1}}{k} \left[1 - \int_0^\infty p_X(x)\, e^{-\frac{\Lambda(R)}{N_{TX}\, x} k}\, dx\right], \quad (47)$$

which can be solved analytically in closed-form by making use of [8] eq. (3.471,9). ■

IV. Numerical Results

Fig. 1 shows the ergodic capacity versus the average SNR for both receive antenna selection only as well as joint transmit and receive antenna selection for a MIMO system with N_{TX} = 4, N_{RX} = 2, and N = 1. As can be seen, there is basically a perfect match between calculated and simulated values, what verifies the validity of our theoretical analysis. Furthermore, it can be seen that especially the lower capacity bound is for the whole SNR range very close to the exact values and that both the upper and the lower bound are really asymptotically tight in the high SNR regime. Finally, Fig. 1 reflects that the capacity with joint transmit and receive antenna selection is always significantly higher than the one with receive antenna selection only, thus implying that the additional (low-rate) feedback required in that case might be worthwhile in practice.

Fig. 2 illustrates the impact of the number of receive antenna elements on the information outage probability of a MIMO system with N_{TX} = 4, N = 1, and for an outage rate of R = 2 bits per channel use. It can be seen that the outage probability is always significantly smaller in case of joint transmit and receive antenna selection and that the diversity order is in both cases the same as it would be in case of a keyhole channel without antenna selection, what has already been proven in [4] and what is reflected by the steepness of the corresponding outage probability curves for high SNRs.

Fig. 3 depicts the capacity gains that can be achieved in the high SNR regime by increasing the number of transmit antennas (and RF chains) compared to a system with one transmit antenna only (upper part) as well as the gain that can be obtained by increasing the number of receive antennas compared to a reference system where the number of receive antennas equals the number of available RF chains (lower part). As can be seen, spending more transmit antenna elements is especially worthwhile in case of joint transmit and receive antenna selection whereas the corresponding capacity gain in case of receive antenna selection only is rather limited. Furthermore, we note that the difference between the two curves in the upper part of Fig. 3 in fact corresponds to the capacity difference between the two antenna selection

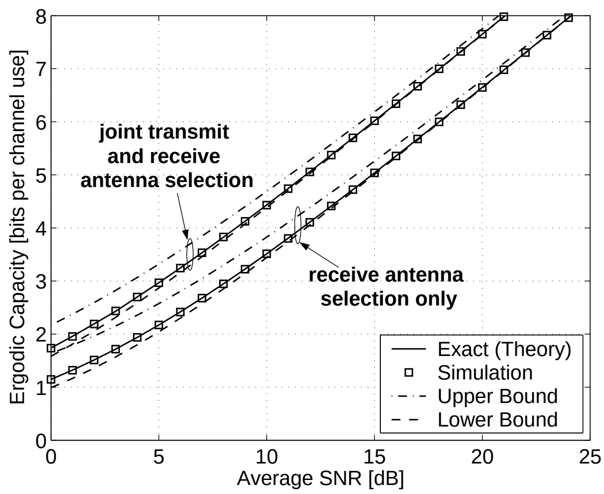

Fig. 1. Ergodic capacity for joint transmit and receive antenna selection as well as receive antenna selection only for N_{TX} = 4, N_{RX} = 2, and N = 1.

strategies in the high SNR regime. From the lower part of Fig. 3, it is getting obvious that spending more receive antenna elements might be especially worthwhile if only very few RF chains are available whereas in case of many RF chains the additional capacity gain is generally rather small.

Finally, Fig. 4 shows as the cdf of the mutual information for a MIMO system with joint transmit and receive antenna selection, N_{RX} = 4, $\bar{\gamma}$ = 10 dB, and for different values of N_{TX} and N. Obviously, if we increase N_{TX} from one to six, the corresponding cdfs basically approach a unit step function, what reflects the increased diversity order that we get this way. However, if the number of receiver RF chains is increased while N_{RX} is fixed, we do not get any additional diversity gain and the corresponding curves are simply right-shifted versions of the curve for N = 1, what can be explained by the fact that we can extract more power from the channel in this case.

V. Conclusion

We have derived exact analytical closed-form expressions for the ergodic capacity as well as the information outage probability of MIMO channels with keyhole in case that receive antenna selection only or joint transmit and receive antenna selection is performed. Aside from the exact ergodic capacity expressions, which have been given as finite sums of weighted Meijer-G functions, we have also derived somewhat simpler upper and lower bounds, which can be expressed by means of elementary functions only and which were proven to be asymptotically tight for high signal-to-noise ratios. Numerical results were shown to be in excellent agreement with simulated values, thus verifying the accuracy of our theoretical analysis.

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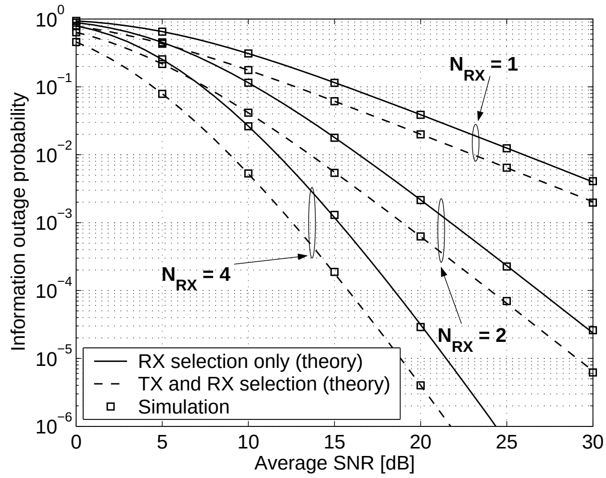

Fig. 2. Impact of the number of receive antennas on the information outage probability for N_{TX} = 4, N = 1, and R = 2 bits per channel use.

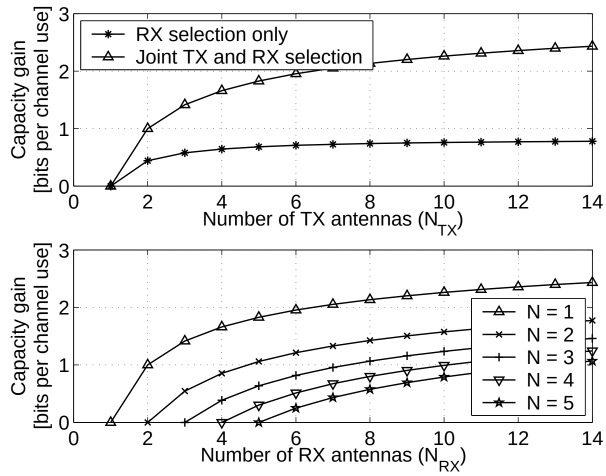

Fig. 3. Capacity gain that can be achieved in the high SNR regime by (a) increasing the number of transmit antennas compared to a system with N_{TX} = 1 (upper part) and (b) increasing the number of receive antennas compared to a reference system with N_{RX} = N (lower part).

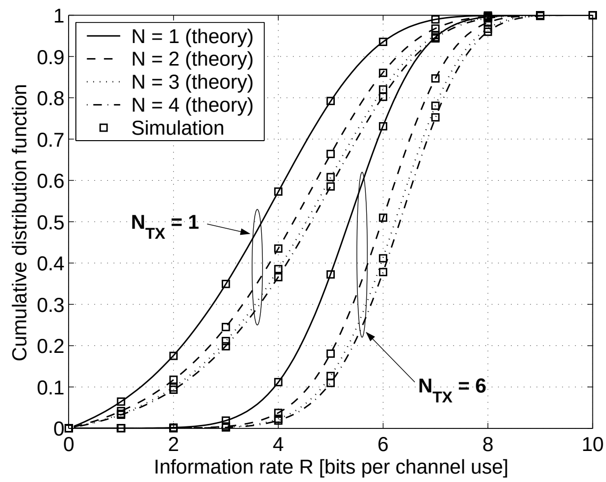

Fig. 4. Cumulative distribution function of the mutual information in case of joint TX and RX selection with N_{RX} = 4 and $\bar{\gamma}$ = 10 dB.