Orthogonal STBC in General Nakagami-m Fading Channels: BER Analysis and Optimal Power Allocation

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***Abstract*— We analyze the performance of multiple-input multiple-output (MIMO) systems employing orthogonal space-time block codes (STBC) in general Nakagami-m fading channels with channel coefficients having not necessarily identical fading parameters and average power gains. First, we derive an exact analytical closed-form expression for the average bit error rate (BER) of M-ary quadrature amplitude modulation. Then, we determine asymptotically tight high and low SNR approximations of the exact BER expression and we quantify the coding gain and diversity order of our system as a function of the channel parameters. Finally, we derive the optimal power allocation strategies for the high and the low SNR regime, assuming that statistical channel knowledge is available at the transmitter-side, and we determine the performance improvement that can be achieved this way compared to uniform power allocation.**

I. Introduction

Multiple-input multiple-output (MIMO) systems are known to offer a wide variety of benefits over conventional single-input single-output (SISO) systems, such as the potential to facilitate significantly higher data rates or to considerably improve the reliability of a wireless link. A promising approach for exploiting the spatial diversity that becomes available if multiple antennas are used is to utilize orthogonal space-time block codes (STBC), for example, which are capable of extracting full diversity gain from a MIMO channel with only moderate encoding and decoding complexity [1], [2].

In this paper, we analyze the performance of MIMO systems employing orthogonal STBC in general Nakagami-m fading channels, where not necessarily all channel coefficients have the same fading level and average power gain, respectively. In practice, fading and power imbalances might occur in case that the distance between the individual antenna elements is relatively large or in case of distributed MIMO systems, where multiple—not necessarily co-located—single-antenna users cooperate in such a way that a virtual MIMO system is established [3]. First of all, we derive an exact analytical closed-form expression for the average bit error rate (BER) of such a system when M-ary quadrature amplitude modulation (M-QAM) is used, thus generalizing and extending the results that have previously been reported in [3] and [4]. In [3], closed-form expressions for the corresponding average symbol error rates have been presented whereas in [4] also the average BER of orthogonal STBC has been considered, but under the simplifying assumption that all channel coefficients are identically distributed. In a next step, we then derive asymptotically tight BER approximations for both the low and the high SNR regime, based on which we determine the optimal power allocation strategies for the case that statistical channel state information (CSI) is available at the transmitter-side. Finally, we explicitly quantify the performance improvement that can be achieved this way compared to uniform power allocation.

The remainder of this paper is organized as follows: In Section II, we introduce our system and channel model and we determine the statistics of the post-combining SNR at the receiver-side. The actual BER analysis follows in Section III whereas the optimal power allocation strategies are derived in Section IV. Finally, numerical results are presented in Section V and some concluding remarks are given in Section VI.

II. System Model and SNR Statistics

A. System and Channel Model

We consider a frequency-flat MIMO communication system with N_{TX} transmit antennas and N_{RX} receive antennas. In the discrete-time equivalent baseband domain, the channel may be modeled by the matrix $\mathbf{H} \in \mathbb{C}^{N_{RX} \times N_{TX}}$, where the (i, j)-th element $h_{i,j}$ corresponds to the channel coefficient between the j-th transmit antenna and the i-th receive antenna. Assuming a block-fading scenario, the input-output relationship of our system may be expressed as

$$\mathbf{Y} = \sqrt{\frac{P_T}{N_{TX}}} \cdot \mathbf{HFC} + \mathbf{N}, \tag{1}$$

where $\mathbf{C} \in \mathbb{C}^{N_{TX} \times L}$ is a space-time codeword matrix with entries having unity average energy, $\mathbf{N} \in \mathbb{C}^{N_{RX} \times L}$ an additive white Gaussian noise (AWGN) matrix whose elements are independent and identically complex Gaussian distributed with zero mean and variance σ_n^2, and $\mathbf{Y} \in \mathbb{C}^{N_{RX} \times L}$ denotes the received signal matrix. Besides, P_T is the total available transmit power and $\mathbf{F} = \mathrm{diag}\left(f_1, \ldots, f_{N_{TX}}\right)$ denotes a (diagonal) power allocation matrix, satisfying the average power constraint

$$\mathrm{trace}\left(\mathbf{FF}^H\right) = \sum_{i=1}^{N_{TX}} |f_i|^2 = N_{TX}. \tag{2}$$

The magnitudes of all channel coefficients $h_{i,j}$ are modeled as Nakagami-m variates with fading levels $m_{i,j}$ and average power gains $\Omega_{i,j}$ whereas the corresponding phases are assumed to be uniformly distributed in $[0; 2\pi)$. Please note that we assume that all $h_{i,j}$ are statistically independent of each other, what is approximately fulfilled if the distance between the individual antenna elements is sufficiently large.

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In addition, we assume that the channel is normalized such that $\mathbb{E}[||\mathbf{H}||_F^2] = N_{TX}\, N_{RX}$ (with $||\cdot||_F$ as the Frobenius norm), or equivalently that $\sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} \Omega_{j,i} = N_{TX}\, N_{RX}$. Finally, we define the average SNR per receive antenna as $\bar{\gamma} = P_T/\sigma_n^2$.

Generally, the orthogonal STBC codeword matrix $\mathbf{C}$ is made up of $W \leq L$ different information symbols, resulting in a code rate equal to $R_C = W/L$. By design, the row vectors of $\mathbf{C}$ are always pairwise orthogonal. Due to this property, it can be shown that after appropriate combining of the received signals, a MIMO system employing orthogonal STBC is equivalent to a set of W parallel scalar channels with effective SNR [5]

$$\gamma = \frac{\bar{\gamma}}{R_C\, N_{TX}}\, ||\mathbf{HF}||_F^2 = \gamma_0 \sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} |f_i|^2\, |h_{j,i}|^2, \quad (3)$$

where we introduced for brevity the short-hand notation $\gamma_0 = \bar{\gamma}/(R_C\, N_{TX})$. In the following, the channel is always assumed to be perfectly known by the receiver while the transmitter has only statistical CSI as given by the particular values of the fading levels $m_{i,j}$ and the average power gains $\Omega_{i,j}$ of the various channel coefficients. Based on this information, the transmitter may then adjust the structure of the power allocation matrix $\mathbf{F}$ in order to optimize transmission. Please note that having only statistical CSI available at the transmitter-side represents an interesting approach since in this case we require only a low-rate feedback channel and the corresponding parameters usually might be accurately estimated during a relatively wide time window as they typically change at a rather slow pace.

B. Post-Combining SNR Statistics

It can easily be seen that the products $|f_i|\, |h_{j,i}|$ in (3) are Nakagami-m distributed with fading parameter $m_{j,i}$ and average power gain $|f_i|^2\, \Omega_{j,i}$. Noting that the square of any Nakagami-m variate is always gamma distributed and exploiting the assumed statistical independence of the individual channel coefficients, the moment-generating function (mgf) of γ according to (3) can easily be shown to be given by

$$M_\gamma(s) = \mathbb{E}\left[e^{\gamma\, s}\right] = \prod_{i=1}^{N_{TX}} \prod_{j=1}^{N_{RX}} \left[\frac{1}{1 - s\, |f_i|^2\, \frac{\Omega_{j,i}\, \gamma_0}{m_{j,i}}}\right]^{m_{j,i}}. \quad (4)$$

Without loss of generality, we assume that there are exactly $Q \leq N_{TX} N_{RX}$ distinct non-zero values of the coefficients $\frac{|f_i|^2\, \Omega_{j,i}}{m_{j,i}}$, which will be denoted by $\phi_1, \phi_2, \ldots, \phi_Q$ in the following. Furthermore, we denote the sum of all fading levels $m_{j,i}$ corresponding to a certain quotient ϕ_ν as α_ν, i.e.,

$$\alpha_\nu = \sum_{m_{j,i} \in \mathbb{M}_\nu} m_{j,i}, \quad \text{where } \mathbb{M}_\nu = \left\{ m_{j,i} \,\middle|\, \frac{|f_i|^2\, \Omega_{j,i}}{m_{j,i}} = \phi_\nu \right\}. \quad (5)$$

Then, $M_\gamma(s)$ according to (4) may be reformulated as

$$M_\gamma(s) = \prod_{\nu=1}^{Q} \frac{1}{(1 - s\, \phi_\nu\, \gamma_0)^{\alpha_\nu}}. \quad (6)$$

Subsequently, we always assume that all coefficients α_ν are natural numbers, which, in fact, does not represent a strong limitation since channels for which this assumption is not fulfilled can usually be accurately approximated by channels for which the assumption holds. If this is the case, we can expand (6) into partial fractions, yielding to

$$M_\gamma(s) = \sum_{\nu=1}^{Q} \sum_{\eta=1}^{\alpha_\nu} \frac{\zeta_{\nu,\eta}}{(1 - s\, \phi_\nu\, \gamma_0)^{\eta}}, \quad (7)$$

where the expansion coefficients $\zeta_{\nu,\eta}$ can be calculated as

$$\zeta_{\nu,\eta} = \frac{(\phi_\nu\, \gamma_0)^{\eta - \alpha_\nu}}{(\alpha_\nu - \eta)!}\, \frac{\partial^{\alpha_\nu - \eta}}{\partial s^{\alpha_\nu - \eta}} \left[\prod_{\substack{l=1 \\ l \neq \nu}}^{Q} \frac{1}{(1 + s\, \phi_l\, \gamma_0)^{\alpha_l}} \right] \Bigg|_{s = \frac{-1}{\phi_\nu\, \gamma_0}}. \quad (8)$$

Based on this expression, we will now determine the average BER of our system when a M-QAM scheme is used.

III. Bit Error Rate Analysis

We directly start with the main result of this section, which is given by the following theorem:

Theorem 1: The average BER of MIMO systems employing orthogonal STBC in general Nakagami-m fading channels with M-QAM modulation and rectangular $I \times J$ signal point constellations (with $I\, J = M$, where I and J are both integer powers of two) based on a Gray mapping is given by

$$P_{\mathrm{b}} = \frac{1}{\log_2 M} \left(\sum_{r=1}^{\log_2(I)} P_{\mathrm{b}}(I; r) + \sum_{l=1}^{\log_2(J)} P_{\mathrm{b}}(J; l) \right), \quad (9)$$

where

$$\begin{aligned} P_{\mathrm{b}}(X; r) \;=\; & \sum_{i=0}^{\eta(r;X)} \sum_{\nu=1}^{Q} \sum_{n=1}^{\alpha_\nu} \xi(i; r; X)\, \zeta_{\nu,n} \\ & \times \left[1 - \sqrt{\frac{\chi_i\, \phi_\nu\, \gamma_0}{I^2 + J^2 - 2 + \chi_i\, \phi_\nu\, \gamma_0}} \right. \\ & \left. \times \sum_{q=0}^{n-1} \binom{2q}{q} \frac{1}{\left[4 \left(1 + \frac{\chi_i\, \phi_\nu\, \gamma_0}{I^2 + J^2 - 2} \right) \right]^q} \right], \end{aligned} \quad (10)$$

and the short-hand notations

$$\eta(r; X) = \left(1 - 2^{-r}\right) X - 1 \quad (11)$$

$$\chi_i = 3\, (2i + 1)^2 \quad (12)$$

$$\xi(i; r; X) = \frac{1}{X}\, (-1)^{\left\lfloor \frac{i\, 2^{r-1}}{X} \right\rfloor} \left(2^{r-1} - \left\lfloor \frac{i\, 2^{r-1}}{X} + \frac{1}{2} \right\rfloor \right), \quad (13)$$

which have been introduced for brevity. In this regard, $\lfloor \cdot \rfloor$ represents the floor operator, i.e., the largest integer value smaller than or equal to the given argument.

Proof: A closed-form expression for the average BER of rectangular M-QAM schemes based on Gray mappings for the case of non-fading AWGN channels has been presented in [6]. Based on this result, the average BER of the considered MIMO system employing orthogonal STBC can easily be obtained by simply averaging this expression over the probability density function of the post-combing SNR γ. Pursuing an mgf-based approach as outlined in [7], we thus obtain (9), where

$$P_{\text{b}}(X;r) = \frac{2}{\pi} \sum_{i=0}^{\eta(r;X)} \xi(i;r;X) \int_0^{\pi/2} M_\gamma \left(\frac{\frac{-\chi_i}{I^2+J^2-2}}{\sin^2(t)} \right) dt. \quad (14)$$

Replacing $M_\gamma(s)$ by the term derived in (7) and solving the integral in closed-form by making use of [8] eq. (77) then finally leads to the desired result provided by Theorem 1. ■

Please note that similar expressions can easily be obtained for other modulation schemes (such as M-ary phase shift keying) as well, but this is not explicitly shown here due to space constraints. In the high and the low SNR regime, the exact average BER according to (9) can be accurately approximated by simpler and more intuitive expressions, which will be presented in the following and which will serve as the basis for the derivation of the optimal power allocation strategies for high and low average SNRs later in Section IV.

Theorem 2: The high SNR asymptotics of the exact average BER expression according to (9) are given by

$$P_{\text{b,high}} = \frac{1}{\log_2 M} \left(\sum_{r=1}^{\log_2(I)} \tilde{P}_{\text{b}}(I;r) + \sum_{l=1}^{\log_2(J)} \tilde{P}_{\text{b}}(J;l) \right), \quad (15)$$

where

$$\tilde{P}_{\text{b}}(X;r) = \frac{\left(\frac{I^2+J^2-2}{\gamma_0}\right)^{\Xi}}{\sqrt{\pi}\prod_{\nu=1}^{Q}\phi_\nu^{\alpha_\nu}} \sum_{i=0}^{\eta(r;X)} \frac{\xi(i;r;X)\,\Gamma\left(\Xi+\frac{1}{2}\right)}{\chi_i^{\Xi}\,\Gamma(\Xi+1)}, \quad (16)$$

with $\Gamma(\cdot)$ as the well-known gamma function [9] and the short-hand notation

$$\Xi = \sum_{i=1}^{Q} \alpha_i. \quad (17)$$

Proof: For deriving the high SNR asymptotics, we pursue an approach similar to the one presented in [10]. Noting that $M_\gamma(s)$ can be reasonably approximated for large SNRs $\bar{\gamma}$ by

$$M_\gamma(s) \approx \frac{(-s)^{-\Xi}}{\prod_{\nu=1}^{Q}(\phi_\nu\,\gamma_0)^{\alpha_\nu}} \quad (18)$$

and using this approximation in (14), we obtain

$$\tilde{P}_{\text{b}}(X;r) = \sum_{i=0}^{\eta(r;X)} \frac{\xi(i;r;X)\left(\frac{I^2+J^2-2}{\chi_i}\right)^{\Xi}}{\frac{\pi}{2}\prod_{\nu=1}^{Q}(\phi_\nu\,\gamma_0)^{\alpha_\nu}} \int_0^{\pi/2} \sin^{2\,\Xi}(t)\,dt, \quad (19)$$

which can be solved in closed-form by capitalizing on the result provided in [9] eq. (3.621,3) that $\int_0^{\pi/2}\sin^{2\,m}x\,dx = \frac{\pi\,(2m-1)!!}{2\,(2m)!!}$, where $m!!$ denotes the double factorial of m. The resulting expression can be further simplified by exploiting that $\frac{(2m-1)!!}{(2m)!!} = \frac{\Gamma(m+1/2)}{\sqrt{\pi}\,\Gamma(m+1)}$ [9], whereby we finally get the result from Theorem 2, what consequently concludes the proof. ■

Corollary 1: The diversity order G_d and the coding gain G_c of the considered system are given by

$$G_d = \Xi = \sum_{\nu=1}^{Q} \alpha_\nu, \quad (20)$$

$$G_c = \left[\left(\sum_{r=1}^{\log_2(I)} \sum_{i=0}^{\eta(r;I)} \frac{\xi(i;r;I)}{\chi_i^{\Xi}} + \sum_{l=1}^{\log_2(J)} \sum_{i=0}^{\eta(l;J)} \frac{\xi(i;l;J)}{\chi_i^{\Xi}} \right) \times \frac{\Gamma\left(\Xi+\frac{1}{2}\right)\left(N_{TX}\,R_C\,\left(I^2+J^2-2\right)\right)^{\Xi}}{\log_2(M)\,\Gamma(\Xi+1)\,\sqrt{\pi}\prod_{\nu=1}^{Q}\phi_\nu^{\alpha_\nu}} \right]^{\frac{-1}{G_d}}. \quad (21)$$

Proof: In the high SNR regime, the average BER can be expressed in terms of the diversity order and the coding gain as $P_b \approx (G_c\,\bar{\gamma})^{-G_d}$ [2]. After bringing (15) together with (16) in this form, G_c and G_d can directly be determined. ■

Please note that if all f_i are larger than zero, the diversity order G_d becomes maximal and corresponds simply to the sum of all fading levels of the individual channel coefficients, i.e., in this case we always have $G_d = G_{d,max} = \sum_{i=1}^{N_{TX}}\sum_{j=1}^{N_{RX}} m_{j,i}$.

Similarly to the derivation of the high SNR asymptotics, we can derive a low SNR approximation P_{low} of the average BER, which can be shown to be asymptotically tight in the sense that $\lim_{\bar{\gamma}\to 0} P_{\text{low}}/P_{\text{b}} = 1$. The corresponding result is given by the following theorem:

Theorem 3: An asymptotically tight low SNR approximation of the average BER according to (9) is given by

$$P_{\text{low}} = \frac{1}{\log_2(M)} \left[\sum_{r=1}^{\log_2(I)} \tilde{\tilde{P}}_{\text{b}}(I;r) + \sum_{l=1}^{\log_2(J)} \tilde{\tilde{P}}_{\text{b}}(J;l) \right], \quad (22)$$

where

$$\tilde{\tilde{P}}_{\text{b}}(X;r) = \sum_{i=0}^{\eta(r;X)} \xi(i;r;X) \left[1 - \sqrt{\frac{1}{1+\frac{I^2+J^2-2}{\gamma_0\,\chi_i\,\vartheta}}} \right], \quad (23)$$

and where we introduced for brevity the short-hand notation

$$\vartheta = \sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} \Omega_{j,i}\,|f_i|^2. \quad (24)$$

Proof: Expanding the denominators of the individual factors in (4) by making use of the binomial theorem and considering only constants and single powers of s in the resulting term, we obtain the low SNR approximation

$$M_\gamma(s) \approx \frac{1}{1-\sum_{i=1}^{N_{TX}}\sum_{j=1}^{N_{RX}} s\,\Omega_{j,i}\,|f_i|^2\,\gamma_0}. \quad (25)$$

Using this approximation in (14) and solving the corresponding integral by making use of [8] eq. (77) again, we finally obtain the expression provided in the theorem. ■

IV. Optimal Power Allocation

Having statistical CSI available at the transmitter-side, the power allocation coefficients f_k might be appropriately adjusted in such a way that the average BER is minimized. In the following, we determine these optimal power allocation coefficients as a function of the fading parameters $m_{i,j}$ and the average power gains $\Omega_{i,j}$ for both the low and the high SNR regime and we quantify the performance improvement that can be achieved this way compared to uniform power allocation. In this regard, we start with the following theorem:

Theorem 4: In the high SNR regime, the optimal power allocation coefficients f_k $(k = 1, \ldots, N_{TX})$ are given by

$$f_k = \sqrt{\frac{N_{TX} \sum_{j=1}^{N_{RX}} m_{j,k}}{\sum_{r=1}^{N_{RX}} \sum_{s=1}^{N_{TX}} m_{r,s}}}. \quad (26)$$

Proof: It is quite obvious that a certain set $\{f_k\}$ of power allocation coefficients is optimal in the high SNR regime if they maximize the coding gain G_c while achieving the maximum diversity order $G_{d,max} = \sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} m_{j,i}$. As can easily be seen from (21), maximizing the coding gain is equivalent to minimizing the product $\prod_{\nu=1}^{Q} \phi_\nu^{\alpha_\nu}$, i.e., we have the constraint optimization problem

$$\{f_k\}_{\text{opt}} = \arg\min_{\{f_k\}} \prod_{i=1}^{N_{TX}} \prod_{j=1}^{N_{RX}} \left(\frac{|f_i|^2 \, \Omega_{j,i}}{m_{j,i}} \right)^{m_{j,i}} \text{ subject to (2),} \quad (27)$$

which can be solved analytically using Lagrange-multipliers. By making the corresponding Lagrangian stationary with respect to all $|f_i|^2$, we find the relationship (a detailed derivation is omitted here due to space constraints)

$$|f_i|^2 \sum_{j=1}^{N_{RX}} m_{j,k} = |f_k|^2 \sum_{j=1}^{N_{RX}} m_{j,i}, \quad (28)$$

which has to be fulfilled for all possible index tuples (i, k). Hence, we can express all power allocation coefficients as a function of f_1, for example, and by plugging these relationships in our side constraint (2), we obtain

$$|f_1|^2 \left(1 + \frac{\sum_{j=1}^{N_{RX}} m_{j,2}}{\sum_{j=1}^{N_{RX}} m_{j,1}} + \ldots + \frac{\sum_{j=1}^{N_{RX}} m_{j,N_{TX}}}{\sum_{j=1}^{N_{RX}} m_{j,1}} \right) = N_{TX}, \quad (29)$$

what finally leads to a closed-form solution for $|f_1|^2$. Based on this result, we can derive the corresponding values for the other $|f_k|^2$ by making use of (28) and the actual power allocation coefficients given in Theorem 4 are then simply the (positive) square roots of these values. Please note that all coefficients f_k according to (26) are larger than zero. Hence, the maximum diversity order $G_{d,max}$ is achieved and consequently we have really found the optimum solution. ■

Obviously, the optimal power allocation coefficients in the high SNR regime depend only on the fading parameters $m_{i,j}$ and they are particularly independent of the average power gains $\Omega_{i,j}$. Furthermore, it can be seen that the amount of power that is allocated to a certain transmit antenna is governed by the relative contribution of this antenna to the overall diversity order of the system. An interesting result that quantifies the performance improvement that can be achieved this way is given by the following corollary:

Corollary 2: In the high SNR regime, the effective SNR gain that we get from performing optimum power allocation compared to uniform power allocation can be calculated as

$$g_{\text{high}} = N_{TX} \prod_{i=1}^{N_{TX}} \left[\frac{\sum_{l=1}^{N_{RX}} m_{l,i}}{\sum_{r=1}^{N_{TX}} \sum_{s=1}^{N_{RX}} m_{s,r}} \right]^{\frac{\sum_{j=1}^{N_{RX}} m_{j,i}}{\sum_{r=1}^{N_{TX}} \sum_{s=1}^{N_{RX}} m_{s,r}}}. \quad (30)$$

Proof: The effective SNR gain in the high SNR regime simply corresponds to the quotient of the corresponding coding gains G_c according to (21). Since the power allocation coefficients only affect the value of the product $\prod_{\nu=1}^{Q} \phi_\nu^{\alpha_\nu}$ in (21), it can easily be shown that g_{high} is generally given by

$$g_{\text{high}} = \frac{G_c|_{\{f_i\}=\{f_k\}_{\text{opt}}}}{G_c|_{f_i=1 \;\forall\; i}} = \left[\prod_{i=1}^{N_{TX}} |f_i|^{2 \sum_{j=1}^{N_{RX}} m_{j,i}} \right]^{\frac{1}{G_d}}. \quad (31)$$

By replacing the power allocation coefficients $|f_i|^2$ with the optimal values according to Theorem 4 and after rearranging the corresponding terms we finally obtain (30). ■

Theorem 5: In the low SNR regime, the optimal power allocation strategy is to allocate all power to the antenna that can convey most power over the channel, i.e., we basically perform antenna selection in that case. In other words, we can say that the index k of the antenna to be selected is given by

$$k = \arg\max_r \sum_{i=1}^{N_{RX}} \Omega_{i,r}. \quad (32)$$

In case that there are multiple antennas which may convey the same maximum power over the channel, an arbitrary one might be chosen.

Proof: It can easily be seen from (22) and (23) that minimizing the average BER in the low SNR regime is equivalent to maximizing ϑ according to (24), i.e., we have

$$\{f_k\}_{\text{opt}} = \arg\max_{\{f_i\}} \sum_{i=1}^{N_{TX}} |f_i|^2 \sum_{j=1}^{N_{RX}} \Omega_{j,i} \quad \text{subject to (2).} \quad (33)$$

Based on this formulation of the optimization problem, it can directly be seen that the corresponding solution is given by the result provided by Theorem 5. ■

Obviously, in the low SNR regime the optimal power allocation coefficients are only governed by the average power gains $\Omega_{i,j}$ and they are particularly independent of the fading parameters $m_{i,j}$. The actual improvement that can be achieved this way is stated in the following corollary:

Corollary 3: In the low SNR regime, the effective SNR gain that we get from performing optimum power allocation compared to uniform power allocation can be calculated as

$$g_{\text{low}} = \frac{\max_k \left(\sum_{j=1}^{N_{RX}} \Omega_{j,k} \right)}{N_{RX}} \leq N_{TX}. \quad (34)$$

Proof: It can easily be seen from (22) and (23) that the effective SNR gain in the low SNR regime for an arbitrary set of power allocation coefficients $\{f_i\}$ is given by

$$g_{\text{low}} = \frac{\vartheta|_{\{f_i\}}}{\vartheta|_{f_i=1 \;\forall\; i}} = \frac{\sum_{i=1}^{N_{TX}} |f_i|^2 \sum_{j=1}^{N_{RX}} \Omega_{j,i}}{\sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} \Omega_{j,i}}. \quad (35)$$

Choosing the power allocation coefficients in the optimal way stated in Theorem 5 and exploiting the assumed normalization that $||\mathbf{H}||_F^2 = \sum_{i=1}^{N_{TX}} \sum_{j=1}^{N_{RX}} \Omega_{j,i} = N_{TX} \, N_{RX}$, we directly obtain (34). The effective SNR gain is always smaller than or equal to N_{TX} since $\sum_{j=1}^{N_{RX}} \Omega_{j,i}$ might be at most $N_{TX} \, N_{RX}$ due to the requested channel normalization. ■

V. Numerical Results

Fig. 1 illustrates the performance improvement that can be achieved by performing optimal rather than uniform power allocation in presence of fading imbalances. In this regard, we consider a simple MIMO system with two transmit antennas and one receive antenna employing the well-known Alamouti scheme presented in [1] as well as 16-QAM modulation. The fading parameter $m_{1,2}$ of the subchannel from the second transmit antenna to the single receive antenna is always set to one whereas the fading parameter $m_{1,1}$ of the other subchannel is varied. As can be seen, optimal power allocation leads to potentially significant performance improvements, particularly if the fading imbalance between the various subchannels is quite high. Furthermore, it can be seen that there is basically a perfect match between simulated and calculated values, what verifies the validity of our theoretical analysis.

Fig. 2 shows a similar comparison, but for scenarios with power imbalances. For that purpose, we consider two different systems, namely one with four transmit antennas and a code rate equal to $1/2$, and one with two transmit antennas employing the well-known Alamouti scheme again. In both cases, we have $N_{RX} = 1$ and a pure Rayleigh-fading channel, i.e., $m_{i,j} = 1 \ \forall \ (i, j)$. Obviously, optimal power allocation leads to a significant performance improvement again, particularly at rather low SNRs. However, it can also be seen that with increasing SNR the performance gain is reduced and it should be noted that depending on the particular channel parameters, optimal power allocation for the low SNR regime might even lead to a worse performance if it is applied at rather high SNRs and vice versa. Therefore, consideration of the average SNR in selecting an appropriate strategy is essential in practice.

VI. Conclusion

We have analyzed the performance of MIMO systems employing orthogonal STBC in general Nakagami-m fading channels with possible power and fading imbalances. In this regard, an exact analytical closed-form expression for the average BER of M-QAM transmission as well as asymptotically tight high and low SNR approximations have been derived. Furthermore, we have determined the optimal power allocation strategies in the high and the low SNR regime, for which the average BER is minimized in case that statistical CSI is available at the transmitter-side. Numerical results were shown to be in perfect agreement with simulated values—thus verifying the accuracy of our theoretical analysis—and illustrated the performance improvement that can be achieved by means of our optimal power allocation strategies.

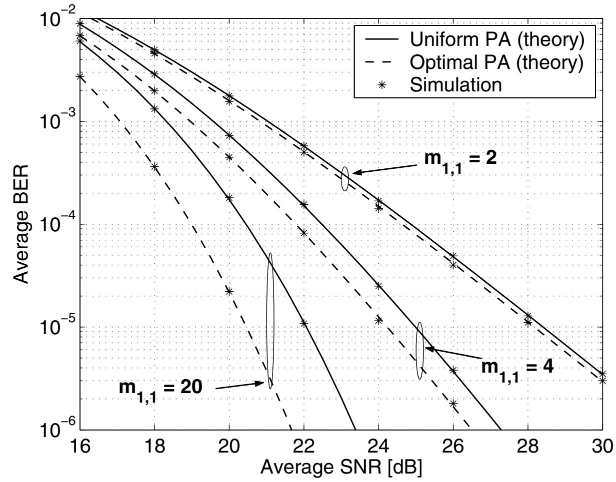

Fig. 1. Comparison between uniform and optimal power allocation (PA) in presence of fading imbalances for $N_{TX} = 2$, $N_{RX} = 1$, $R_C = 1$, 16-QAM modulation, unity average power gains, and $m_{1,2} = 1$.

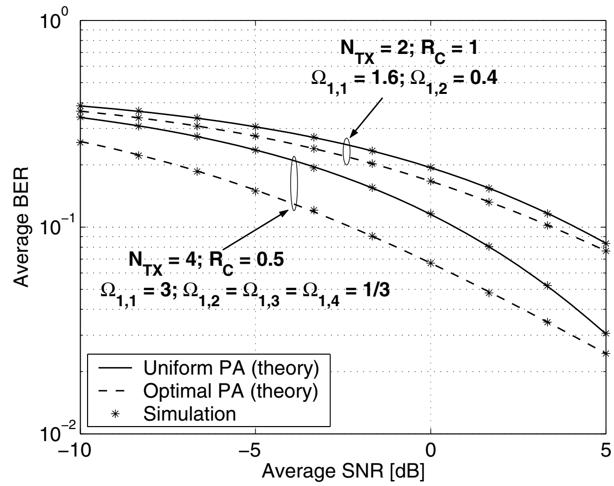

Fig. 2. Comparison between uniform and optimal power allocation (PA) in presence of power imbalances for $N_{RX} = 1$, 4-QAM modulation, equal fading parameters $m_{i,j} = 1 \ \forall \ (i, j)$, and two different orthogonal STBCs.

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