Investigation of Fast and Efficient Adaptation Algorithms for Linear Transversal and Decision-Feedback Equalizers in High-Bitrate Optical Communication Systems

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Abstract

We investigate fast and efficient adaptation algorithms for linear transversal feed-forward (FFE) and decision feedback (DFE) equalizers in intensity modulated optical communication systems with direct detection (IM/DD). First, we employ the well-known minimum mean square error (MMSE) adaptation criterion and give a benchmark with respect to minimal bit error rate (BER). We further take the least-mean-square algorithm (LMS) as stochastic implementation of the MMSE scheme and derive some simplified versions which use coarsely quantized signals. Consequently, the complexity of a digitally implemented adaptation circuit is reduced and application to bitrates of 40-100 Gbit/s becomes more feasible. The performance of these simplified versions is assessed with respect to adaptation speed and accuracy in comparison to the MMSE equalizer. It is shown that sufficient fast and almost accurate adaptation of the equalizer can be achieved.

1 Introduction

Emerging services like triple play and increasing internet traffic force service providers to upgrade their networks. Therefore, better exploitation of the installed fiber infrastructure by dynamic bandwidth allocation in the backbone and metropolitain domain is one driver towards carrier grade ethernet. These requirements will directly influence the physical layer equipment of optical networks which have to cope with dynamic and time-varying effects. They may either directly originate from time-variations of the optical channel or the amount of dispersion affecting the signal may change abruptly due to switching events in a self-managing optical network. Electrical signal processing in the receiver electronics is one approach to meet these requirements while supporting additional cost benefits. Equalization schemes ranging from linear feed-forward (FFE) and decision feedback equalizers (DFE) [1] - [4], maximum likelihood sequence estimation (MLSE) by means of Viterbi equalizers [5][6] or maximum a-posteriori (MAP) turbo detection [7] have been investigated for their capability to mitigate transmission impairments like chromatic (CD) or polarization-mode dispersion (PMD). While the latter two equalization schemes offer superior performance with respect to improvement of dispersion tolerance, dispersion compensation using FFE and DFE provides a lower complexity and thus lower cost alternative, especially for the aggregational network domain or for adaptive post-compensation in optically precompensated long-haul transmission.

Among all publications on FFE and DFE for optical communications, very few treat the issue of coefficient adaptation and the suitability of the adaptation criterion in more detail [8]-[10]. In this paper, we investigate different adaptation algorithms not only with respect to adaptation speed and accuracy but also with focus on implementation complexity which is an important issue at bitrates of 40-100 Gbit/s. First of all, we compare an MMSE equalizer with an equalizer which is designed to minimize BER. Then, we review the basics of LMS adaptation, and we introduce simplified algorithms which lead to reduced computational complexity. Finally, simulation results for the adaptation speed and accuracy are discussed.

2 System Configuration

2.1 Optical System Model

Fig. 1 shows the principal block diagram of the optical link. The bit sequence b_k is taken at discrete time

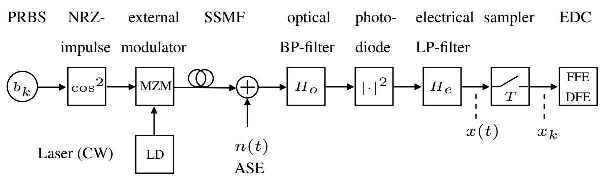

Fig. 1. Model of the optical NRZ-IM/DD system

instants $kT_B (k \in \mathcal{Z})$ out of a pseudo-random bit sequence (PRBS) generator of length 2^{11}. The bit rate is $R_B = 42.7$ Gbit/s including 6.75% FEC overhead.

The b_k are fed in a time domain raised cosine impulse shaper with roll-off 0.35 for non-return-to-zero (NRZ) impulse generation. The optical tx-signal is provided by an external Mach-Zehnder modulator (MZM) with extinction ratio of 13dB. When propagating along the standard single mode fiber (SSMF), the optical tx-signal is affected by CD or first order PMD. Since PMD is the dominant time-varying effect in an SSMF, we only focus on the transient adaptation behavior caused by first order PMD with a power split ratio of 0.5. The transient adaptation can be considered as a worst-case scenario when the optical system is switched. The noise introduced by the optical amplifier at the receiver is modeled by the amplified spontaneous emission (ASE) noise process $n(t)$. At the receiver, a 2nd order Gaussian optical bandpass filter with a 3dB-banwidth $\Delta B_{3dB} \sim 5.0R_B$ is applied, which serves as WDM demultiplexer and as noise reduction filter. After square-law detection by the photodiode, its lowpass characteristic is taken into account by a 3rd order Bessel filter with 3dB-cut-off-frequency $f_{3dB} \sim 0.5R_B$. Finally, the received signal $x(t)$ is sampled at time instants $t = t_0 + kT$ and the samples x_k are handed to the electronic dispersion compensation (EDC) unit, which performs FFE and DFE equalization and which provides estimates $\hat{b}_k$ of the transmitted bit sequence b_k.

2.2 Electronic Equalization

In **Fig. 2** the principal block diagram of the equalizer $M = 5$ FFE coefficients and $N = 1$ DFE coefficient is depicted. The adaptation mechanism of the equalizer coefficients is indicated by dashed lines. Different adaptation criteria are designed in the following subsections. The output of the equalizer can be written by

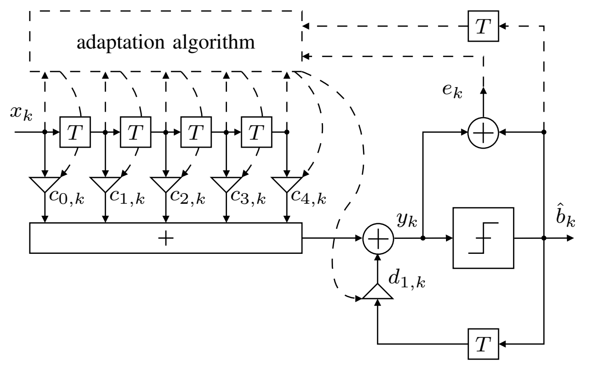

Fig. 2. FFE and DFE (adaptation loop indicated by dashed lines)

$$y_k = \sum_{i=0}^{M-1} c_i x_{k-i} + \sum_{j=1}^{N} d_j \hat{b}_{k-j} \qquad (1)$$

if the temporal index of the equalizer coefficients is omitted since the coefficients do not vary with time. For convenience, we take the input samples $x_k, \ldots, x_{k-M+1}$ and estimates $\hat{b}_{k-1}, \ldots, \hat{b}_{k-N}$ to form the vector $\mathbf{x}_k = (x_k, \ldots, x_{k-M+1}, \hat{b}_{k-1}, \ldots, \hat{b}_{k-N})^T$ and the equalizer coefficients $c_0, \ldots, c_{M-1}$ and $d_1, \ldots, d_N$ to form the coefficient vector $\mathbf{c} = (c_0, \ldots, c_{M-1}, d_1, \ldots, d_N)^T$. Thus, from eq. (1) follows

$$y_k = \mathbf{c}^T \mathbf{x}_k \qquad (2)$$

where bold letters denote column vectors.

3 Accuracy of the MSE Criterion

In this section we discuss the theoretical performance of an FFE and DFE. The performance gap between the MMSE and the minimal BER equalizer design is studied with respect to dispersion tolerance. Both algorithms are not relevant for practical implementation since they would require long training sequences and consume high signal processing power.

3.1 Minimal BER vs. MMSE Equalization

Minimizing the BER is the desirable goal in designing a digital communication systems. The setup of the equalizer according to minimal BER P_b can be generally formulated as a multidimensional optimization problem:

$$\mathbf{c}_{P_b} = \arg\min_{\mathbf{c}} \{P_b(\mathbf{c})\} \qquad (3)$$

Since no tractable analytical expressions for $P_b(\mathbf{c})$ can be given, which also include signal-to-noise ratio (SNR) and intersymbol interference (ISI), we must conduct some brute search algorithm and evaluate the $P_b(\mathbf{c})$ in order to determine the optimum equalizer coefficients. In literature, there exist some approches which support more sophisticated search strategies. They make use of gradient or topology information of the multidimensional surface described by the function $P_b(\mathbf{c})$. Our generated results are based on the latter approach and we have found an optimum equalizer solution with the *Downhill Simplex Method* [11], which converged in resonable time.

The equalizer adjustment based on minimizing the mean squared error (MSE) $E[|e_k|^2] = E[|b_k - y_k|^2]$ has been treated by various textbooks and publications [12] and leads to the so called *Wiener-Hopf Equation*

$$\underbrace{E[\mathbf{x}\mathbf{x}^T]}_{\mathbf{R}_{\mathbf{xx}}} \mathbf{c}_{MMSE} = \underbrace{E[b_k \mathbf{x}]}_{\mathbf{p}_{b_k \mathbf{x}_k}} \qquad (4)$$

which requires inversion of the autocorrelation matrix $\mathbf{R}_{\mathbf{xx}}$ to solve for $\mathbf{c}_{MMSE}$. The advantage of the MSE is the existence of an analytical expression which directly connects the target function $E[|e_k|^2]$ to the equalizer coefficients (cf. eq. (4)). Noise and ISI are included in the signal samples x_k which are used to determine $\mathbf{R}_{\mathbf{xx}}$ and $\mathbf{p}_{b_k \mathbf{x}_k}$. The familiy of LMS algorithms which is treated in the next section exploits this property for solving eq. (4) iteratively.

In **Fig. 3** the required optical signal-to-noise ratio (OSNR) at BER of $1.0 \cdot 10^{-3}$ is depicted as a function of differential group delay (DGD) for minimal BER and MMSE coefficient adjustment. It can be seen that

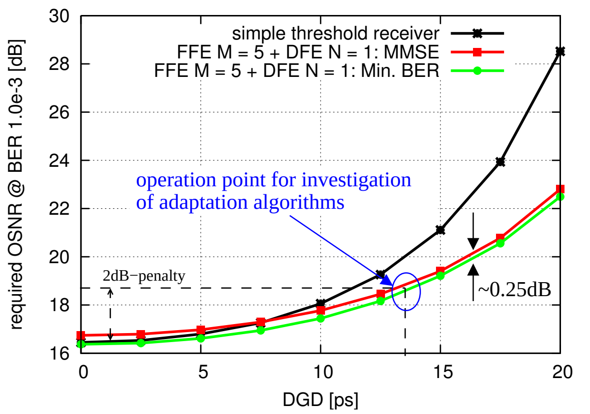

Fig. 3. PMD dispersion tolerance improvement with respect to minimal BER and MMSE coefficient adjustment

the MMSE criterion results in an almost small constant offset of about 0.25dB compared to minimal BER adjustment. Thus, it can be stated that the MMSE criterion is quite in line with minimum BER and is therfore suitable for adjusting FFE and DFE in an optical communication system with dominant optical noise.

4 Adaptive Equalization

4.1 The LMS Algorithm

We shortly review the LMS algorithm which is the starting point for simplified versions in the next subsection. The optimization goal of the LMS algorithm is to minimize $E[|e_k|^2]$ and to solve eq. (4) iteratively. Thus, the idea of how to reach the optimum coefficients in the MSE sense starts with an initial coefficient vector $\mathbf{c}_0$ at $k = 0$ and applies the *Method of Steepest Descent*:

$$\mathbf{c}_{k+1} = \mathbf{c}_k - \mu \, \nabla_{\mathbf{c}_k} E[|e_k|^2] \tag{5}$$

Here, we indicate the temporal dependency of the coefficient vector and label it with the discrete time instant k. Since $E[|e_k|^2]$ is a convex function of $\mathbf{c}_k$, the *Method of Steepest Descent* means going into negative direction of the gradient $\nabla_{\mathbf{c}_k} E[|e_k|^2]$ scaled by a constant step size factor μ towards the optimum coefficient vector, i.e. the minimum of $E[|e_k|^2]$.

Since the expected value of the gradient $\nabla_{\mathbf{c}_k} E[|e_k|^2]$ is hard to determine in practical system application, we further rely on the so-called *Stochastic Gradient* approach where we replace the expectation by its argument [12]. This leads us to the LMS coefficient update equation:

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \mu \, e_k \mathbf{x}_k \tag{6}$$

4.2 Simplified LMS Algorithms

Although the LMS algorithm in (6) has a rather simple structure, its application in a digital signal processing unit at 40-100 Gbit/s imposes several challenges especially concerning the required multiplications. Despite of using fixed-point representation for μ, e_k and $\mathbf{x}_k$, the multiplications among these represent still a bottleneck to execution speed and power consumption of the adaptation loop. Our idea is to coarsly quantize e_k and $\mathbf{x}_k$ in order to replace their multiplication by simple sign correlations which are easy to implement by comparators.

4.2.1 The Slice-LMS Algorithm

As a first approximation for eq. (6), we quantize the the first M components of the vector $\mathbf{x}_k$. Since $x_k, \ldots, x_{k-M+1}$ consist of samples of a unipolar signal taking the sign would make no sense. Instead, we apply a slicing operation as indicated in **Fig. 4**. From Fig. 4

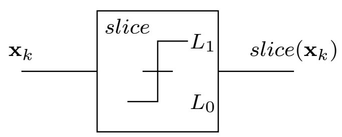

Fig. 4. Slicing operation executed with the vector $\mathbf{x}_k$

we call the resulting simplified LMS algorithm Slice-LMS algorithm and the update equation can be written as:

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \mu \, e_k slice(\mathbf{x}_k) \tag{7}$$

where the $slice(\cdot)$-operation is applied to the first M vector components.

4.2.2 The Sign-Slice-LMS Algorithm

A further step to reduce computational complexity is to apply coarse quantization to the error signal e_k in (7). We just employ sign operation since e_k is bipolar. Consequently, eq. (7) is further simplified to:

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \mu \, sign(e_k) slice(\mathbf{x}_k) \tag{8}$$

If we choose the step size parameter μ as an integer power of 2, the multiplication of $(sign(e_k)slice(\mathbf{x}_k))$ by μ will become a simple shift operation in a digital processor.

4.3 Simulation Results

In **Figs. 5-7** simulation results for the three LMS algorithms in eqs. (6),(7) and (8) are given for $M = 5$ and $N = 1$. The 2dB penalty point in Fig. 3 is the basis. The results are generated by averaging over 50 adaptation runs to get rid of random noise effects. Each figure shows the temporal change of the equalizer coefficients. For comparison fixed coefficients according to the MMSE criterion are indicated by dashed lines.

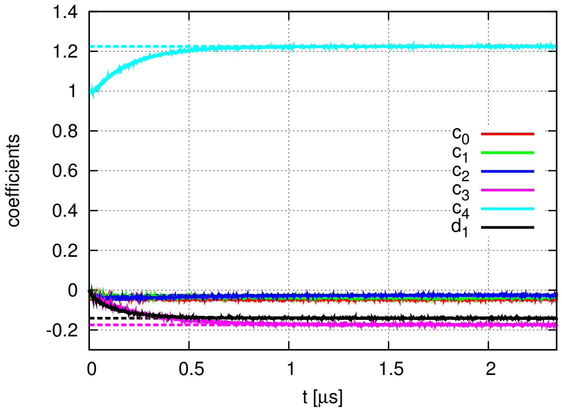

Fig. 5. Equalizer coefficients as a function of time for LMS

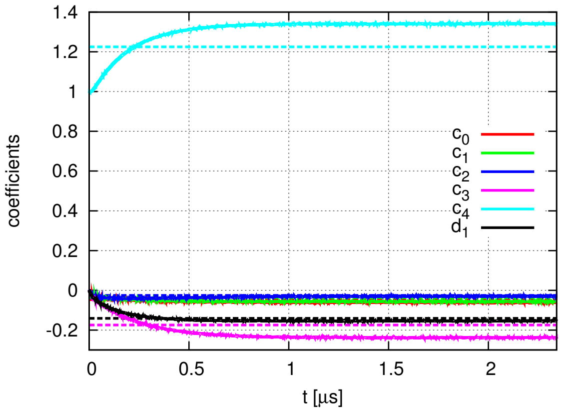

Fig. 6. Equalizer coefficients as a function of time for Slice-LMS

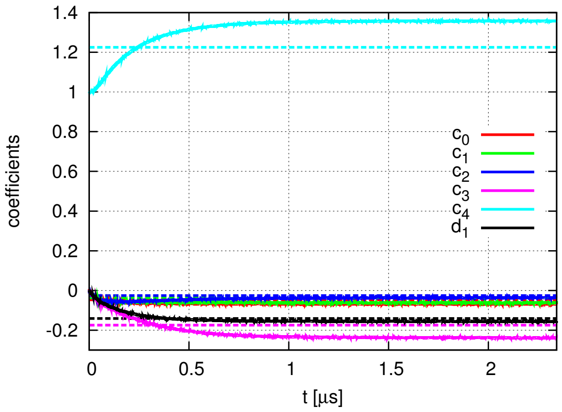

Fig. 7. Equalizer coefficients as a function of time for Sign-Slice-LMS

The LMS and Slice-LMS algorithm converge within 1μs to their final values. Quantization of e_k in the Sign-Slice-LMS algorithm leads to slightly larger convergence time of about 1.5μs. If we consider the adaptation accuracy, the LMS algorithm converges almost exactly to the MMSE solution (dashed lines). The Slice-LMS and Sign-Slice-LMS do not converge exactly to the MMSE solution. This is due to the fact that the coarse quantization of $\mathbf{x}_k$ allows only distinct directions of the gradient vector in eqs. (7) and (8), respectively. This results in an offset compared to the MMSE solution. Since the final coefficients of the Slice-LMS and Sign-Slice-LMS algorithm do not differ very much, we can conclude that the quantization of e_k in the Sign-Slice-LMS algorithm does not further affect accuray.

The table below shows the resulting BER at the 2dB penalty point after the final adjustment of the equalizer coefficients. It confirms the findings of Figs. 5-7.

Algorithm	Min. BER	MMSE	LMS
BER	0.774e-3	0.913e-3	0.937e-3
Algorithm	Slice-LMS	Sign-Slice-LMS	
BER	1.3e-3	1.41e-3	

5 Conclusion

In this paper, we have investigated simple but efficient adaptation algorithms for electronic equalizers in optical communication systems. We have shown that the computational complexity can be reduced by coarse quantization of the signals required for adaptation. We have studied the original LMS algorithm and derived the computationally simpler versions Slice-LMS and Sign-Slice-LMS algorithm. Computer simulations show that the equalizer coefficients can be adjusted in about 1-1.5μs at 42.7 Gbit/s. The achieved BER is slightly increased by less than factor 2, e.g. from $0.9 \cdot 10^{-3}$ to $1.4 \cdot 10^{-3}$ using the proposed algorithms with the advantage of a significant reduction of computational complexity.

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