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Performance limits of multiple-input multiple-output keyhole channels with antenna selection†

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SUMMARY

We present a comprehensive analysis of the ergodic capacity and information outage probability of multiple-input multiple-output (MIMO) keyhole channels with antenna selection. In this regard, we consider several different scenarios with perfect channel state information (CSI) at the receiver and no, partial or full CSI at the transmitter. For all cases, we derive exact analytical closed-form expressions for the corresponding outage probability and ergodic capacity, where the latter one is always given as a finite sum of weighted Meijer G-functions, which might be easily evaluated numerically using standard mathematical software packages. Furthermore, we derive somewhat simpler upper and lower capacity bounds, which are proven to be asymptotically tight in the high SNR regime. Numerical results are shown to be in perfect agreement with results obtained from Monte Carlo simulations, thus verifying the accuracy of our theoretical analysis.

1. INTRODUCTION

It is well known that the spectral efficiency of wireless communication systems can be significantly increased and the transmission reliability greatly improved if multiple antenna elements are deployed on both sides of a wireless link. In particular, it has been shown in Reference [1] that in case of independent and identically distributed (i.i.d.) Rayleigh-fading scenarios, the ergodic capacity of such multiple-input multiple-output (MIMO) channels scales linearly with the minimum out of the number of transmit and receive antenna elements, respectively, thus promising great enhancements compared to conventional single-antenna transmission. For that reason, MIMO technology is expected to become an integral part of future wireless communication standards and products.

In real-world propagation scenarios, however, the actual capacity gains are often much smaller than those reported in Reference [1], what can basically be attributed to two different factors: on the one hand to the detrimental effects of spatial fading correlation, which generally might occur if the individual antenna elements are not placed far enough apart from each other and/or due to lack of a rich-scattering environment, so that there are not enough scattering objects to decorrelate the transmit and receive signals, respectively. On the other hand, the actual capacity gains might be smaller due to possible degenerate channel phenomena caused by so-called keyholes or pinholes [2, 3]. Keyholes generally characterise rank-deficient MIMO channels, which may have sufficient scattering around the transmitter and the receiver to obtain uncorrelated or at least only weakly correlated signals, but due to other propagation effects, such as certain diffraction or waveguiding phenomena, the channel matrix might nevertheless exhibit only low rank. This effect has first been predicted theoretically in References [2, 3] and could be verified experimentally by various different measurement campaigns, see for example Reference [4]. The impact of spatial correlation on the ergodic capacity of MIMO channels has been well studied in literature over the past few years, see for example

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References [5, 6]. Only recently, the capacity of keyhole channels and the joint effects of spatial correlation and keyholes on the properties of the mutual information of MIMO channels have been extensively analysed as well [5, 7–10]. In fact, it is well known that keyhole channels offer always only a single degree of freedom and therefore do not provide any spatial multiplexing gain, but only a potential diversity gain [3, 7].

Unfortunately, the promising advantages of MIMO systems are not completely free, because using multiple antenna elements on both sides of a wireless link might considerably increase the signal processing complexity and entails significantly higher hardware costs. While the costs for the additional antenna elements usually are more or less negligible, the costs for the additional radio frequency (RF) chains required for amplifying, up- and down-converting as well as digital to analogue and analogue to digital converting the corresponding transmit and receive signals generally might be rather high and therefore represent a limiting factor for the production and wide-spread deployment of inexpensive MIMO devices. An attractive approach for partially alleviating these drawbacks of MIMO systems is to perform antenna selection, that is, to select always only a subset of the available transmit and receive antenna elements, respectively. This way, the number of required RF chains as well as the signal processing complexity can be reduced while most of the benefits offered by MIMO systems can be retained—provided that the antenna selection is done in an appropriate way [11–13].

In the past few years, a lot of research effort has been devoted to the development and performance analysis of efficient antenna selection techniques for MIMO systems, see for example References [12, 13] and references therein. In this regard, most previous studies have only focused on MIMO systems with non-degenerate channels, whereas only little work has been presented for antenna selection in presence of keyhole channels so far. However, in fact selecting only a subset of the available antenna elements seems to be especially attractive in case of keyhole channels since the performance degradation compared to a full-complexity system normally should be much smaller than for non-keyhole channels due to the general lack of spatial multiplexing gain. Only recently, a first analysis of the performance of antenna selection in the presence of keyhole channels has been presented by Sanayei and Nosratinia in Reference [14], but in their paper the authors basically only prove that the diversity order that can be obtained this way is exactly the same as the diversity order of the same channel without antenna selection and they present some results for the corresponding ergodic capacity, which, however, have only been obtained by means of Monte Carlo simulations.

In order to further enhance the understanding of the impact of antenna selection in presence of keyhole channels and for quantifying the corresponding performance limits, this paper presents a comprehensive analysis of the associated ergodic capacity and information outage probability, where we consider three different scenarios with different kinds of channel state information (CSI) at the transmitter side. In the first scenario, the transmitter has no CSI at all, wherefore opportunistic antenna selection can only be performed at the receiver side. In the second scenario, the transmitter has only partial CSI as given by the indices of the antenna elements to be selected, which might be efficiently fed back by the receiver using a low-rate feedback link. Finally, in the last scenario the transmitter has perfect CSI, so that not only the best antennas can be selected, but also optimal power allocation is feasible. For all cases, we derive exact analytical closed-form expressions for the ergodic capacity and information outage probability as well as somewhat simpler upper and lower capacity bounds, which are shown to be asymptotically tight for high signal-to-noise ratios (SNR).

The remainder of this paper is organised as follows: in Section 2, we introduce our system and channel model, and we review the special structure of keyhole channels. Afterwards, the actual analysis of the ergodic capacity and information outage probability for different kinds of CSI at the transmitter side (CSIT) is done in Section 3 whereas some numerical results illustrating the corresponding performance limits are presented in Section 4. Finally, a short summary and our main conclusions are given in Section 5.

2. SYSTEM AND CHANNEL MODEL

We consider a frequency-flat MIMO system with N_{TX} transmit antennas and N_{RX} receive antennas. The discrete-time equivalent baseband representation of the channel is modelled by the matrix $\mathbf{H} \in \mathbb{C}^{N_{\mathrm{RX}} \times N_{\mathrm{TX}}}$, which is assumed to be constant during the transmission of one codeword but to change independently from one codeword to another. This is approximately fulfilled if the channel coherence time is in the order of the codeword length [11]. Besides, we assume that the only way for radio waves to propagate from the transmitter to the receiver is to pass through a keyhole, which reradiates all the captured energy. Please note that such a keyhole channel might occur in presence of a hallway acting as a single-mode waveguide, for example, or if the radii within which scattering objects are distributed around the transmitter and the receiver are small compared to the distance between both entities [2, 3]. The channel can then be considered as a concatenation of a multiple-input single-output (MISO) channel from the individual transmit antennas to the keyhole and a (statistically independent) single-input multiple-output (SIMO) channel from the keyhole to the various receive antennas [2], that is, the channel matrix $\mathbf{H}$ can be modelled as:

$$\mathbf{H} = \mathbf{h}_1 \mathbf{h}_2^H \tag{1}$$

where $(\cdot)^H$ denotes the conjugate transpose of a vector or matrix and $\mathbf{h}_1 \in \mathbb{C}^{N_{\mathrm{RX}}}$ as well as $\mathbf{h}_2 \in \mathbb{C}^{N_{\mathrm{TX}}}$ model the aforementioned SIMO and MISO channels, respectively. All elements of these vectors are assumed to be i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance one, thus corresponding to a spatially uncorrelated Rayleigh-fading scenario.

When performing antenna selection, we generally select M out of the available N_{TX} transmit antennas and N out of the available N_{RX} receive antennas. Due to the special structure of the considered keyhole channels according to Equation (1), it can easily be seen that the joint transmit and receive antenna selection problem actually can be decomposed into two independent antenna selection problems at the transmitter and the receiver side, respectively [14]. The effective channel matrix $\mathbf{H}_{\mathrm{eff}}$ denoting the channel coefficients between all active transmit-receive antenna pairs hence can be written as:

$$\mathbf{H}_{\mathrm{eff}} = \mathbf{x}\mathbf{y}^H \tag{2}$$

where $\mathbf{x} \in \mathbb{C}^N$ and $\mathbf{y} \in \mathbb{C}^M$ denote the *effective* SIMO and MISO channels, which we get after selecting the N and M strongest elements of $\mathbf{h}_1$ and $\mathbf{h}_2$, respectively. The input–output relationship of our system is then given by

$$\mathbf{r} = \mathbf{H}_{\mathrm{eff}}\mathbf{s} + \mathbf{n} \tag{3}$$

where $\mathbf{r} \in \mathbb{C}^N$ denotes the received signal vector, $\mathbf{s} \in \mathbb{C}^M$ the transmitted signal, and $\mathbf{n} \in \mathbb{C}^N$ an additive noise vector, whose elements are assumed to be i.i.d. circularly symmetric complex Gaussian distributed with zero mean and variance one. Furthermore, the transmit signals $\mathbf{s}$ are assumed to be complex Gaussian distributed as well, with zero mean and covariance matrix $\mathbf{R}_{ss} = \mathbb{E}[\mathbf{s}\mathbf{s}^H]$, where $\mathrm{trace}(\mathbf{R}_{ss}) = \bar{\gamma}$ with $\bar{\gamma}$ denoting the average SNR per receive antenna. Finally, the channel is always assumed to be perfectly known by the receiver whereas we consider three different kinds of CSIT, namely no CSI, perfect CSI, as well as partial CSI as given by the indices of the antenna elements to be selected. These indices might be fed back by the receiver using a low-rate feedback channel and therefore this approach represents an attractive solution for practical implementations.

3. CAPACITY AND OUTAGE PROBABILITY

Under the previously outlined assumptions, it is well known from literature that for a certain realisation of the effective channel matrix $\mathbf{H}_{\mathrm{eff}}$, the mutual information between the transmit signal $\mathbf{s}$ and the corresponding received signal $\mathbf{r}$ is generally given by [1]

$$I(\mathbf{s}, \mathbf{r}) = \ln \det \left(\mathbf{I}_N + \mathbf{H}_{\mathrm{eff}}\mathbf{R}_{ss}\mathbf{H}_{\mathrm{eff}}^H\right) \tag{4}$$

where $\mathbf{I}_n$ denotes the identity matrix of dimension $n \times n$. Please note that throughout this paper both the mutual information and the ergodic channel capacity are for notational convenience always given in nats per channel use, but a conversion to the more common unit bits per channel use can easily be performed by dividing the corresponding expressions by ln 2. By decomposing the input covariance matrix $\mathbf{R}_{ss}$ as $\mathbf{R}_{ss} = \mathbf{R}_{ss}^{\frac{H}{2}}\mathbf{R}_{ss}^{\frac{1}{2}}$ with $\mathbf{X}^{\frac{1}{2}}$ as the matrix root of $\mathbf{X}$, exploiting the special structure of the effective channel matrix $\mathbf{H}_{\mathrm{eff}}$ according to Equation (2), and further making use of the well-known determinant identity $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$ for arbitrary matrices $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{AB}$ is square, Equation (4) can easily be shown to be equivalent to [7]:

$$I(\mathbf{s}, \mathbf{r}) = \ln(1 + \Xi) \tag{5}$$

where $\Xi = XY$ with $X = \|\mathbf{x}\|^2$, $Y = \|\mathbf{R}_{ss}^{\frac{1}{2}}\mathbf{y}\|^2$ and $\|\cdot\|^2$ as the squared Euclidean norm. Obviously, maximising the mutual information is equivalent to maximising both X and Y. Since the receiver is assumed to have perfect CSI, it is always possible to select the N best receive antennas for which X gets maximal. Hence, X simply corresponds to the sum of the N strongest out of N_{RX} i.i.d. χ^2 random variables with two degrees of freedom. The problem of finding the distribution of X has already been solved in the context of the performance analysis of generalised selection combining systems and by making use of the results presented in Reference [15], for example, we can directly determine its

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probability density function (pdf) as:

$$p_X(x) = \binom{N_{\mathrm{RX}}}{N} \left[x^{N-1} \frac{e^{-x}}{\Gamma(N)} + \sum_{i=1}^{N_{\mathrm{RX}}-N} \beta_i e^{-x} \times \left(e^{-\frac{ix}{N}} - \sum_{j=1}^{N-1} \eta_{i,j} x^{j-1} \right) \right], \quad x \geqslant 0 \quad (6)$$

where we have introduced for brevity the short-hand notations

$$\beta_i = (-1)^i \binom{N_{\mathrm{RX}} - N}{i} \left(-\frac{N}{i} \right)^{N-1} \quad (7)$$

$$\eta_{i,j} = \frac{1}{\Gamma(j)} \left(-\frac{i}{N} \right)^{j-1} \quad (8)$$

and where $\Gamma(\cdot)$ denotes the well-known gamma function [16]. At the transmitter side, however, the optimal antenna selection strategy as well as the optimal structure of the input covariance matrix $\mathbf{R}_{ss}$ both depend on the type of CSI that is available. For that reason, we will separately analyse the considered cases with no, partial or full CSIT in the following subsections.

3.1. No CSI at the transmitter

If the transmitter does not have any CSI, we naturally have to randomly select M out of the available N_{TX} antenna elements and it is well known that without any CSI it is optimal to allocate the same amount of power to each selected antenna, that is, we set $\mathbf{R}_{ss} = \frac{\bar{\gamma}}{M} \mathbf{I}_M$ [1]. However, please note that a random selection actually leads to exactly the same performance that we would get for a system without antenna selection but only M transmit antennas. Hence, the only way to obtain full diversity gain is to use all antenna elements, that is, to set $M = N_{\mathrm{TX}}$. The ergodic capacity that we obtain without CSI at the transmitter is given by the following theorem:

Theorem 1. *The ergodic channel capacity with perfect CSI at the receiver and no CSI at the transmitter is given by*

$$C_I = \frac{1}{\Gamma(M)} \binom{N_{\mathrm{RX}}}{N} \left[\frac{1}{\Gamma(N)} G_{2,4}^{4,1} \left[\frac{M}{\bar{\gamma}} \middle| \begin{matrix} 0, 1 \\ N, M, 0, 0 \end{matrix} \right] + \sum_{i=1}^{N_{\mathrm{RX}}-N} \beta_i \left[\frac{N}{N+i} G_{1,3}^{3,1} \left[\frac{M}{\bar{\gamma} \frac{N}{N+i}} \middle| \begin{matrix} 0 \\ M, 0, 0 \end{matrix} \right] - \sum_{j=1}^{N-1} \eta_{i,j} G_{2,4}^{4,1} \left[\frac{M}{\bar{\gamma}} \middle| \begin{matrix} 0, 1 \\ j, M, 0, 0 \end{matrix} \right] \right] \right] \quad (9)$$

with β_i and $\eta_{i,j}$ according to Equations (7) *and* (8), *respectively, and where $G_{p,q}^{m,n}[\cdot|\cdot]$ denotes the Meijer G-function* [16].

Proof. If M out of the available N_{TX} antenna elements are randomly selected and uniform power allocation is performed, it can easily be seen that Y simply corresponds to the sum of M i.i.d. χ^2 random variables with two degrees of freedom, multiplied by $\bar{\gamma}/M$. Consequently, Y follows a gamma distribution with pdf

$$p_Y(y) = \frac{1}{\Gamma(M)} \left(\frac{M}{\bar{\gamma}} \right)^M y^{M-1} \exp\left(-\frac{yM}{\bar{\gamma}} \right), \quad y \geqslant 0 \quad (10)$$

The pdf of the product $\Xi = XY$ can then be determined by exploiting the statistical independence of X and Y as:

$$p_\Xi(\xi) = \int_0^\infty p_X(x) p_Y\left(\frac{\xi}{x} \right) \frac{1}{x} \, dx, \quad \xi \geqslant 0 \quad (11)$$

Inserting the expressions for $p_X(x)$ and $p_Y(y)$ according to Equations (6) and (10) and making use of Reference [16, equation (3.471,9)], this integral can be solved analytically in closed-form, yielding to

$$p_\Xi(\xi) = \frac{2}{\Gamma(M)} \binom{N_{\mathrm{RX}}}{N} \left(\frac{M}{\bar{\gamma}} \right)^{\frac{M}{2}} \left[\frac{1}{\Gamma(N)} \xi^{\frac{M+N}{2}-1} \left(\frac{M}{\bar{\gamma}} \right)^{\frac{N}{2}} \times K_{N-M}\left(2\sqrt{\frac{\xi M}{\bar{\gamma}}} \right) + \sum_{i=1}^{N_{\mathrm{RX}}-N} \beta_i \times \left[\left(\frac{N+i}{N} \xi \right)^{\frac{M-1}{2}} \sqrt{\frac{M}{\bar{\gamma}}} K_{M-1}\left(2\sqrt{\frac{\xi M}{\bar{\gamma}} \frac{N+i}{N}} \right) - \sum_{j=1}^{N-1} \eta_{i,j} \xi^{\frac{M+j}{2}-1} \left(\frac{M}{\bar{\gamma}} \right)^{\frac{j}{2}} K_{j-M}\left(2\sqrt{\frac{\xi M}{\bar{\gamma}}} \right) \right] \right] \quad (12)$$

where $K_\nu(\cdot)$ denotes the νth order modified Bessel function of the second kind [16]. The ergodic capacity can then simply be calculated as:

$$C_I = \int_0^\infty \ln(1+\xi) \, p_\Xi(\xi) \, d\xi \quad (13)$$

By inserting Equation (12) in this formula, it turns out that the problem of calculating the ergodic capacity actually can be traced back to the problem of solving integrals of the form

$$\mathcal{I} = \int_0^\infty \ln(1+x) x^p K_n(2\sqrt{bx})\, dx \qquad (14)$$

where b is a positive constant and n as well as p are both non-negative. However, due to the relatively complicated structure of the integrand, it is—to the best of our knowledge—not possible to find an analytical closed-form solution for this integral for general values of b, n and p by means of conventional integration methods. For that reason, we resort to an approach based upon so-called Meijer G-functions instead, which has already been used in References [5, 7, 8]. Meijer G-functions are very general functions, which contain virtually all-known elementary functions as special cases [16]. In particular, it is well known that the logarithm in Equation (13) might be expressed in terms of a Meijer G-function as (*cf.* Reference [17, equation (8.4.6,5)]):

$$\ln(1+x) = G_{2,2}^{1,2}\left[x \,\middle|\, \begin{matrix} 1, & 1 \\ 1, & 0 \end{matrix} \right] \qquad (15)$$

With this identity, Equation (13) can be expressed in a somewhat more general form for which we can invoke a well-known integration result given in Reference [16, equation (7.821,3)] and hence finally solve the integral (13) in closed form as:

$$\mathcal{I} = \frac{1}{2b^{p+1}} G_{4,2}^{1,4}\left[\frac{1}{b} \,\middle|\, \begin{matrix} -p-\frac{n}{2}, & -p+\frac{n}{2}, & 1, & 1 \\ 1, & 0 & & \end{matrix} \right] \qquad (16)$$

After performing some simple mathematical manipulations and exploiting some fundamental properties of Meijer G-functions (see e.g. Reference [16, Equations (9.31,1) and (9.31,2)]), we finally get the result given in Equation (9). □

Please note that the general expression provided by Theorem 1 contains the ergodic capacity of MIMO keyhole channels without antenna selection, that is, where always all available antenna elements are used, as a special case. The corresponding formula can easily be obtained from Equation (9) by setting $N = N_{\mathrm{RX}}$ as well as $M = N_{\mathrm{TX}}$, yielding to

$$C_{\text{no selection}} = \frac{1}{\Gamma(N_{\mathrm{TX}})\Gamma(N_{\mathrm{RX}})} \times G_{2,4}^{4,1}\left[\frac{N_{\mathrm{TX}}}{\bar{\gamma}} \,\middle|\, \begin{matrix} 0, 1 \\ N_{\mathrm{RX}}, N_{\mathrm{TX}}, 0, 0 \end{matrix} \right] \qquad (17)$$

This result is perfectly in line with References [5] and [7], where this special case has already been considered before.

Even though the exact capacity expression according to Equation (9) might be easily evaluated numerically since Meijer G-functions are readily available in standard mathematical software packages, these functions are generally not very well known and particularly not very intuitive, that is, it is hard to analyse the impact of various different parameters on the capacity solely based on this result. For that reason, we present somewhat simpler upper and lower bounds in the following, which are quite close to the exact ergodic capacity and which can be expressed by means of elementary functions only. In this regard, a very tight lower bound is provided by the following theorem:

Theorem 2. *A lower bound on the exact ergodic capacity C_I according to Equation* (9) *is given by*

$$C_{\mathrm{I,low}} = \ln\left(1 + \frac{\bar{\gamma}}{M} e^{\psi(M)+\zeta}\right) \qquad (18)$$

with

$$\zeta = \binom{N_{\mathrm{RX}}}{N}\left[\psi(N) - \sum_{i=1}^{N_{\mathrm{RX}}-N} \beta_i \left[\sum_{j=1}^{N-1} \left(\frac{-i}{N}\right)^{j-1} \psi(j) + \frac{N}{N+i}\left(\epsilon + \ln\frac{N+i}{N}\right)\right]\right] \qquad (19)$$

β_i according to Equation (7)*, and $\psi(\cdot)$ as Euler's ψ-function, which can be calculated for integer arguments n as:*

$$\psi(n) = -\epsilon + \sum_{k=1}^{n-1} \frac{1}{k} \qquad (20)$$

where $\epsilon = 0.577215\ldots$ *denotes the Euler–Mascheroni constant* [16].

Proof. The derivation of the lower bound can be done similarly to the derivation of a lower bound on the ergodic capacity of spatially correlated keyhole channels without antenna selection already presented in Reference [9]. Exploiting that $\ln(1+e^x)$ is a convex function in x, rewriting the general capacity formula as $C_I = \mathbb{E}[\ln(1 + \exp(\ln X + \ln Y))]$ and making use of Jensen's inequality, we obtain:

$$C_I \geqslant C_{\mathrm{I,low}} = \ln\left(1 + e^{\mathbb{E}_X[\ln X] + \mathbb{E}_Y[\ln Y]}\right) \tag{21}$$

Capitalising on Reference [16, equation (4.352,1)] and the known pdfs of X and Y according to Equations (6) and (10), we find $\mathbb{E}_X[\ln X] = \zeta$ as well as $\mathbb{E}_Y[\ln Y] = \ln(\frac{\bar{\gamma}}{M}) + \psi(M)$. □

Theorem 3. *An upper bound on the exact ergodic capacity* C_I *according to Equation* (9) *is given by*

$$C_{\mathrm{I,up}} = \ln\left(\frac{\bar{\gamma}}{M}\right) + \zeta + \psi(M) + \sqrt{\frac{M}{\bar{\gamma}}}\frac{\Gamma\left(M-\frac{1}{2}\right)}{\Gamma(M)}\chi \tag{22}$$

with ζ *according to Equation* (19) *and the short-hand notation:*

$$\chi = \binom{N_{\mathrm{RX}}}{N}\left[\frac{\Gamma\left(N-\frac{1}{2}\right)}{\Gamma(N)} + \sum_{i=1}^{N_{\mathrm{RX}}-N}\beta_i \times\left[\sqrt{\frac{\pi N}{N+i}} - \sum_{j=1}^{N-1}\eta_{i,j}\Gamma\left(j-\frac{1}{2}\right)\right]\right] \tag{23}$$

which has been introduced for brevity.

Proof. It can easily be checked that for any non-negative x, we have $\ln(1+x) \leqslant \ln(x) + x^{-1/2}$. Hence, the ergodic capacity $C_I = \mathbb{E}[\ln(1+XY)]$ is upper-bounded by

$$C_I \leqslant \mathbb{E}[\ln(XY)] + \mathbb{E}\left[\frac{1}{\sqrt{XY}}\right] \tag{24}$$

The first term in Equation (24) might be rewritten as $\mathbb{E}[\ln(XY)] = \mathbb{E}_X[\ln(X)] + \mathbb{E}_Y[\ln(Y)]$ and the second term by exploiting the previously assumed statistical independence of X and Y as $\mathbb{E}[(XY)^{-1/2}] = \mathbb{E}_X[X^{-1/2}]\mathbb{E}_Y[Y^{-1/2}]$. The expectations of $\ln(X)$ and $\ln(Y)$ have already been determined before and are given by ζ according to Equation (19) and $\psi(M) + \ln\frac{\bar{\gamma}}{M}$, respectively. The expectations of $1/\sqrt{X}$ and $1/\sqrt{Y}$, on the other hand, can directly be calculated based on Equations (6) and (10) by making use of Reference [16, equation (3.381,4)] as well as the well-known relationship that $\Gamma(1/2) = \sqrt{\pi}$ [18], which finally leads to the expression given in Equation (22). □

Corollary 1. *Both, the upper bound* $C_{\mathrm{I,up}}$ *as well as the lower bound* $C_{\mathrm{I,low}}$ *are asymptotically tight in the high SNR regime and the actual high SNR asymptotes are given by*

$$C_{\mathrm{I,asymp}} = \ln\left(\frac{\bar{\gamma}}{M}\right) + \zeta + \psi(M) \tag{25}$$

with ζ *according to Equation* (19).

Proof. It can easily be seen from Equations (18) and (22) that for $\bar{\gamma} \to \infty$ both $C_{\mathrm{I,up}}$ and $C_{\mathrm{I,low}}$ approach $C_{\mathrm{I,asymp}}$ according to Equation (25). Since $C_{\mathrm{I,low}} \leqslant C_I \leqslant C_{\mathrm{I,up}}$ holds in general, this implies that $C_I \to C_{\mathrm{I,asymp}}$ for $\bar{\gamma} \to \infty$ as well. □

Please note that while the ergodic capacity that has been considered so far is generally suitable for characterising ergodic fading channels, it is usually more expedient to consider the information outage probability in case of non-ergodic fading channels instead. The information outage probability actually denotes the probability that for an arbitrary channel realisation a certain transmission rate R cannot be guaranteed and hence it simply corresponds to the cumulative distribution function (cdf) of the mutual information according to Equation (5). The corresponding result for uninformed transmitters is given by the following theorem:

Theorem 4. *The probability that the instantaneous mutual information with perfect CSI at the receiver and no CSI at the transmitter falls below a certain information rate* R *is given by*

$$P_I = \binom{N_{\mathrm{RX}}}{N}\left[1 - \frac{\Phi_1(N,0)}{\Gamma(N)} - \sum_{i=1}^{N_{\mathrm{RX}}-N}\beta_i\left[\Phi_1\left(1,\frac{i}{N}\right) - \frac{N}{N+i} + \sum_{j=1}^{N-1}\eta_{i,j}\left[\Gamma(j) - \Phi_1(j,0)\right]\right]\right] \tag{26}$$

with the short-hand notations

$$\Phi_1(a,b) = 2\sum_{k=0}^{M-1}\frac{1}{k!}\left(\Lambda(R)\frac{M}{\bar{\gamma}}\right)^{\frac{a+k}{2}}(1+b)^{\frac{k-1}{2}} \times K_{a-k}\left(2\sqrt{\Lambda(R)\frac{M}{\bar{\gamma}}(1+b)}\right) \quad (27)$$

and

$$\Lambda(R) = e^R - 1 \quad (28)$$

Proof. It can easily be seen that $P_I = \text{Prob}[I(\mathbf{s},\mathbf{r}) \leqslant R] = \text{Prob}[\Xi \leqslant \Lambda(R)]$, with $\Lambda(R)$ as defined in Equation (28). Hence, P_I generally can be calculated as $P_I = \int_0^{\Lambda(R)} p_\Xi(\xi)\,d\xi$. Inserting the generic expression for $p_\Xi(\xi)$ according to Equation (11) and changing the order of integration, we obtain:

$$P_I = \int_0^\infty \frac{1}{x}p_X(x)\int_0^{\Lambda(R)} p_Y\left(\frac{\xi}{x}\right)d\xi\,dx \quad (29)$$

with $p_X(x)$ and $p_Y(y)$ according to Equations (6) and (10), respectively. After combining Equation (29) with these expressions, the inner integral can be solved analytically in closed form by making use of Reference [16, equation (3.381,1)]. If we then exploit that the lower incomplete gamma function $\gamma(n,x)$ that we obtain this way can be calculated for integer arguments n as [18]:

$$\gamma(n,x) = \Gamma(n)\left[1 - e^{-x}\sum_{k=0}^{n-1}\frac{x^k}{k!}\right] \quad (30)$$

and capitalise on Reference [16, equation (3.381,4) and (3.471,9)], the outer integral can be solved in closed form as well, thus leading after some basic rearrangements to the expression provided in Equation (26). □

3.2. Partial CSI at the transmitter

In the previously considered case without any CSI at the transmitter, the transmit antennas can only be selected in a random fashion, wherefore the diversity order that we obtain this way generally should be smaller than the diversity order of a full-complexity system, where always all available antenna elements are used. Since perfect CSIT might be hard to realise in practice, we alternatively could think of a system where the receiver (which is still assumed to have perfect CSI) feeds only the indices of the M strongest transmit antenna elements back to the transmitter, which then allocates the same amount of power to each selected antenna. However, it has recently been shown in Reference [14] that if the transmitter knows only the indices of the strongest antenna elements and performs uniform power allocation, it is actually optimal to feed back only the index of the single best transmit antenna and to allocate all available transmit power to it. Hence, we can achieve the optimal performance for this approach based on partial CSI with minimal feedback load because we consequently require only $\lceil \log_2 N_{\text{TX}} \rceil$ feedback bits for each channel realisation. For that reason, we restrict in the following to exclusively considering this case and the main result in this regard is given by the following theorem:

Theorem 5. *The ergodic capacity of keyhole channels with antenna selection based on perfect CSI at the receiver as well as partial CSI at the transmitter is given by*

$$C_{II} = \binom{N_{\text{RX}}}{N}\sum_{k=0}^{N_{\text{TX}}-1}\frac{\alpha_k}{1+k}\left[\frac{1}{\Gamma(N)}G_{3,1}^{1,3}\left[\frac{\bar{\gamma}}{1+k}\middle|\begin{matrix}1-N,1,1\\1\end{matrix}\right] + \sum_{i=1}^{N_{\text{RX}}-N}\beta_i\left[\frac{N}{N+i}G_{3,1}^{1,3}\left[\frac{\bar{\gamma}N}{(1+k)(N+i)}\middle|\begin{matrix}0,1,1\\1\end{matrix}\right] - \sum_{j=1}^{N-1}\eta_{i,j}G_{3,1}^{1,3}\left[\frac{\bar{\gamma}}{1+k}\middle|\begin{matrix}1-j,1,1\\1\end{matrix}\right]\right]\right] \quad (31)$$

where the coefficients α_k can be calculated as:

$$\alpha_k = \frac{N_{\text{TX}}}{\bar{\gamma}}\binom{N_{\text{TX}}-1}{k}(-1)^k \quad (32)$$

and with β_i and $\eta_{i,j}$ according to Equations (7) *and* (8)*, respectively.*

Proof. If always the single strongest transmit antenna is selected, we have $Y = \bar{\gamma}\max\{h_{2,1}, h_{2,2}, \ldots, h_{2,N_{TX}}\}$, where $h_{2,i}$ denotes the ith component of the MISO channel $\mathbf{h}_2$ from all transmit antennas to the keyhole. Hence, the cdf of Y can easily be shown to be given by

$$F_Y(y) = \left(1 - \exp\left(-\frac{y}{\bar{\gamma}}\right)\right)^{N_{\mathrm{TX}}} \tag{33}$$

and deriving this expression with respect to y then yields the corresponding pdf

$$p_Y(y) = \frac{\partial}{\partial y} F_Y(y) = \sum_{k=0}^{N_{\mathrm{TX}}-1} \alpha_k \exp\left(-\frac{y}{\bar{\gamma}}(1+k)\right) \tag{34}$$

where we made use of the binomial theorem and with the short-hand notation α_k according to Equation (31). The pdf of X, in contrast, is still the same as before and given by Equation (6). In order to get the pdf of the product $\Xi = XY$, we then insert Equations (6) and (33) in Equation (11) and after solving the corresponding integral similarly to the proof of Theorem 1 in closed form, we finally obtain

$$\begin{aligned} p_\Xi(\xi) = 2\binom{N_{\mathrm{RX}}}{N} \sum_{k=0}^{N_{\mathrm{TX}}-1} \alpha_k \Bigg[\frac{1}{\Gamma(N)} \left(\frac{\xi(1+k)}{\bar{\gamma}}\right)^{\frac{N-1}{2}} \\ \times K_{N-1}\left(2\sqrt{\frac{\xi(1+k)}{\bar{\gamma}}}\right) \\ + \sum_{i=1}^{N_{\mathrm{RX}}-N} \beta_i \Bigg[K_0\left(2\sqrt{\frac{\xi(1+k)(N+i)}{N\bar{\gamma}}}\right) \\ - \sum_{j=1}^{N-1} \eta_{i,j} \left(\frac{\xi(1+k)}{\bar{\gamma}}\right)^{\frac{j-1}{2}} K_{j-1}\left(2\sqrt{\frac{\xi(1+k)}{\bar{\gamma}}}\right)\Bigg]\Bigg] \end{aligned} \tag{35}$$

The actual ergodic capacity can then be determined as $C_{II} = \int_0^\infty \ln(1+\Xi) p_\Xi(\xi)\, d\xi$. This integral can be solved analytically in closed form by making use of Meijer G-functions as well as Reference [16, equation (7.821,3)] again, what eventually leads to the expression given in Equation (31). □

As before, the exact expression for the ergodic capacity with partial CSI at the transmitter corresponds to a finite sum of weighted Meijer G-functions and hence is not very intuitive. Therefore, we present in the following somewhat simpler upper and lower bounds again, which both can be expressed by means of elementary functions only.

Corollary 2. *A lower bound on the exact ergodic capacity C_{II} according to Equation* (31) *is given by*

$$C_{\mathrm{II,low}} = \ln(1 + \bar{\gamma} \exp(\zeta + \theta)) \tag{36}$$

where

$$\theta = \sum_{k=0}^{N_{\mathrm{TX}}-1} \frac{\alpha_k}{1+k}(\epsilon + \ln(1+k)) \tag{37}$$

and with ζ as already defined in Equation (19).

Proof. The proof can be done in a rather straightforward manner similarly to the proof of Theorem 2 by exploiting that $\mathbb{E}_X[\ln X] = \zeta$ as well as $\mathbb{E}_Y[\ln Y] = \theta + \ln\bar{\gamma}$, where the latter result can be calculated by capitalising on the pdf of Y according to Equation (34) as well as Reference [16, equation (4.331,1)]. □

Corollary 3. *An upper bound on the exact ergodic capacity C_{II} according to Equation* (31) *is given by*

$$C_{\mathrm{II,up}} = \ln(\bar{\gamma}) + \zeta + \theta + \frac{1}{\sqrt{\bar{\gamma}}}\chi\phi \tag{38}$$

with ζ, θ and χ according to Equations (19), (37) *and* (23), *respectively, as well as*

$$\phi = \sum_{k=0}^{N_{\mathrm{TX}}-1} \alpha_k \sqrt{\frac{\pi}{1+k}} \tag{39}$$

Proof. The basic idea of the proof is the same as the one that has already been used in Theorem 3, but now we have $\mathbb{E}_Y[\ln Y] = \theta + \ln\bar{\gamma}$ as well as $\mathbb{E}_Y[1/\sqrt{Y}] = \phi/\sqrt{\bar{\gamma}}$ with ϕ according to Equation (39). □

Corollary 4. *The upper bound $C_{\mathrm{II,up}}$ as well as the lower bound $C_{\mathrm{II,low}}$ are both asymptotically tight in the high SNR regime and the actual high SNR asymptotes are given by*

$$C_{\mathrm{II,asymp}} = \ln(\bar{\gamma}) + \zeta + \theta \tag{40}$$

with ζ and θ according to Equations (19) *and* (37), *respectively.*

Proof. See proof of corollary 1. □

Finally, we determine the information outage probability again, which is provided by the following theorem:

Theorem 6. *The outage probability with perfect CSI at the receiver and partial CSI at the transmitter is given by*

$$P_{II} = \sum_{k=1}^{N_{\text{TX}}} \frac{\alpha_{k-1}}{k} \left[1 - \binom{N_{\text{RX}}}{N} \left[\frac{\Phi_2(N,0,k)}{\Gamma(N)} + \sum_{i=1}^{N_{\text{RX}}-N} \beta_i \left[\Phi_2\left(1, \frac{i}{N}, k\right) - \sum_{j=1}^{N-1} \eta_{i,j} \Phi_2(j,0,k) \right] \right] \right] \quad (41)$$

with the short-hand notation

$$\Phi_2(a,b,k) = 2 \left(\frac{\Lambda(R)k}{\bar{\gamma}(1+b)} \right)^{\frac{a}{2}} K_a \left(2\sqrt{\frac{\Lambda(R)(1+b)k}{\bar{\gamma}}} \right) \quad (42)$$

and $\Lambda(R)$ according to Equation (28).

Proof. As for the proof of Theorem 4, we generally have:

$$P_{II} = \int_0^\infty \frac{1}{x} p_X(x) \int_0^{\Lambda(R)} p_Y\left(\frac{\xi}{x}\right) d\xi\, dx \quad (43)$$

where the inner integral can easily be shown to be given by

$$\int_0^{\Lambda(R)} p_Y\left(\frac{\xi}{x}\right) d\xi = \sum_{k=1}^{N_{\text{TX}}} \frac{\alpha_{k-1}}{k} x\bar{\gamma} \left[1 - e^{-\frac{\Lambda(R)}{\bar{\gamma}x}k} \right] \quad (44)$$

Hence, we obtain for P_{II} by combining Equation (43) with Equation (44):

$$P_{II} = \sum_{k=1}^{N_{\text{TX}}} \frac{\alpha_{k-1}\bar{\gamma}}{k} \left[1 - \int_0^\infty p_X(x) e^{-\frac{\Lambda(R)}{\bar{\gamma}x}k} dx \right] \quad (45)$$

which can be solved analytically in closed form by making use of Reference [16, equation (3.471,9)]. □

3.3. Perfect CSI at the transmitter

With perfect CSI at the transmitter, we cannot only select the most favorable antennas, but we can also allocate the available power among these antennas in the optimal way. Actually, the input covariance matrix $\mathbf{R}_{ss}$ only affects the value of $Y = \|\mathbf{R}_{ss}^{1/2}\mathbf{y}\|^2$ while the squared norm X of the effective SIMO channel from the keyhole to the receiver is independent of $\mathbf{R}_{ss}$. Since the effective MISO channel $\mathbf{y}$ is obtained by selecting the M strongest elements of $\mathbf{h}_2$, it can easily be shown that—just like for conventional MISO channels—the optimal transmit strategy is to perform beamforming in the direction of the keyhole, that is, the optimal input covariance matrix is given by

$$\mathbf{R}_{ss,\text{opt}} = \frac{\bar{\gamma}}{\|\mathbf{y}\|^2} \mathbf{y}\mathbf{y}^H \quad (46)$$

Hence, we have $Y = \bar{\gamma}\|\mathbf{y}\|^2$ in this case. Consequently, Y corresponds to the sum of the M largest out of N_{TX} χ^2 random variables with two degrees of freedom, multiplied by the average SNR $\bar{\gamma}$. Hence, its pdf can similarly to the pdf of X be determined as:

$$p_Y(y) = \frac{1}{\bar{\gamma}} \binom{N_{\text{TX}}}{M} \left[\left(\frac{y}{\bar{\gamma}}\right)^{M-1} \frac{e^{-\frac{y}{\bar{\gamma}}}}{\Gamma(M)} + \sum_{i=1}^{N_{\text{TX}}-M} \kappa_i \times e^{-\frac{y}{\bar{\gamma}}} \left(e^{-\frac{iy}{M\bar{\gamma}}} - \sum_{j=1}^{M-1} \vartheta_{i,j} \left(\frac{y}{\bar{\gamma}}\right)^{j-1} \right) \right] \quad (47)$$

with the short-hand notations

$$\kappa_i = (-1)^i \binom{N_{\text{TX}} - M}{i} \left(-\frac{M}{i}\right)^{M-1} \quad (48)$$

$$\vartheta_{i,j} = \frac{1}{\Gamma(j)} \left(-\frac{i}{M}\right)^{j-1} \quad (49)$$

Inserting this expression together with the previously derived pdf of X according to Equations (6) and (11), we can determine the pdf of the product Ξ of X and Y again by exploiting Reference [16, equation (3.471,9)], yielding to Equation (50), which is shown at the top of the next page. Based on this result, it is then straightforward to calculate the ergodic capacity since the occuring integrals are identical to the ones that have already been solved before by means of Meijer G-functions as well as Reference [16, equation (7.821,3)]. Besides, the outage probability can be obtained in closed form from Equations (6) and (47) as well by using an approach as in Equation (43). However, since the resulting expressions are rather lengthy, we do not explicitly present them here due to space constraints, but instead we just provide somewhat simpler upper and lower bounds again, which are more compact and which can be expressed in terms of elementary functions only.

$$p_\Xi(\xi) = \binom{N_{\mathrm{RX}}}{N}\binom{N_{\mathrm{TX}}}{M}\frac{2}{\bar{\gamma}}\left[\frac{1}{\Gamma(N)\Gamma(M)}\left(\frac{\xi}{\bar{\gamma}}\right)^{\frac{N+M}{2}-1} K_{N-M}\left(2\sqrt{\frac{\xi}{\bar{\gamma}}}\right)\right.$$
$$+ \sum_{i=1}^{N_{\mathrm{RX}}-N}\beta_i\left[\sum_{k=1}^{N_{\mathrm{TX}}-M}\kappa_k\left[K_0\left(2\sqrt{\frac{\xi(M+k)(N+i)}{\bar{\gamma}MN}}\right) - \sum_{\nu=1}^{M-1}\vartheta_{k,\nu}\left(\frac{\xi(N+i)}{\bar{\gamma}N}\right)^{\frac{\nu-1}{2}} K_{1-\nu}\left(2\sqrt{\frac{\xi(N+i)}{\bar{\gamma}N}}\right)\right]\right.$$
$$- \sum_{j=1}^{N-1}\eta_{i,j}\left[\sum_{k=1}^{N_{\mathrm{TX}}-M}\kappa_k\left[\left(\frac{\xi(M+k)}{\bar{\gamma}M}\right)^{\frac{j-1}{2}} K_{j-1}\left(2\sqrt{\frac{\xi(M+k)}{\bar{\gamma}M}}\right) - \sum_{\nu=1}^{M-1}\vartheta_{k,\nu}\left(\frac{\xi}{\bar{\gamma}}\right)^{\frac{j+\nu}{2}-1} K_{j-\nu}\left(2\sqrt{\frac{\xi}{\bar{\gamma}}}\right)\right]\right]$$
$$+ \sum_{i=1}^{N_{\mathrm{RX}}-N}\frac{\beta_i}{\Gamma(M)}\left[\left(\frac{\xi(N+i)}{\bar{\gamma}N}\right)^{\frac{M-1}{2}} K_{M-1}\left(2\sqrt{\frac{\xi(N+i)}{\bar{\gamma}N}}\right) - \sum_{j=1}^{N-1}\eta_{i,j}\left(\frac{\xi}{\bar{\gamma}}\right)^{\frac{j+M}{2}-1} K_{M-j}\left(2\sqrt{\frac{\xi}{\bar{\gamma}}}\right)\right]$$
$$\left. + \sum_{i=1}^{N_{\mathrm{TX}}-M}\frac{\kappa_i}{\Gamma(N)}\left[\left(\frac{\xi(M+i)}{\bar{\gamma}M}\right)^{\frac{N-1}{2}} K_{N-1}\left(2\sqrt{\frac{\xi(M+i)}{\bar{\gamma}M}}\right) - \sum_{j=1}^{M-1}\vartheta_{i,j}\left(\frac{\xi}{\bar{\gamma}}\right)^{\frac{N+j}{2}-1} K_{N-j}\left(2\sqrt{\frac{\xi}{\bar{\gamma}}}\right)\right]\right] \quad (50)$$

Corollary 5. *A lower bound on the ergodic capacity with perfect CSI at both, the transmitter and the receiver side, is given by*

$$C_{\mathrm{III,low}} = \ln(1 + \bar{\gamma}\exp(\zeta + \phi)) \quad (51)$$

with ζ *according to Equation* (19) *and*

$$\phi = \left[\psi(M) - \sum_{i=1}^{N_{\mathrm{TX}}-M}\kappa_i\left[\sum_{j=1}^{M-1}\left(\frac{-i}{M}\right)^{j-1}\psi(j) + \frac{M}{M+i}\left(\epsilon + \ln\frac{M+i}{M}\right)\right]\right]\binom{N_{\mathrm{TX}}}{M} \quad (52)$$

whereas an upper bound can be calculated as:

$$C_{\mathrm{III,up}} = \ln\bar{\gamma} + \zeta + \phi + \sqrt{\frac{1}{\bar{\gamma}}}\chi\lambda \quad (53)$$

with χ *according to Equation* (23) *and the short-hand notation*

$$\lambda = \binom{N_{\mathrm{TX}}}{M}\left[\frac{\Gamma(M-\frac{1}{2})}{\Gamma(M)} + \sum_{i=1}^{N_{\mathrm{TX}}-M}\kappa_i \times\left[\sqrt{\frac{\pi M}{M+i}} - \sum_{j=1}^{M-1}\vartheta_{i,j}\Gamma\left(j-\frac{1}{2}\right)\right]\right] \quad (54)$$

Proof. The proof can be done in a rather straightforward manner similarly to the proof of Corollary 1. □

Please note that it can easily be shown again that both bounds are asymptotically tight in the high SNR regime, where the actual high SNR asymptotes are given by

$$C_{\mathrm{III,asymp}} = \ln\bar{\gamma} + \zeta + \phi \quad (55)$$

4. NUMERICAL RESULTS

Figure 1 depicts the ergodic capacity *versus* the average SNR for different kinds of CSIT and a MIMO system with four transmit antennas and two receive antennas, out of which always only the best one is selected. With perfect CSIT, always two transmit antennas are selected whereas in case of no CSIT always all available transmit antenna elements are used. Clearly, the number of required RF chains is different for the various cases, but it should be noted that selection based on perfect CSIT with $M = 1$ is identical to selection based on partial CSIT while in the latter case we would even with two RF chains at the transmitter always select only the single best antenna element. If we have no CSIT, we assume $M = 4$ RF chains at the transmitter side because only this way the maximum diversity order can be achieved, yielding to the best performance, which, however, is obviously still significantly worse compared to the other two cases with fewer RF chains. With perfect CSIT and $M = 2$, the capacity is generally about 1.5 bits per channel use

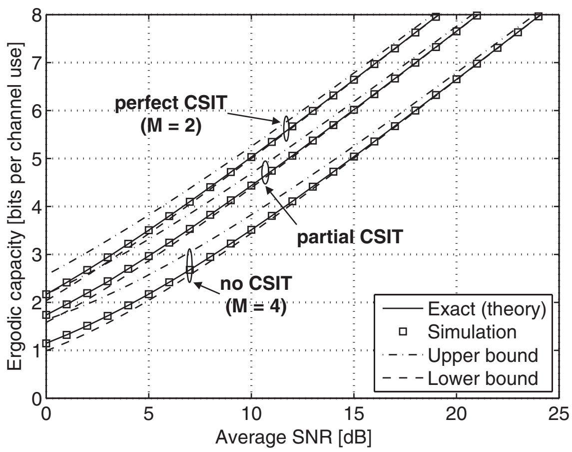

Figure 1. Exact ergordic capacity as well as capacity bounds as a function of the average SNR for different kinds of CSIT with $N_{\mathrm{TX}} = 4$, $N_{\mathrm{RX}} = 2$ and $N = 1$.

higher than without CSIT whereas in case of partial CSIT with only one RF chain at the transmitter side, we are already quite close to the curve with perfect CSIT. This suggests that this approach might be very interesting for practical systems since it represents a good tradeoff between required feedback load, achievable performance and required hardware at the transmitter side. Finally, it can be seen that there is basically a perfect match between the analytically calculcated ergodic capacities as well as results obtained from Monte Carlo simulations, what verifies the validity of our theoretical analysis.

Figure 2 shows the information outage probability as a function of the average SNR for different numbers of receive antenna elements and a MIMO system with $N_{\mathrm{TX}} = 4$, $N = 1$ and $R = 2$ bits per channel use. Again, we observe that there is a perfect match between calculated and simulated values, thus verifying the accuracy of our analysis. Besides, it can be seen that we achieve in all cases the same diversity order, which actually corresponds to the diversity order that we would obtain for a full-complexity system where always all available antenna elements are used. This has already been formally proven in Reference [14] and is reflected by the slopes of the corresponding outage probability curves in the high SNR regime. Finally, it is getting obvious that there is independent of the number of receive antennas basically a constant gap between the curves that we obtain for different kinds of CSIT, which reflects the different array gains that can be realised at the transmitter side for the various cases.

The impact of the number of available transmit antenna elements on the ergodic channel capacity is illustrated in Figure 3 for an average SNR of $\bar{\gamma} = 15$ dB. In particular, we consider full-complexity systems, where always all available transmit antennas are used, as well as systems with transmit antenna selection based on partial and perfect CSIT, respectively. Please note that in case of a full-complexity system with perfect CSIT, which clearly should exhibit the highest capacity, we perform transmit beamforming with all antenna elements in the direction of the keyhole whereas in case of the system with antenna selection and perfect CSIT, always only the two best transmit antennas are selected for that purpose. As can be seen, without CSIT, the gain that we get by increasing the number of transmit antennas is more or less negligible and saturates for $N_{\mathrm{TX}} \to \infty$. However, in all other cases,

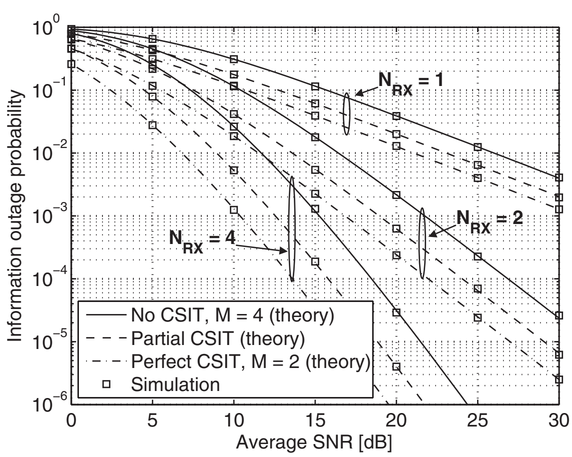

Figure 2. Information outage probability it versus the average SNR for different kinds of CSIT and different numbers of receive antennas with $N_{\mathrm{TX}} = 4$, $N = 1$ and $R = 2$ bits per channel use.

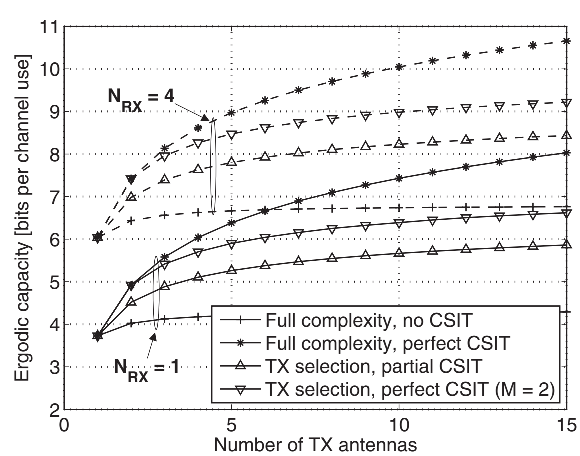

Figure 3. Ergodic capacity as a function of the number of transmit antennas for an average SNR of $\bar{\gamma} = 15$ dB. At the receiver side, always all available antenna elements are used.

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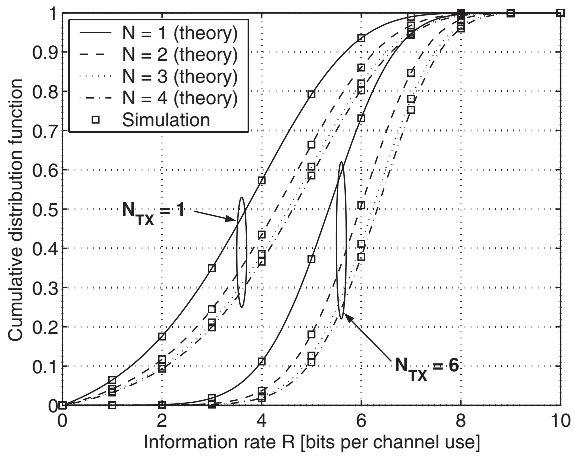

Figure 4. Cumulative distribution function of the mutual information in case of partial CSIT with $N_{\mathrm{RX}} = 4$ and $\bar{\gamma} = 10\,\mathrm{dB}$.

significant improvements can be achieved by spending more transmit antenna elements, where the gain is approximately the same for the systems with transmit antenna selection based on partial and perfect CSIT, respectively. Please note that especially for practically relevant numbers of transmit antennas up to about $N_{\mathrm{TX}} = 5$, the gap between the full-complexity system with perfect CSIT and the system with antenna selection based on perfect CSIT actually is rather small, thus indicating that the performance loss due to the usage of only a subset of the available antenna elements might be almost negligible.

Finally, Figure 4 shows as an example the cdf of the mutual information for a MIMO system with transmit antenna selection based on partial CSIT, $N_{\mathrm{RX}} = 4$, $\bar{\gamma} = 10\,\mathrm{dB}$, and for different values of N_{TX} and N. Obviously, if the number of transmit antennas is increased from 1 to 6, the corresponding cdfs basically approach a unit step function, what reflects the increased diversity order that we obtain this way since the variations of the instantaneous mutual information become less severe. However, if the number N of RF chains at the receiver side is increased while the number of receive antennas itself is fixed, we do not get any additional diversity gain and the corresponding curves are simply right-shifted versions of the curve for $N = 1$, what can be explained by the fact that we can extract more power from the channel in this case.

5. CONCLUSION

We have derived exact analytical closed-form expressions for the ergodic capacity as well as the information outage probability of MIMO keyhole channels with antenna selection, considering several different cases with perfect CSI at the receiver side and various kinds of CSI at the transmitter side. The exact ergodic capacity expressions have been given as finite sums of several weighted Meijer G-functions, which might be easily evaluated numerically. Nevertheless, we additionally derived somewhat simpler upper and lower bounds, which are asymptotically tight for high SNR and which can be expressed by means of elementary functions only. Finally, numerical results were shown to be in excellent agreement with simulated values, thus verifying the accuracy of our theoretical analysis, and they illustrated the impact of various parameters on the capacity and outage probability of the considered MIMO channels.

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