

Long-Term Power Allocation for Multihop Communication Systems with Regenerative Relays

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Abstract—We consider wireless multihop systems with regenerative relaying, where the data transmission between two peers is realized with the help of a certain number of intermediate estimate-and-forward relays. In this regard, we specifically deal with the problem of allocating appropriate transmit power levels to the source node and the various relay stations subject to both individual power constraints as well as an overall sum power constraint. In particular, we derive the optimal long-term power allocation strategy which minimizes the average end-to-end symbol or bit error rate provided that the average SNR on all hops is sufficiently high. Our approach is very practical since it requires only knowledge of the statistical properties of all hops and it is very generic since it can be directly applied to a wide variety of different channel distributions and modulation schemes, including cases where the channel distributions of different hops do not belong to the same family or where on different hops different modulation schemes are used.

I. INTRODUCTION

Relayed transmission recently has received a considerable amount of research attention due to its impressive potential to significantly improve the performance of current and future wireless communication systems [1], [2]. In such systems, the data transmission between two peers is realized on a hop-by-hop basis with the help of a certain set of intermediate relay stations, which process the received signals in an appropriate way before forwarding them towards the actual destination. In this regard, one has to distinguish between conventional multihop systems, where every relay node processes and forwards only the signals received from its immediate predecessor node, and more general cooperative diversity systems, where the signals received from all preceding nodes might be combined in an appropriate way in order to further improve the performance [1], [3]. In this paper, we focus exclusively on pure multihop transmission, which has been extensively investigated during the past few years and which has among others the big advantage compared to cooperative diversity systems of a significantly lower implementation complexity as well as reduced power consumption [4]–[7].

Aside from the obvious importance for wireless ad-hoc and sensor networks, where relayed transmission represents a key enabling technology due to the usual absence of a permanently deployed infrastructure network, intelligent relaying is likely to play also a central role in the future evolution of cellular systems, such as WiMax or the 3GPP LTE [2]. In fact, certain relay nodes might be deployed by the network operators as part of the network infrastructure in order to extend the cell range and data rate coverage in a cost-effective way since the

complexity and hence the costs of relay stations are assumed to be much smaller than the costs for conventional base stations.

In this paper, we address the important problem of assigning appropriate transmit power levels to the source node and the involved relay stations of a wireless multihop system with regenerative relays subject to an overall sum power constraint as well as individual per-node power constraints with the goal to minimize the average end-to-end error rate. In general, the power allocation problem for relayed transmission has already been considered in literature before, albeit in a clearly different manner. In [5], [8], [9], for example, the authors determine optimal power allocation strategies for regenerative and non-regenerative multihop and cooperative diversity systems aiming at minimizing the end-to-end outage probability. Besides, the maximization of the corresponding capacity has been considered in [10], [11] and references therein. In [12], the authors derive a power allocation strategy which minimizes the average end-to-end symbol error rate (SER) in the high SNR regime for a special cooperation protocol with one regenerative relay only and in [13] the minimization of the average bit error rate (BER) is considered for non-regenerative cooperative diversity systems with a single relay station only as well. In contrast to most previous works, we focus on power allocation based on statistical channel state information (csi) only, what is assumed to be a promising approach for practical systems due to relatively minor feedback requirements and since statistical csi usually can be estimated in a rather reliable way. Besides, we consider a pure multihop system with an *arbitrary* number of regenerative relay stations and we aim at minimizing the average end-to-end BER or SER. As will be seen, our approach is very generic since it is directly applicable to a wide variety of different fading distributions and modulation schemes.

The remainder of this paper is structured as follows: First, our system model as well as some fundamental assumptions and prerequisites are outlined in Section II. Then, the actual power allocation strategy is derived and analyzed in Section III, followed by selected performance results in Section IV. Finally, our conclusions are given in Section V.

II. SYSTEM MODEL

We consider a general wireless multihop system, where the data transmission between a source node and a corresponding destination node is realized via a set of $(N - 1)$ intermediate relay stations. The actual data to be transmitted is forwarded

from the source to the destination on a hop-by-hop basis during N different time intervals of equal length, where the relay stations are assumed to operate in a regenerative estimate-and-forward mode, i.e., they perform always hard decisions on the received symbols before forwarding them to their respective successor node. In this regard, always only the signals received from the direct predecessor node are evaluated, without considering any signals that might be received from other (preceding) nodes of the end-to-end transmission chain. This way, both the complexity and the power consumption of the involved relay stations can be kept limited, what consequently reduces the costs of such devices. Apart from that, it is expected that in most practical systems the channel conditions between two neighboring nodes generally should be much better than the channel conditions between two more distant nodes, wherefore the additional gain that could be obtained by combining all received signals in an appropriate way is anyway assumed to be relatively small in most cases.

In order to keep our analysis as general as possible, we do not assume any specific channel distribution or modulation scheme and it might even be the case that on different hops different modulation schemes are used or that different hops have completely different fading statistics. The only prerequisite we have is that for sufficiently high average SNR $\bar{\gamma}_i$, the average BER or SER—depending on what is considered—on the i -th hop can be reasonably approximated by

$$\bar{P}_{e,i} \approx (\eta_i \bar{\gamma}_i)^{-\delta_i}, \quad i = 1, \dots, N \quad (1)$$

where $\eta_i > 0$ and $\delta_i \geq 1$ denote the corresponding coding gain and diversity order, respectively [14], [15]. Please note that this does not represent a big limitation since this approximation is well-known to be valid for most commonly employed coherent and differential modulation schemes as well as most standard multipath fading models, including Rayleigh, Nakagami- m , and Rician fading [14]. In fact, in [15] a systematic approach for determining the parameters η_i and δ_i has been presented, which is basically almost always applicable if the probability density function (pdf) of the SNR before a hard-decision is performed can be expanded into a Maclaurin series. Since in multihop systems the average SNR on all hops is usually relatively high as the detrimental effects of path loss and shadowing can be at least partially mitigated by properly placing/selecting the involved relay stations, (1) generally becomes rather accurate and therefore might be reasonably used as the basis for our further analysis.

Assuming that the average power gain of the i -th hop is given by Ω_i and denoting the transmit power level of the i -th node when the node is active by P_i ¹, (1) can be rewritten as

$$\bar{P}_{e,i} \approx \left(\mu_i \frac{P_i}{\sigma^2} \right)^{-\delta_i}, \quad i = 1, \dots, N, \quad (2)$$

where we have introduced for brevity the short-hand notation

$$\mu_i = \eta_i \Omega_i \quad (3)$$

¹Please note that since every node is only active during one out of N time intervals of equal length, the actual *average* transmit power level of the i -th node is actually P_i/N .

and where σ^2 denotes the variance of the additive white Gaussian noise (AWGN), which is—without loss of generality—assumed to be the same for all hops².

III. POWER ALLOCATION STRATEGY

If the coding gains η_i , the diversity orders δ_i as well as the average power gains Ω_i of all hops are known, this knowledge can be exploited to optimize the power allocation among the involved nodes provided that the total power that might be used for an end-to-end transmission is limited, for example due to the regulatory restrictions. Since all three parameters depend only on the used modulation schemes and the prevalent channel distributions and hence usually change only at a rather slow pace, they generally can be estimated rather reliably and signaled using a very low-rate control channel. Therefore, such a statistical long-term power allocation algorithm represents a very attractive approach for practical implementations.

For notational convenience, let us introduce a reference SNR $\bar{\gamma}$ in the following, which is defined as

$$\bar{\gamma} = \frac{P_{\text{avg}}}{\sigma^2}, \quad (4)$$

where $P_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N P_i$ corresponds to the average transmit power of an active node. Using this parameter, we can express all transmit power levels P_i as

$$P_i = p_i P_{\text{avg}}, \quad i = 1, \dots, N \quad (5)$$

where p_i denotes the fraction of P_{avg} allocated to the i -th node when the i -th node is active. The overall sum power constraint, which ensures that the total power in the system is fixed, hence can be written as

$$\sum_{i=1}^N p_i = N. \quad (6)$$

Aside from this sum power constraint, it might be necessary in practice to take additional per-node power constraints into account, thus further limiting the range of allowed values for the coefficients p_i to

$$0 < p_i \leq p_{i,\text{max}}, \quad i = 1, \dots, N \quad (7)$$

with $p_{i,\text{max}}$ as the corresponding maximum power levels. If we denote the average end-to-end BER/SER³ as a function of the power allocation coefficients p_i by $\bar{P}_e(p_1, \dots, p_N)$, our optimization problem consequently can be written as

$$\{p_1, \dots, p_N\}_{\text{opt}} = \arg \min_{\substack{\{p_1, \dots, p_N\} \\ \sum_{i=1}^N p_i = N \\ 0 < p_i \leq p_{i,\text{max}}}} \bar{P}_e(p_1, \dots, p_N). \quad (8)$$

In this regard, the average end-to-end error rate \bar{P}_e can be reasonably approximated by

$$\bar{P}_e \approx \sum_{i=1}^N \bar{P}_{e,i}, \quad (9)$$

²The case with unequal noise variances can easily be traced back to the considered case by adjusting the average power gains Ω_i appropriately.

³If the average end-to-end SER is considered, on all hops the same modulation order should be used since otherwise the definition of an end-to-end SER does not make any real sense.

with $\bar{P}_{e,i}$ as the average error rate of the i -th hop according to (1). Please note that (9) actually represents only an upper bound on the average end-to-end error rate since it is based on the assumption that an end-to-end transmission is always erroneous if at least one error occurs on any of the N hops. However, in fact multiple errors occurring on different hops might compensate each other, thus also leading to error-free end-to-end transmission and hence to an improved performance compared to (9) [7]. Nevertheless, provided that the average SNR on every hop is sufficiently high—what should be usually the case for multihop transmission—the expression according to (9) becomes very accurate since in this case the probability that multiple errors occur during one end-to-end transmission is virtually negligible compared to the error probability of each hop. As a simple example, assume that we employ binary phase shift keying (BPSK) and that the error probability on each hop is 10^{-3} . Then the probability that a bit is erroneously received on two hops so that the errors could compensate each other is already $10^{-6} \ll 10^{-3}$ and the probability of three erroneous per-hop transmissions would be as small as 10^{-9} . Using (8) together with (9) and (2), our optimization problem consequently can be written as

$$\{p_1, \dots, p_N\}_{\text{opt}} = \arg \min_{\substack{\{p_1, \dots, p_N\} \\ \sum_{i=1}^N p_i = N \\ 0 < p_i \leq p_{i,\max}}} \sum_{i=1}^N (\mu_i p_i \bar{\gamma})^{-\delta_i}. \quad (10)$$

In this regard, we have to distinguish two different cases. If the sum of all individual power constraints $p_{i,\max}$ is smaller than or equal to the overall sum power constraint, i.e., if

$$\sum_{i=1}^N p_{i,\max} \leq N, \quad (11)$$

the solution is trivial and the optimum performance can be achieved if every node transmits at its maximum allowed power. For determining the solution of (10) in case that (11) is not fulfilled, we first of all consider a slightly modified optimization problem by neglecting all individual power constraints. The corresponding Lagrangian clearly is given by

$$\mathcal{L}(p_1, \dots, p_N, \lambda) = \sum_{k=1}^N (\mu_k p_k \bar{\gamma})^{-\delta_k} + \lambda \left(\sum_{j=1}^N p_j - N \right), \quad (12)$$

where λ denotes the Lagrange multiplier. Making this expression stationary with respect to all p_i , we obtain as the solution of this modified problem

$$p'_{i,\text{opt}} = \frac{1}{\mu_i \bar{\gamma}} \sqrt{\frac{\delta_i \mu_i \bar{\gamma}}{\lambda}}, \quad i = 1, \dots, N. \quad (13)$$

The additional individual power constraints can then easily be taken into account by clipping the power allocation coefficients $p'_{i,\text{opt}}$ to their respective maximum values $p_{i,\max}$, whereby optimality is well-known to be preserved [5]. Moreover, the parameter λ can be determined from the overall sum power

constraint (6), so that we finally obtain the general solution

$$p_{i,\text{opt}} = \begin{cases} p_{i,\max} & \text{if } \sum_{i=1}^N p_{i,\max} \leq N \\ \min \left\{ p_{i,\max}, \frac{1}{\mu_i \bar{\gamma}} \sqrt{\frac{\delta_i \mu_i \bar{\gamma}}{\lambda}} \right\} & \text{otherwise} \end{cases}, \quad (14)$$

where the parameter λ is implicitly given by

$$\sum_{i=1}^N \min \left\{ p_{i,\max}, \frac{1}{\mu_i \bar{\gamma}} \sqrt{\frac{\delta_i \mu_i \bar{\gamma}}{\lambda}} \right\} = N. \quad (15)$$

In the general case with arbitrary diversity orders δ_i , it unfortunately seems to be impossible to determine the parameter λ analytically in closed-form. However, this generally can be efficiently done numerically. Introducing a function $g(\lambda)$ as

$$g(\lambda) = \sum_{i=1}^N \min \left\{ p_{i,\max}, \frac{1}{\mu_i \bar{\gamma}} \sqrt{\frac{\delta_i \mu_i \bar{\gamma}}{\lambda}} \right\} - N, \quad (16)$$

solving (15) is actually equivalent to solving the standard root-finding problem $g(\lambda) = 0$, where $g(\lambda)$ obviously is a continuous function of λ . Assuming the non-trivial case where $\sum_{i=1}^N p_{i,\max} > N$, we have $\lim_{\lambda \rightarrow 0} g(\lambda) > 0$ and at the same time $\lim_{\lambda \rightarrow \infty} g(\lambda) = -N$. Furthermore, we find

$$\frac{\partial}{\partial \lambda} \min \left\{ p_{i,\max}, \frac{1}{\mu_i \bar{\gamma}} \sqrt{\frac{\delta_i \mu_i \bar{\gamma}}{\lambda}} \right\} \leq 0 \quad \forall i, \quad (17)$$

where equality holds only if hard-clipping to $p_{i,\max}$ occurs. However, if $\sum_{i=1}^N p_{i,\max} > N$, there is at least one coefficient p_i which is not clipped to its maximum value and consequently $g(\lambda)$ is the sum of a finite number of non-increasing functions and at least one steadily decreasing function, wherefore $g(\lambda)$ itself is steadily decreasing as well. Putting these facts together implies that there exists always a *unique* λ which solves (15) and by exploiting the strictly decreasing nature of $g(\lambda)$, a very accurate numerical solution can always be efficiently obtained.

Please note that for the special case without (active) individual power constraints and the same diversity order on all hops, i.e., if $p_{i,\max} > N \quad \forall i$ and $\delta_i = \delta \quad \forall i$, λ can be determined in closed-form based on (15) and can easily be shown to be given by

$$\lambda = \frac{\delta}{\bar{\gamma}^\delta} \left(\frac{1}{N} \sum_{i=1}^N \mu_i^{-\frac{\delta}{\delta+1}} \right)^{\delta+1}. \quad (18)$$

Plugging this expression in (14), we hence obtain for the optimal power allocation coefficients in this case

$$p_{i,\text{opt}}|_{\delta_i=\delta \quad \forall i} = \frac{\mu_i^{-\frac{\delta}{\delta+1}}}{\sum_{k=1}^N \mu_k^{-\frac{\delta}{\delta+1}}} N, \quad i = 1, \dots, N. \quad (19)$$

IV. PERFORMANCE RESULTS

We illustrate the performance of our power allocation strategy by considering a four-hop system with BPSK transmission and coherent detection, where all hops undergo independent but not necessarily identically distributed Nakagami- m fading. In this regard, we denote the fading parameter of the i -th hop

by m_i , which is assumed to be an integer value and which also corresponds to the diversity order δ_i in this case. Besides, the corresponding coding gains η_i have been shown in [15] to be given by

$$\eta_i = \sqrt[m]{\frac{2\Gamma(m)\sqrt{\pi}}{m^{m-1}\Gamma(m+\frac{1}{2})}}, \quad (20)$$

with $\Gamma(\cdot)$ as the well-known gamma function. For the considered case with binary modulation, the exact average end-to-end BER for a certain set of power allocation coefficients $p_k > 0$ actually can be calculated analytically in closed-form by noting that a certain bit is transmitted erroneously from source to destination if the number of intermediate hops on which an error occurs is odd. This can be expressed as

$$\bar{P}_{e,\text{bpsk}} = \sum_{\nu=1}^{\lfloor \frac{N+1}{2} \rfloor} \sum_{\substack{\mathbb{A} \cup \bar{\mathbb{A}} = \{1, 2, \dots, N\} \\ |\mathbb{A}| = 2\nu - 1}} \prod_{m \in \mathbb{A}} \bar{P}_{b,m} \prod_{n \in \bar{\mathbb{A}}} (1 - \bar{P}_{b,n}), \quad (21)$$

where the summation is over all possibilities for partitioning the set $\{1, \dots, N\}$ into two disjoint subsets \mathbb{A} and $\bar{\mathbb{A}}$ subject to the constraint that the cardinality of \mathbb{A} is an odd number and where $\bar{P}_{b,k}$ is the average BER on the k -th hop. For our case with BPSK transmission and Nakagami- m fading on all hops, $\bar{P}_{b,k}$ can easily be shown to be given by [16]

$$\bar{P}_{b,k} = \frac{1}{2} \left[1 - \sqrt{\frac{p_k \Omega_k \bar{\gamma}}{m_k + p_k \Omega_k \bar{\gamma}}} \right] \times \sum_{i=0}^{m_k-1} \binom{2i}{i} \left(\frac{1}{4} \left(1 - \frac{p_k \Omega_k \bar{\gamma}}{m_k + p_k \Omega_k \bar{\gamma}} \right) \right)^i. \quad (22)$$

Fig. 1 shows the average end-to-end BER versus the average SNR assuming unity average power gains on all hops and unequal fading parameters m_k for our optimal power allocation scheme without individual power constraints as well as conventional uniform power allocation with $p_k = 1 \forall k$, which is optimal in case that all hops are identically distributed. Aside from the exact BER curves, which have been calculated based on (21), Fig. 1 shows the BER approximation that has been used as the basis for our derivation according to (9) and (2), as well as results obtained from Monte Carlo simulations. First, we note that the difference between the exact BER curves and the approximation according to (9) and (2) is almost negligible, even for low to moderate average SNRs, what justifies the pursued approach. Furthermore, it can be seen that there is an excellent match between calculated and simulated BERs, what verifies the validity of (21). Most importantly, however, it can be seen that with our optimal power allocation strategy significant performance gains can be achieved, reaching up to about 6 dB if the average SNR is sufficiently high.

The impact of individual power constraints on the system performance is illustrated in Figs. 2 and 3. Here, we consider exactly the same setup as before, but now the individual power allocation coefficients p_k are limited to $p_{k,\text{max}} = p_{\text{max}} \forall k$. As one would expect, the performance gain is obviously the smaller the stricter the imposed individual power constraints

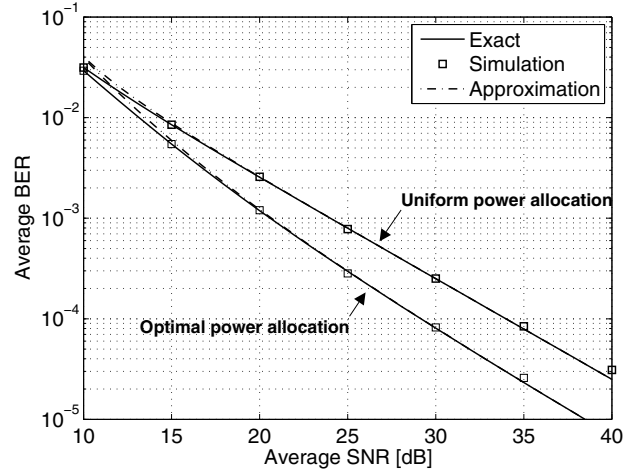


Fig. 1. Average BER as a function of the average SNR for a four-hop system with BPSK transmission, Nakagami- m fading with unity average power gains on all hops and fading parameters $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, and $m_4 = 4$. Individual power constraints are not considered.

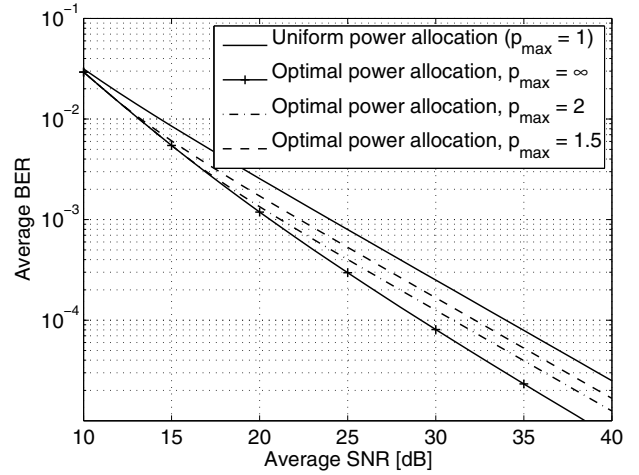


Fig. 2. Average BER as a function of the average SNR for a four-hop system with BPSK transmission, Nakagami- m fading with unity average power gains on all hops and fading parameters $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, and $m_4 = 4$. The individual power constraints are set to $p_{k,\text{max}} = p_{\text{max}} \forall k$.

and for $p_{\text{max}} = 1$, our optimal power allocation strategy actually reduces to conventional uniform power allocation. Besides, the impact of individual power constraints is clearly more significant in the high SNR regime, what can be explained by the fact that in the low SNR regime these constraints are simply not effective since the power assigned to any node approaches approximately one. This can be seen from Fig. 3, for example. In fact, for rather small average SNRs, the power allocation coefficients are actually identical to those that we would obtain without individual power constraints and with increasing SNR, more and more power is assigned to the first hop with minimal diversity order. However, once a certain value of $\bar{\gamma}$ is exceeded, $p_{1,\text{opt}}$ is hard-clipped to $p_{1,\text{max}}=1.5$ and if the SNR is further increased, most remaining power

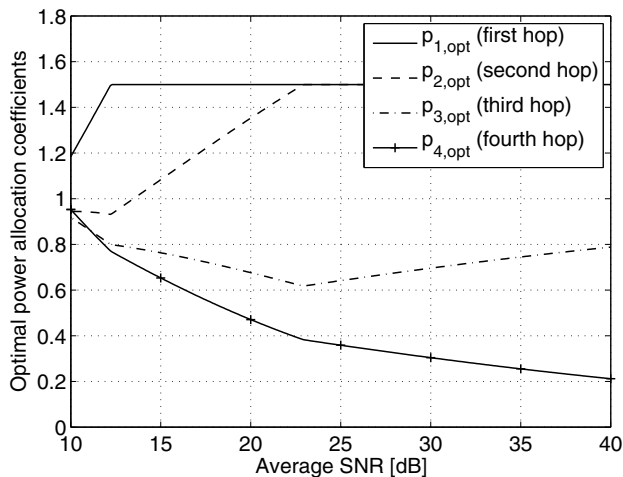


Fig. 3. Optimal power allocation coefficients over the average SNR for a four-hop system with BPSK transmission, Nakagami- m fading with unity average power gains on all hops and fading parameters $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, and $m_4 = 4$. All individual power constraints are set to $p_{\max} = 1.5$.

is concentrated on the hop with the second smallest diversity order until also its maximum power level has been reached and so on. This is reasonable since once a certain node transmits at maximum allowed power, the performance of the corresponding hop cannot be improved anymore and hence the overall end-to-end performance optimization reduces to the optimization of the power allocation among the remaining hops subject to the original sum power constraint minus the power already allocated to the already saturated nodes.

Having considered a scenario with different diversity orders on different hops so far, Fig. 4 shows the average BER as a function of the average SNR for a system with identical fading parameters and hence diversity orders on all hops, but unequal average power gains. It can be seen that the performance can be significantly improved again by means of our optimal power allocation strategy, yielding an impressive effective SNR gain of up to about 4 dB in the high SNR regime.

V. CONCLUSION

We have derived a long-term power allocation algorithm for distributing a given power budget among the source and the relay nodes of a wireless multihop system with regenerative relays, taking also potential per-node power constraints into account. The algorithm is very generic since it can be readily applied to a wide variety of different fading distributions and modulation schemes and it is optimal in terms of minimizing the average end-to-end error rate in the high SNR regime. Since the proposed scheme is based upon statistical CSI only, it requires solely a very low-rate feedback channel, thus making it a very attractive approach for practical implementations. The performance gains over conventional uniform power allocation have been illustrated considering a four-hop system with not necessarily identically distributed Nakagami- m fading on all hops and BPSK transmission as an example and they were shown to be quite significant, even for moderate average SNRs.

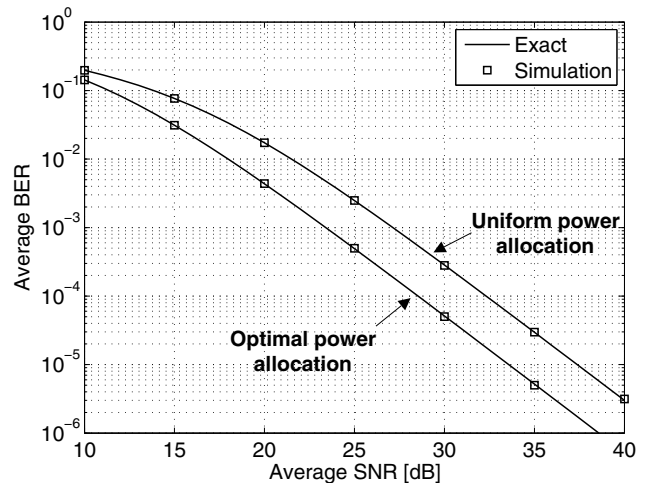


Fig. 4. Average BER for a four-hop system with BPSK transmission and Nakagami- m fading with $m_k = 2 \forall k$ and $\Omega_1 = 0.05$ as well as $\Omega_2 = \Omega_3 = \Omega_4 = 1$. Individual power constraints are not considered.

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