# Adaptive Modulation for Wireless Multihop Systems with Regenerative Relays

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***Abstract*— We present and analyze several adaptive modulation schemes for wireless multihop systems with regenerative relays. In particular, we consider both rate adaptation based on perfect channel state information (csi) of all hops as well as suboptimal strategies based on partial csi, which require only a very limited amount of feedback information and hence represent attractive approaches for practical systems. We thoroughly investigate the performance of all proposed schemes in terms of the average spectral efficiency as well as the average bit error rate, considering the important case of a Rayleigh-fading environment and $M$-ary quadrature amplitude modulation as a concrete example. Numerical results are shown to be in perfect agreement with simulation results—thus verifying the accuracy of our analysis—and illustrate the performance of our schemes in various situations.**

## I. Introduction

In wireless multihop systems, data transmission between a source and the corresponding destination generally is realized on a hop-by-hop basis via a certain set of intermediate relay stations, which always process the signals received from their immediate predecessor nodes in an appropriate way before forwarding them on the next hop [1]–[3]. These relay stations might be either dedicated devices being part of the network infrastructure or other collaborating users nearby. By properly placing/selecting the relay nodes, the channel conditions on the individual hops might be significantly better than the direct link between source and destination and hence considerable performance gains might be achieved [2], [4]. While multihop transmission evidently represents a key enabling technology for wireless mesh and ad-hoc networks, this principle might be beneficially applied to conventional cellular networks as well, for example for enhancing the data rate coverage and extending the cell range in a cost-effective way. Besides, since users at the cell edge usually benefit most of it, relayed transmission might improve the system fairness and by means of a spatial reuse of the available resources through simultaneous transmissions by sufficiently separated relays in the same cell, substantial throughput gains are possible [1]–[3].

The analysis and optimization of multihop systems recently has attracted a lot of research attention, but in most cases it has been assumed that all transmission parameters and in particular the utilized modulation schemes are always fixed [5]–[9]. In the following, we therefore propose and analyze various adaptive modulation schemes for multihop transmission, where the used modulation scheme is always dynamically selected based on the current channel conditions in order to maximize the average spectral efficiency subject to a certain bit error rate (BER) constraint. While adaptive modulation for conventional point-to-point links has been well-studied during the past decade, see for example [10], [11], only very few results have been reported for more general multihop systems so far [12]. However, rate adaptation for relayed transmission generally represents a significantly more complex task since not only a single channel has to be considered, but ideally rather all individual hops have be taken into account for that purpose. The performance of our proposed schemes will be thoroughly analyzed in terms of the average spectral efficiency as well as the average BER, where we always focus on the important case of adaptive $M$-ary quadrature amplitude modulation (M-QAM) and Rayleigh fading channels on all hops.

The remainder of this paper is structured as follows: In Section II, we introduce our system and channel model whereas the actual relay selection strategies are presented and analyzed in Section III. Afterwards, some performance results are given in Section IV, followed by our main conclusions in Section V.

## II. System and Channel Model

We consider a wireless multihop system with $K$-1 regenerative relay stations, which always perform hard decisions on the received symbols before forwarding them to their respective successor node. All relay stations are assumed to work in a half-duplex mode, i.e., they cannot transmit and receive data at the same time, wherefore an end-to-end transmission takes $K$ times the time needed for a direct transmission between source and destination. Besides, we assume independent but not necessarily identically distributed flat fading on all hops, so that a symbol received by the $(\nu+1)$-th node can be expressed in the discrete-time equivalent baseband domain as

$$y_{\nu+1} = \sqrt{P_\nu}\, h_\nu\, x_\nu + n_\nu, \qquad \nu = 1, \ldots, K \tag{1}$$

where $P_\nu$ and $x_\nu$ denote the transmit power and transmit symbol of the $\nu$-th node, respectively, $h_\nu$ the channel coefficient of the $\nu$-th hop, and $n_\nu$ models additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_\nu^2$. For all relay nodes, we have $x_\nu = \hat{x}_{\nu-1}$ ($\nu = 2, \ldots, K-1$), where $\hat{x}_{\nu-1}$ denotes the estimate of the symbol transmitted by the ($\nu$-1)-th node. The instantaneous SNR on the $\nu$-th hop is hence given by

$$\gamma_\nu = \frac{P_\nu}{\sigma_\nu^2}\, |h_\nu|^2, \qquad \nu = 1, \ldots, K. \tag{2}$$

Please note that for simplicity we focus on uncoded transmission in the following, but it should be kept in mind that the

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end-to-end performance might be further improved if additionally sophisticated coding schemes are used. Furthermore, we assume that every node processes only the signals received from its immediate predecessor node, what allows for the production of low-complexity and hence low-cost devices.

## III. Rate Adaptation Strategies

We assume that there are in total $N$ different modulation schemes available, where with the $i$-th scheme always $k_i$ bits are mapped to one complex symbol. Besides, we focus on the case where for one end-to-end transmission always the same modulation scheme is used on all hops, what relaxes the buffer requirements of the relay stations and avoids the need for possible segmentations of transport blocks in practice. The design and analysis of schemes with decoupled rate adaptation on every hop is left for further studies. The goal of the proposed strategies generally is to maximize the average spectral efficiency by selecting an appropriate modulation scheme based on the available csi in such a way that the instantaneous end-to-end bit error probability (BEP) conditioned on the available csi does not exceed a certain target value $\delta_0$.

### *A. Rate Selection Based on Perfect CSI of All Hops*

If we have perfect instantaneous csi of all hops, it is possible to calculate the end-to-end BEP for all available modulation schemes and hence to simply select the one with the highest spectral efficiency which satisfies the imposed BEP constraint. Given the instantaneous SNR values $\gamma_1, \ldots, \gamma_K$ of all hops, the selection strategy can be formulated mathematically as

$$\phi(\gamma_1, \ldots, \gamma_K) = \arg \max_{i \in \{1,\ldots,N\} | P_{b,i}(\gamma_1,\ldots,\gamma_K) \le \delta_0} k_i, \qquad (3)$$

where $\phi$ denotes the index of the modulation scheme to be selected and $P_{b,i}(\gamma_1, \ldots, \gamma_K)$ the average end-to-end BEP in AWGN for the $i$-th available modulation scheme with SNRs $\gamma_1, \ldots, \gamma_K$ on the individual hops. This BEP generally can be reasonably approximated by

$$P_{b,i}(\gamma_1, \gamma_2, \cdots, \gamma_K) \approx \sum_{\nu=1}^{K} P_{b,i}(\gamma_\nu) \qquad (4)$$

where $P_{b,i}(\gamma)$ denotes the average BEP on a single hop with SNR $\gamma$ if the $i$-th available modulation scheme is used. Please note that (4) actually represents only an upper bound on the average end-to-end BEP since it is based on the assumption that a bit is always erroneously transmitted from source to destination if at least one error occurs on any of the $K$ hops. However, in practice multiple errors on different hops might compensate each other, thus leading to error-free end-to-end transmission as well. Nevertheless, if the target BER is small enough, the probability of multiple errors on different hops which compensate each other is negligible compared to the error probability of one hop so that (4) in fact becomes quite accurate. As a simple example, assume that the per-hop BEP of one bit is $10^{-2}$. Then, the probability of two mutually compensating bit errors on different hops would be as small as $10^{-4} \ll 10^{-2}$ and hence might be reasonably neglected.

In the following, we always consider the case where the rate adaptation is performed based on binary phase shift keying (BPSK) and square M-QAM constellations. For BPSK, the average per-hop BEP is well-known to be given by $P_{b,\mathrm{BPSK}}(\gamma)$ = $\frac{1}{2}\mathrm{erfc}\sqrt{\gamma}$, with $\mathrm{erfc}(\cdot)$ as the complementary error function, whereas for square M-QAM with Gray mapping we have [13]

$$P_{b,i}(\gamma) = \frac{M_i^{-1/2}}{\log_2\sqrt{M_i}} \sum_{k=1}^{\log_2\sqrt{M_i}} \sum_{\nu=0}^{(1-2^{-k})\sqrt{M_i}-1} \Bigg\{ (-1)^{\left\lfloor \frac{\nu\, 2^{k-1}}{\sqrt{M_i}} \right\rfloor} \times \left( 2^{k-1} - \left\lfloor \frac{\nu\, 2^{k-1}}{\sqrt{M_i}} + \frac{1}{2} \right\rfloor \right) \mathrm{erfc}\left( \sqrt{\frac{3\,(2\,\nu+1)^2\,\gamma}{2(M_i-1)}} \right) \Bigg\}, \qquad (5)$$

where $M_i = 2^{k_i}$ denotes the number of available constellation symbols for the $i$-th modulation scheme.

### *B. Rate Selection Based on Limited Feedback*

*1) Mode of Operation:* The optimal rate adaptation strategy outlined in the previous section requires perfect csi of all hops, what is considered to be unrealistic in practical systems. Therefore, we propose a more practical approach in the following, in which every node only has to know the channel from its preceding station, which is already required for coherent detection. The basic idea is to break down the end-to-end BER constraint $\delta_0$ into several per-hop BER constraints $\delta_1, \ldots, \delta_K$ and to select the maximum modulation scheme for which all these per-hop constraints are satisfied. For that purpose, the destination first of all determines based on $\gamma_K$ the highest modulation order that might be used on the $K$-th hop without exceeding the corresponding BER target $\delta_K$ and feeds back the associated index to the preceding relay station. Likewise, this relay then determines the highest modulation order that might be used on the (K-1)-th hop without violating the respective per-hop BER target $\delta_{K-1}$, compares the obtained modulation order with the modulation order desired by the destination and feeds back the minimum of both values to its predecessor. This process is then continued in the same way until the actual source node is reached. Please note that with this approach every node only has to feed back $\lceil \mathrm{ld}\ N \rceil$ bits denoting the supported modulation scheme on the corresponding hop to its preceding station. Based on (4), it can easily be seen that the per-hop BER constraints have to satisfy

$$\sum_{\nu=1}^{K} \delta_\nu \le \delta_0, \qquad (6)$$

what guarantees that the end-to-end BER constraint is always fulfilled. As will be shown in Section IV, the actual values of $\delta_\nu$ ($\nu = 1, \ldots, K$) might be appropriately chosen based on the average SNRs of all hops in order to optimize the system performance. Similarly to adaptive modulation for conventional single-hop systems, the maximum modulation order that might be used on the $\nu$-th hop can be easily determined by comparing the corresponding SNR $\gamma_\nu$ with precalculated switching thresholds $\vartheta_{\nu,i}$ ($i = 1, \ldots, N+1$), which depend on the available modulation schemes and the per-hop BER constraint $\delta_\nu$ [10], [11]. In fact, if $\vartheta_{\nu,i} \le \gamma_\nu <$

$\vartheta_{\nu,i+1}$, the $i$-th available modulation scheme should be chosen. In this regard, we always set $\vartheta_{\nu,N+1} = \infty \; \forall \; \nu$ as well as

$$\vartheta_{\nu,i} = P_{b,i}^{-1}(\delta_\nu), \qquad \nu = 1, \ldots, K; \quad i = 1, \ldots, N \tag{7}$$

where $P_{b,i}^{-1}$ denotes the inverse BEP function for the $i$-th modulation scheme, which might be determined numerically. Please note that if $\gamma_\nu < \vartheta_{\nu,1}$, the BER target $\delta_\nu$ would be violated even with the most robust modulation scheme, wherefore transmission should be suspended in this case.

*2) Analysis:* The probability that the $i$-th available modulation scheme is chosen is generally given by

$$P_i = \text{Prob}\left[\bigwedge_{\nu=1}^{K} (\gamma_\nu \geq \vartheta_{\nu,i})\right] - \text{Prob}\left[\bigwedge_{\nu=1}^{K} (\gamma_\nu \geq \vartheta_{\nu,i+1})\right] \tag{8}$$

$$= \prod_{\nu=1}^{K} \left(1 - F_{\gamma_\nu}(\vartheta_{\nu,i})\right) - \prod_{\eta=1}^{K} \left(1 - F_{\gamma_\eta}(\vartheta_{\eta,i+1})\right), \tag{9}$$

where $F_{\gamma_\nu}(x)$ denotes the cumulative distribution function (cdf) of $\gamma_\nu$. The average spectral efficiency for a general set of switching thresholds $\{\vartheta_{\nu,i}\}$ is then simply given by

$$\Phi = \frac{1}{K} \sum_{i=1}^{N} k_i \, P_i, \tag{10}$$

where the factor $\frac{1}{K}$ is due to the rate loss caused by the fact that the multihop transmission takes $K$ times the time needed for a direct transmission between source and destination. For the important case of Rayleigh fading on all hops, we have $F_{\gamma_i}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}_i}\right)$, $\gamma \geq 0$, with $\bar{\gamma}_i$ as the average SNR of the $i$-th hop. Hence, (10) simplifies in this case to

$$\Phi = \frac{1}{K} \sum_{i=1}^{N} k_i \left[\exp\left(-\sum_{\nu=1}^{K} \frac{\vartheta_{\nu,i}}{\bar{\gamma}_\nu}\right) - \exp\left(-\sum_{\eta=1}^{K} \frac{\vartheta_{\eta,i+1}}{\bar{\gamma}_\eta}\right)\right]. \tag{11}$$

The average BER, on the other hand, is given by the average number of erroneously received bits at the destination divided by the average spectral efficiency and hence given by

$$\bar{P}_e = \frac{1}{K\,\Phi} \sum_{i=1}^{N} k_i \left[\int_{\vartheta_{1,i}}^{\infty} \cdots \int_{\vartheta_{K,i}}^{\infty} P_{b,i}(\gamma_1, \cdots, \gamma_k) \prod_{\nu=1}^{K} p_{\gamma_\nu}(\gamma_\nu) d\gamma_\nu - \int_{\vartheta_{1,i+1}}^{\infty} \cdots \int_{\vartheta_{K,i+1}}^{\infty} P_{b,i}(\gamma_1, \cdots, \gamma_K) \prod_{\nu=1}^{K} p_{\gamma_\nu}(\gamma_\nu) d\gamma_\nu\right], \tag{12}$$

with $p_{\gamma_\nu}(x)$ as the probability density function of $\gamma_\nu$. By combining (12) with (4), we obtain after some manipulations

$$P_e \approx \frac{1}{K\Phi} \sum_{i=1}^{N} k_i \sum_{\nu=1}^{K} \left[\mathcal{I}_{\nu,i}(\vartheta_{\nu,i}) \prod_{\substack{j=1 \\ j \neq \nu}}^{K} \left(1 - F_{\gamma_j}(\vartheta_{j,i})\right) - \mathcal{I}_{\nu,i}(\vartheta_{\nu,i+1}) \prod_{\substack{j=1 \\ j \neq \nu}}^{K} \left(1 - F_{\gamma_j}(\vartheta_{j,i+1})\right)\right], \tag{13}$$

where we have introduced for brevity the short-hand notation

$$\mathcal{I}_{k,i}(x) = \int_x^{\infty} P_{b,i}(\gamma_k) p_{\gamma_k}(\gamma_k) d\gamma_k. \tag{14}$$

In case of Rayleigh-fading on all hops, we have $p_{\gamma_k}(\gamma_k) = \frac{1}{\bar{\gamma}_k} \exp\left(-\gamma_k/\bar{\gamma}_k\right)$, $\gamma_k \geq 0$. Plugging this expression together with the known BEP expressions for BPSK and square M-QAM in (14), it can be seen that calculating (13) reduces in both cases to solving integrals of the form

$$\Xi = \int_x^{\infty} \text{erfc}\left(a\sqrt{\gamma}\right) \exp\left(-b\,\gamma\right) d\gamma, \qquad a, b, > 0. \tag{15}$$

By making use of partial integration and exploiting [14, eqs. (3.361,1), (3.361,2)], (15) can be solved in closed-form as

$$\Xi = \frac{1}{b}\left[e^{-bx}\text{erfc}\left(a\sqrt{x}\right) - \frac{a}{\sqrt{a^2+b}}\text{erfc}\left(\sqrt{(a^2+b)x}\right)\right]. \tag{16}$$

Hence, for BPSK (16) reduces to

$$\mathcal{I}_{k,1}(x) = \exp\left(-\tfrac{x}{\bar{\gamma}_k}\right) \text{erfc}\left(\sqrt{x}\right) - \sqrt{\tfrac{\bar{\gamma}_k}{\bar{\gamma}_k+1}}\, \text{erfc}\left(\sqrt{\tfrac{x(1+\bar{\gamma}_k)}{\bar{\gamma}_k}}\right) \tag{17}$$

and likewise we get for square M-QAM

$$\mathcal{I}_{k,i}(x) = \frac{M_i^{-1/2}}{\log_2\sqrt{M_i}} \sum_{k=1}^{\log_2\sqrt{M_i}} \sum_{\nu=0}^{(1-2^{-k})\sqrt{M_i}-1} \left\{(-1)^{\left\lfloor \frac{\nu\, 2^{k-1}}{\sqrt{M_i}} \right\rfloor} \times \left(2^{k-1} - \left\lfloor \frac{\nu\, 2^{k-1}}{\sqrt{M_i}} + \frac{1}{2} \right\rfloor\right) \left[\exp\left(-\frac{x}{\bar{\gamma}_k}\right) \text{erfc}\left(\sqrt{\lambda_{\nu,i}\, x}\right) - \sqrt{\frac{\lambda_{\nu,i}}{\lambda_{\nu,i} + \frac{1}{\bar{\gamma}_k}}}\, \text{erfc}\left(\sqrt{\left(\frac{1}{\bar{\gamma}_k} + \lambda_{\nu,i}\right) x}\right)\right]\right\}, \tag{18}$$

where $\lambda_{\nu,i} = \frac{3(2\nu+1)^2}{2(M_i-1)}$. Plugging (17) and (18) in (13) then finally yields the desired analytical closed-form expression for the average BER of our rate-adaptive system.

### *C. Rate Selection Based on Statistical/Perfect CSI*

The last proposed scheme is illustrated and analyzed considering the important case of dual-hop transmission with a single relay station. Basically it would be easily possible to generalize this scheme to an arbitrary number of hops as well, but we expect that this is not of real significance for practical systems wherefore we do not consider this generalization in more detail here. The basic idea is that for one hop only statistical csi is available (e.g., for the relay-to-destination link) while the other hop is perfectly known. Since statistical information can be fed back over a rather low-rate feedback channel, the required feedback load for this approach is very small. This statistical information might be the average BEPs $\bar{P}_{2,i} = \mathcal{I}_{2,i}(0)$ for the available modulation schemes on the second hop or the corresponding channel distribution, based on which these average BERs can be easily determined. The optimal selection strategy subject to the instantaneous end-to-end BER constraint $\delta_0$ (conditioned on the available csi) can then be expressed as

$$\phi\left(\gamma_1, \{\bar{P}_{2,i}\}_{i=1}^{N}\right) = \arg \max_{i \in \{1,\ldots,N\} | P_{b,i}(\gamma_1, \bar{P}_{2,i}) \leq \delta_0} k_i, \tag{19}$$

where similarly to (4) we have

$$P_{b,i}(\gamma_1, \bar{P}_{2,i}) \approx P_{b,i}(\gamma_1) + \bar{P}_{2,i}. \tag{20}$$

For a given average BER $\bar{P}_{2,i}$ on the second hop, we hence can determine a maximum allowed BEP $\delta_{1,i}$ on the first hop, which is obviously given by $\delta_{1,i} = \delta_0 - \bar{P}_{2,i}$. Based on the set of all these $\delta_{1,i}$ ($i = 1, \ldots, N$), we can then determine switching thresholds for the first hop again as

$$\vartheta_{1,i} = \begin{cases} P_{b,i}^{-1}\left(\delta_0 - \bar{P}_{2,i}\right), & \text{if } \bar{P}_{2,i} < \delta_0 \\ \infty & \text{else} \end{cases}, \quad i = 1, \ldots, N. \tag{21}$$

For a given set of switching thresholds $\{\vartheta_{1,i}\}_{i=1}^{N+1}$, the average spectral efficiency can then be calculated as

$$\Phi = \frac{1}{2}\sum_{i=1}^{N} k_i \left[F_{\gamma_1}\left(\vartheta_{1,i+1}\right) - F_{\gamma_1}\left(\vartheta_{1,i}\right)\right], \tag{22}$$

while the corresponding average BER is given by

$$\begin{aligned} \bar{P}_e &\approx \frac{1}{2\Phi}\sum_{i=1}^{N} k_i \int_{\vartheta_{1,i}}^{\vartheta_{1,i+1}} P_{b,i}(\gamma_1, \bar{P}_{2,i})\, p_{\gamma_1}(\gamma_1)\, d\gamma_1 \\ &= \frac{1}{2\Phi}\sum_{i=1}^{N} k_i \left[\left(F_{\gamma_1}(\vartheta_{1,i+1}) - F_{\gamma_1}(\vartheta_{1,i})\right)\bar{P}_{2,i}\right. \\ &\quad \left. + \mathcal{I}_1(\vartheta_{1,i}) - \mathcal{I}_1(\vartheta_{1,i+1})\right]. \end{aligned} \tag{23}$$

As before, for the considered Rayleigh-fading scenario with rate adaptation based on BPSK and square M-QAM constellations, $\mathcal{I}_{k,i}(\cdot)$ can be given analytically in closed-form according to (17) and (18), respectively.

## IV. Performance Results

In the following, we always focus on dual-hop transmission with a single relay station only and we assume that four different modulation schemes are available, namely BPSK, QPSK, 16-QAM and 64-QAM. Fig. 1 depicts the average spectral efficiency versus the average SNR on the source-to-relay link for different average SNRs on the relay-to-destination link and $\delta_0 = 10^{-2}$. As can be seen, rate adaptation based on limited csi leads in all cases to almost the same performance as adaptation based on perfect csi, even though the required feedback load is much lower in this case. Besides, the performance of rate adaptation based on perfect csi of the source-to-relay link and statistical csi of the relay-to-destination link strongly depends on the average SNR of the second hop. For $\bar{\gamma}_2 = 15$ dB, for example, the average BER of the second hop with QPSK already exceeds the end-to-end BER target $\delta_0$, wherefore always at most BPSK might be used, no matter how good the first link is. With $\bar{\gamma}_2 = 35$ dB, however, the average spectral efficiency in this case is even slightly higher than for adaptation based on perfect csi. This is because we always adapt the modulation scheme such that the instantaneous BEP *conditioned* on the available csi does not exceed $\delta_0$. If, however, for the second hop only statistical csi is available, the actual BEP (conditioned on perfect csi) sometimes might exceed the set target, thus facilitating a higher spectral efficiency. Nevertheless, the imposed average BER constraint is always satisfied. Finally, it can be seen that there is basically a perfect match between our theoretical and simulated values, what verifies the accuracy of our analysis.

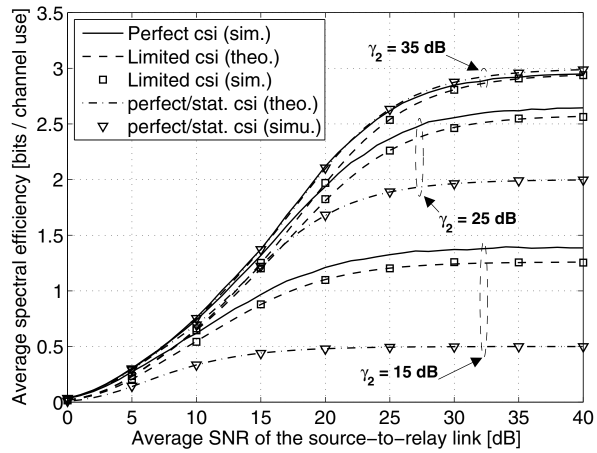


Fig. 1. Average spectral efficiency versus the average SNR on the source-to-relay link for an end-to-end BEP target of $\delta_0 = 10^{-2}$. In case of adaptation based on limited csi, the per-hop BEP targets are set to $\delta_1 = \delta_2 = 0.005$.

Fig. 2 illustrates the average BER versus the average SNR of the source-to-relay link for two different BER constraints and $\bar{\gamma}_2 = 25$ dB. Obviously, there is an excellent match between theoretical and simulation results again and it can be seen that in all cases the average BER is well below the corresponding BER target $\delta_0$. Especially in case of adaptation based on perfect or limited csi, there is a considerable gap, which results from the fact that we always impose a rather conservative instantaneous BER constraint instead of an average one.

Fig. 3 compares the performance of a rate-adaptive system with direct transmission between source and destination with a dual-hop system, where one relay station is placed exactly in the middle between both. The relay uses the same transmit power as the source and we perform rate adaptation based on limited csi. Besides, we assume an exponential path loss model with a path loss exponent of four, so that the average SNRs on the source-to-relay and relay-to-destination links are always 12 dB higher than the average SNR of the single-hop system indicated on the abscissa of Fig. 3. Obviously, in the low SNR regime our dual-hop system outperforms the single-hop system, but at high SNRs, it is the other way around. This is due to the restriction to a finite set of modulation schemes and the general rate loss of one half for dual-hop transmission.

Finally, Fig. 4 illustrates the optimization potential of adaptation based on limited csi if the per-hop BER targets are appropriately chosen based on the corresponding average SNRs. In all cases, the average spectral efficiency is normalized to the respective maximum value, i.e., the value that can be achieved in the optimal case. On the x-axis, we depict $\delta_1$, which implicitly determines $\delta_2$ as well. As can be seen, if both average SNRs are the same, we obtain as expected the best performance for $\delta_1 = \delta_2 = 0.005$. However, if the relay-to-destination link is much better, we should generally relax the per-hop BER constraint on the source-to-relay link a bit in order to get optimal results. This is reasonable since in such a case the source-to-relay link represents the bottleneck of the system. For $\bar{\gamma}_2 = 30\,\mathrm{dB}$, for example, the average spectral efficiency with the standard value $\delta_1 = \delta_0/2$ is only about $88\%$ of the spectral efficiency that can be achieved in the best case.

## V. Conclusion

We have proposed and analyzed various adaptation modulation strategies for wireless multihop systems with regenerative relays. The proposed schemes require different kinds of csi and hence exhibit different performance-complexity tradeoffs. For adaptation based on limited as well as perfect/statistical csi, we have derived exact analytical closed-form expressions for the corresponding average spectral efficiency as well as very accurate approximations for the average bit error rate, considering a Rayleigh-fading scenario with BPSK and square $M$-QAM transmission as a concrete example. Simulation results were shown to be in excellent agreement with theoretically calculated values, thus verifying the accuracy of our analysis.

## References


[1] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80 – 89, Sept. 2004.

[2] Ö. Oyman, J. N. Laneman, and S. Sandhu, "Multihop relaying for broadband wireless mesh networks: from theory to practice," *IEEE Commun. Mag.*, vol. 45, no. 11, pp. 116 – 122, Nov. 2007.

[3] H. Yanikomeroglu, "Fixed and mobile relaying technologies for cellular networks," in *Proc. Second Workshop on Applications and Services in Wireless Networks*, July 2002.

[4] A. Müller and J. Speidel, "Relay-selection in dual-hop transmission systems: selection strategies and performance results," in *Proc. IEEE Int. Conf. on Commun. (ICC)*, May 2008.

[5] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126 – 1131, Sept. 2003.

[6] ——, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130 – 135, Jan. 2004.

[7] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-$m$ fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18 – 22, Jan. 2006.

[8] A. Müller and J. Speidel, "Exact symbol error probability of M-PSK for multihop transmission with regenerative relays," *IEEE Commun. Lett.*, vol. 11, no. 12, pp. 952 – 954, Dec. 2007.

[9] ——, "Symbol error probability of M-QAM in multihop communication systems with regenerative relays," in *Proc. IEEE Veh. Technol. Conf.*, May 2008.

[10] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Trans. Commun.*, vol. 49, no. 9, pp. 1561 – 1571, Sept. 2001.

[11] M.-S. Alouini and A. J. Goldsmith, "Adaptive modulation over Nakagami fading channels," *Wireless Personal Commun.*, vol. 13, no. 1, pp. 119 – 143, May 2000.

[12] Z. Lin, E. Erkip, and M. Ghosh, "Rate adaptation for coded cooperative systems," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2006.

[13] K. Cho and D. Yoon, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1074 – 1080, July 2002.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 6th ed. New York: Academic Press, 2000.


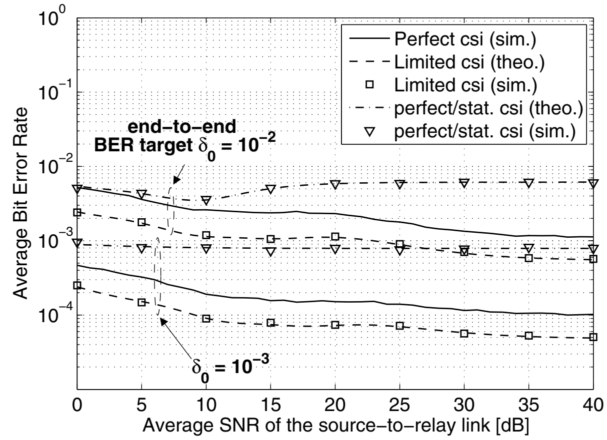


Fig. 2. Average end-to-end BER as a function of the average SNR on the source-to-relay link for $\delta_1 = \delta_2 = \delta_0/2$ and an average SNR on the relay-to-destination link of $\bar{\gamma}_2 = 25\,\mathrm{dB}$.

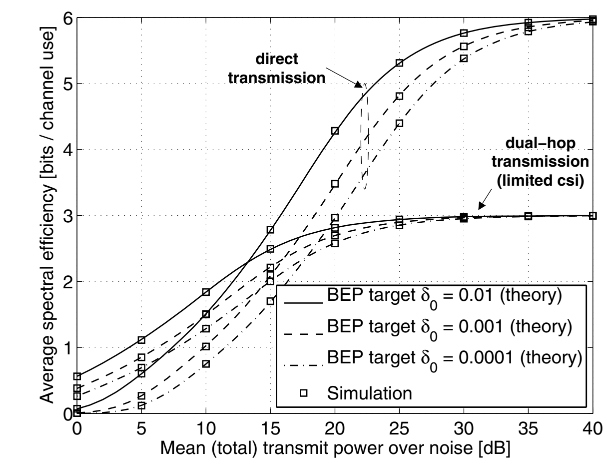


Fig. 3. Average spectral efficiency as a function of the mean (total) transmit power over noise. In case of dual-hop transmission, the per-hop BEP targets are set to $\delta_1 = \delta_2 = \delta_0/2$ and a path loss exponent of $\nu = 4$ is assumed.

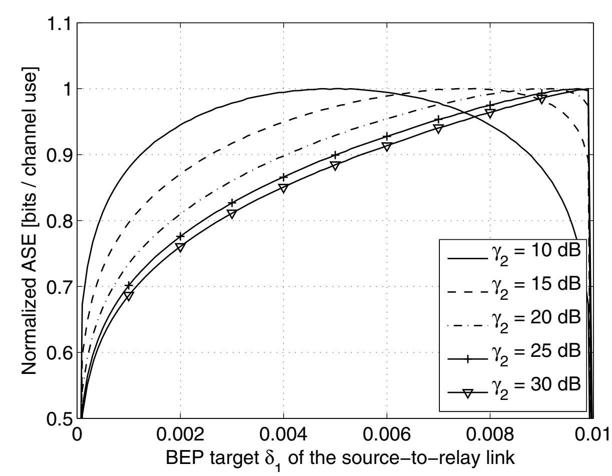


Fig. 4. Average spectral efficiency normalized to the respective maximum value versus the BEP target $\delta_1$ on the source-to-relay link for $\bar{\gamma}_1 = 10\,\mathrm{dB}$, $\delta_0 = 0.01$, and different average SNRs of the relay-to-destination link.