BER Analysis and Optimization of Generalized Spatial Modulation in Correlated Fading Channels

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***Abstract*—We propose and analyze a generalized spatial modulation scheme, where depending on the bits to be transmitted a certain data symbol as well as a certain beamforming vector is chosen. The receiver then has to estimate both the used beamforming vector and the transmitted data symbol for being able to reconstruct the originally transmitted bit sequence. We analyze our scheme theoretically by deriving a very tight upper bound on the average bit error rate (BER) for arbitrary symbol constellations and beamforming codebooks in correlated Rayleigh fading channels. Based on this bound, we determine a design criterion for optimizing the used beamforming codebook in presence of spatial correlation at the transmitter-side. Simulation results are shown to be in excellent agreement with our analytical calculated BERs and they illustrate the significant performance gains that can be obtained with our generalized scheme compared to conventional spatial modulation.**

I. Introduction

The ever increasing demand for higher data rates in wireless communication systems requires innovative transmission schemes achieving a high spectral efficiency. In this regard, multiple-input multiple-output (MIMO) systems have become a key technology for achieving this target and a great variety of corresponding transmission schemes has been proposed and thoroughly analyzed in literature within the past few years. With the well-known vertical Bell Labs layered space-time (V-BLAST) architecture, for example, M complex data symbols are simultaneously transmitted using M different antenna elements, thus directly increasing the achievable data rates by a factor of M in the ideal case [1]. However, due to the inherent inter-channel interference (ICI) of such systems, rather complex receiver structures like maximum likelihood (ML) or ordered successive interference cancelation (OSIC) receivers are required for achieving a good performance.

In order to mitigate the computational requirements at the receiver-side while still achieving a good performance, recently so-called spatial modulation has been proposed [2], [3]. Spatial modulation is a fundamentally novel transmission technique for MIMO systems, where depending on the input bits both a single transmit antenna element as well as a complex data symbol is chosen. Hence, the actual information to be transmitted is contained in the transmitted data symbol as well as in the selected transmit antenna element. Since only one transmit antenna is active per point in time, ICI is completely avoided, thus allowing significantly less complex receiver structures, which now have to estimate both the active transmit antenna and the actual data symbol [2], [3].

The performance of spatial modulation has been thoroughly investigated for certain low-complexity receiver structures in [3] and it has been shown that it is possible to achieve a comparable performance to conventional V-BLAST systems with a remarkably lower complexity. Besides, the optimal detection algorithm of a spatial modulation system has recently been presented in [4] and analytical expressions for the average BER have been obtained based on the corresponding pairwise error probabilities (PEP). However, the derived results are only valid for real-valued signal point constellations and scenarios without any spatial correlation at the transmitter- or receiver-side. Recently, in [5] the authors proposed so-called generalized space-shift keying as a special case of spatial modulation, where the data to be transmitted is exclusively encoded in the selection of a certain set of transmit antennas, without performing any symbol modulation.

In this paper, we first generalize the original spatial modulation scheme introduced in [2] by not selecting a single transmit antenna element based on the bits to be transmitted, but rather a certain beamforming vector from a given codebook. In a next step, we analyze the corresponding performance by deriving a very tight upper bound on the average BER, which in contrast to the results in [4] is valid for arbitrary complex-valued data symbols as well as arbitrarily correlated channels. Based on this result, we determine a simple criterion for optimizing the BER performance by choosing an appropriate long-term beamforming codebook depending on the spatial correlation properties at the transmitter-side. Simulation results show that it is possible to reduce the average BERs tremendously compared to a conventional spatial modulation system.

The paper is organized as follows: Section II introduces the system model and the detection algorithm. In Section III, we derive the tight BER bound, whereas the optimization criterion is presented in Section IV. Simulation results are shown in Section V and Section VI finally concludes the paper.

Throughout this paper, we use the following notation. Bold lower case letters represent vectors, while bold upper case letters denote matrices. The Hermitian transposed, the trace and the determinant of a matrix $\mathbf{A}$ are indicated by $\mathbf{A}^H$, $tr(\mathbf{A})$ and $det(\mathbf{A})$, respectively. We use $\otimes$ for the Kronecker product and $vec(\mathbf{A})$ stacks all columns of $\mathbf{A}$ into a vector. Furthermore, $E[.]$ represents the expected value of a random variable (RV) and a complex Gaussian distribution with mean m and variance σ^2 is denoted by $\mathcal{CN}(m, \sigma^2)$. The n-th partial derivative with respect to variable s is indicated by $\frac{\partial^n}{\partial s^n}$.

II. System Model

Figure 1 shows the considered MIMO system with M transmit and N receive antennas. The (conventional) spatial mapper maps a certain number of input bits $\mathbf{b}$ onto a vector symbol $q\mathbf{e}_i$, where $\mathbf{e}_i = [0...0\ 1\ 0...0]^T$ denotes the i-th unit vector having one non-zero entry at the i-th position.The modulation symbol q can be drawn from any real- or complex-valued symbol constellation. For illustrating the basic principle in detail, an exemplary mapping process is shown in Fig. 2 for a MIMO system with four transmit antennas and a QPSK constellation. Before every transmission interval, four bits are taken from the input stream, where the first two bits are used for selecting a unit vector $\mathbf{e}_i$ and hence one out of four available antennas whereas the other two bits are mapped to a complex QPSK symbol q. In the case of conventional spatial modulation as introduced in [2], the signal $q\mathbf{e}_i$ would then be directly transmitted, thus having always only one active transmit antenna. The number of simultaneously transmitted bits per channel use or equivalently the spectral efficiency is clearly given by

$$\eta = \eta_{symb} + \log_2(M), \tag{1}$$

where η_{symb} denotes the number of bits per data symbol q and depends on the applied signal point constellation.

As a generalization, we propose to select a general beamforming vector $\mathbf{p}_i$ from a given codebook based on the first input bits rather than an unit vector $\mathbf{e}_i$. Hence, the actual transmit signal is generally given by $q\mathbf{p}_i$ and the symbol q is not necessarily transmitted over only one single antenna element. This approach can be easily implemented by multiplying the output of the conventional spatial mapper with a precoding matrix $\mathbf{P} = [\mathbf{p}_1, ..., \mathbf{p}_M]$ as depicted in Fig. 1, which in fact represents the codebook of available beamforming vectors. Clearly, if $\mathbf{P}$ is chosen as the identity matrix of size M, our generalized scheme reduces to conventional spatial modulation as a special case. Besides, it is quite obvious that the linear transformation done by $\mathbf{P}$ needs to be invertible (i.e. $\det(\mathbf{P}) \neq 0$) and that the total mean transmit power must not be affected by $\mathbf{P}$ (i.e. $tr\left(\mathbf{P}^H\mathbf{P}\right) = M$).

In the following, we always assume that the vector $\mathbf{s} = q\mathbf{p}_i$ is transmitted through a correlated frequency-flat Rayleigh fading channel $\mathbf{H} = \mathbf{A}^H\mathbf{H}_W\mathbf{B}$, where the matrices $\mathbf{A}$ and $\mathbf{B}$ are the square roots of the correlation matrices $\mathbf{R}_{rx} = \mathbf{A}^H\mathbf{A}$ and $\mathbf{R}_{tx} = \mathbf{B}^H\mathbf{B}$, respectively. Furthermore, the entries of $\mathbf{H}_W$ are modeled as independent, complex Gaussian RVs according to $\mathcal{CN}(0,1)$. It is well-known that autocorrelation matrix $\mathbf{R_{HH}}$ of $\mathbf{H}$ is then given by $\mathbf{R_{HH}} = \mathbf{R}_{tx}^* \otimes \mathbf{R}_{rx}$. The elements of the AWGN vector $\mathbf{n}$ are assumed to be independent and identically distributed according to $\mathcal{CN}\left(0, \sigma_n^2\right)$. The received signal vector $\mathbf{r}$ is then given by $\mathbf{r} = \mathbf{Hs} + \mathbf{n}$ and a maximum likelihood (ML) detector [4] estimates the transmitted symbol vectors according to $(\hat{q}, \hat{\mathbf{e}}_i) = \arg\min_{(q,\mathbf{e}_i)} \|\mathbf{r} - \mathbf{HP}q\mathbf{e}_i\|^2$. Provided that the mean symbol power is normalized (i.e., $E\left[|q|^2\right] = 1$) and that $\mathbf{P}$ fulfills the mentioned prerequisites, the average SNR per receive antenna is generally given by $\frac{1}{\sigma_n^2}$.

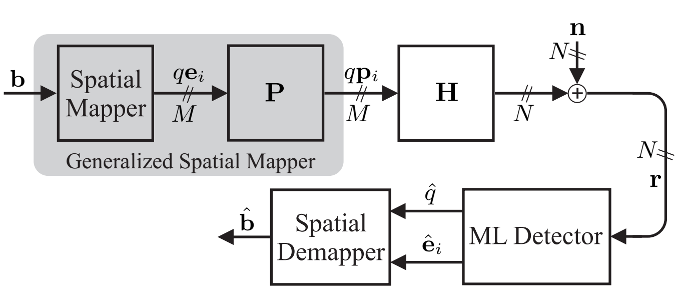

Fig. 1. Considered system model.

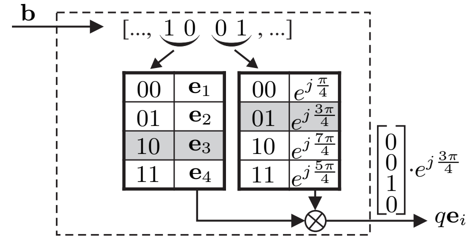

Fig. 2. Exemplary mapping process inside the Spatial Mapper (M=4, QPSK).

Due to the insertion of the prefilter matrix $\mathbf{P}$, generally more than one TX antenna is active per point in time, resulting in a higher transmitter and receiver complexity[1]. However, this approach has the big advantage that the available transmit power can be equally distributed among all antenna elements while conventional spatial modulation assigns the power to only one antenna element. This has for example the potential to significantly reduce the requirements on the used amplifiers. In fact, if $\mathbf{P}$ is chosen as a discrete Fourier transform matrix

$$[\mathbf{P}]_{p,q} = \exp\left(j\frac{2\pi}{M}(p-1)(q-1)\right) \quad p, q = 1, ..., M\ , \tag{2}$$

every antenna element is always active and transmits at the same constant power level. Clearly, without spatial correlation at the transmitter-side, any unitary matrix $\mathbf{P}$ achieves the same performance as conventional spatial modulation since the statistical properties of $\mathbf{H}_W$ and $\mathbf{H}_W\mathbf{P}$ are identical. However, in presence of spatial correlation, this is generally no longer the case and $\mathbf{P}$ might be chosen appropriately in order to achieve a more reliable transmission. We follow this idea in Section IV, where we derive a design criterion for the matrix $\mathbf{P}$ for such a scenario.

III. Performance Analysis

In the following, we derive a tight upper bound on the average BER of our generalized spatial modulation scheme based on the PEP $P(\mathbf{x} \to \hat{\mathbf{x}})$, which denotes the probability that the receiver decides more likely in favor of a vector symbol $\hat{\mathbf{x}}$ than in favor of the actually transmitted vector symbol $\mathbf{x}$. Having a ML detector at the receiver-side, this PEP can be formulated as

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \text{Prob}\left[\|\mathbf{r_x} - \mathbf{Hx}\|^2 > \|\mathbf{r_x} - \mathbf{H}\hat{\mathbf{x}}\|^2\right], \tag{3}$$

[1]Please note that the receiver complexity is higher than in [4], since $\mathbf{r}$ needs to be compared to $q\mathbf{Hp}_i$ instead of $q\mathbf{h}_i$.

where $\mathbf{r_x} = \mathbf{Hx} + \mathbf{n}$ denotes the noisy received signal. Please note that the definition of the PEP in (3) takes only two vector symbols $\mathbf{x}$ and $\hat{\mathbf{x}}$ into account. As the Euclidean distance from $\mathbf{r_x}$ to all other vector symbols $\mathbf{x} \neq \hat{\mathbf{x}}$ of the signal space is not considered, there might actually be another vector symbol which is more likely detected than $\hat{\mathbf{x}}$.

Following the analysis in [6], the PEP according to (3) conditioned on $\mathbf{H}$ can be rewritten as

$$P\left(\mathbf{x} \to \hat{\mathbf{x}} \,|\mathbf{H}\right) = Q\left(\sqrt{\frac{\left\|\mathbf{H}\left(\mathbf{x}-\hat{\mathbf{x}}\right)\right\|^2}{2\sigma_n^2}}\right), \tag{4}$$

where $Q(\cdot)$ denotes the Gaussian Q-function. Averaging (4) over the distribution of $\mathbf{H}$ may be accomplished using an alternative expression for the Q-function as in [7], where it is shown that the unconditional PEP can be obtained by

$$P\left(\mathbf{x} \to \hat{\mathbf{x}}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Phi\left(-\frac{1}{4\sigma_n^2 \sin^2\theta}\right) d\theta, \tag{5}$$

with $\Phi\left(s\right)$ denoting the moment-generating function (MGF) of the RV $\left\|\mathbf{H}\delta\right\|^2$, where $\delta = \mathbf{x} - \hat{\mathbf{x}}$. Clearly, $\left\|\mathbf{H}\delta\right\|^2$ can be expressed as a complex quadratic form like in [7] as

$$\left\|\mathbf{H}\delta\right\|^2 = vec\left(\mathbf{H}_W^H\right)^H \mathbf{R}_s^{\frac{H}{2}}\left(\mathbf{I}_N \otimes \delta\delta^H\right)\mathbf{R}_s^{\frac{1}{2}} vec\left(\mathbf{H}_W^H\right), \tag{6}$$

with the short-hand notation $\mathbf{R}_s = \mathbf{R}_{rx} \otimes \mathbf{R}_{tx}$. Using the results derived in [8], the MGF of (6) can then be expressed as

$$\Phi(s) = \frac{1}{\det\left(\mathbf{I}_{NM} - s\mathbf{R}_s^{\frac{H}{2}}\left(\mathbf{I}_N \otimes \delta\delta^H\right)\mathbf{R}_s^{\frac{1}{2}}\right)}. \tag{7}$$

Mathematical manipulations as in [7, eqs. (7)-(9)] lead to

$$\Phi(s) = \prod_{i=1}^{M}\prod_{j=1}^{N} \frac{1}{1 - s\psi_i\xi_j'}, \tag{8}$$

where the parameters ψ_i and ξ_j' denote the eigenvalues of the matrices $\delta\delta^H\mathbf{R}_{tx}$ and $\mathbf{R}_{rx}$, respectively. Clearly, $\delta\delta^H\mathbf{R}_{tx}$ has for any δ and $\mathbf{R}_{tx}$ always rank one due to the factor $\delta\delta^H$. For that reason, there is generally only one single non-zero eigenvalue ψ, which can be determined by

$$\psi = \sum_i^N \psi_i = tr\left(\delta\delta^H\mathbf{R}_{tx}\right) = \delta^H\mathbf{R}_{tx}\delta, \tag{9}$$

so that $\Phi\left(s\right)$ can be rewritten as

$$\Phi\left(s\right) = \prod_{j=1}^{P} \frac{1}{\left(1 - s\psi\xi_j\right)^{M_j}}, \tag{10}$$

where $\xi_1, \ldots, \xi_P$ denote the $P \leq N$ *distinct* eigenvalues of $\mathbf{R}_{rx}$, which occur with multiplicities $M_1, \ldots, M_P$. Combining (10) with (5), it is eventually possible to derive exact analytical closed-form expressions for the corresponding PEPs. Expanding $\Phi\left(s\right)$ according to (10) into partial fractions, we obtain

$$\Phi\left(s\right) = \sum_{j=1}^{P}\sum_{m=1}^{M_j} \frac{A_{j,m}}{\left(1 - s\psi\xi_j\right)^m} \tag{11}$$

where the parameters $A_{j,m}$ denote the corresponding expansion coefficients, which can generally be determined as

$$A_{j,m} = \frac{1}{\left(M_j - m\right)!}\frac{1}{\left(-\psi\xi_j\right)^{M_j - m}} \times \frac{\partial^{M_j - m}}{\partial s^{M_j - m}}\left[\prod_{\substack{\nu=1\\ \nu\neq j}}^{P} \frac{1}{\left(1 - s\psi\xi_\nu\right)^{M_\nu}}\right]\Bigg|_{s=\frac{1}{\psi\xi_j}}. \tag{12}$$

Combining (11) with (5) and using the integration result presented in [9, App. B], we finally obtain the exact analytical closed-form expression for the PEP

$$P\left(\mathbf{x} \to \hat{\mathbf{x}}\right) = \sum_{j=1}^{P}\sum_{m=1}^{M_j} \frac{A_{j,m}}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\psi\xi_j}{4\sigma_n^2}}\right)^m d\theta \tag{13}$$

$$= \sum_{j=1}^{P}\sum_{m=1}^{M_j} A_{j,m}\ \mu\left(\frac{\psi\xi_j}{4\sigma_n^2}\right)^m \times \sum_{\nu=0}^{m-1}\binom{m-1+\nu}{\nu}\left[1 - \mu\left(\frac{\psi\xi_j}{4\sigma_n^2}\right)\right]^\nu, \tag{14}$$

where the short-hand notation $\mu\left(x\right)$ is defined as

$$\mu\left(x\right) = \frac{1}{2}\left(1 - \sqrt{\frac{1}{1 + \frac{1}{x}}}\right), \qquad x \geq 0. \tag{15}$$

If we have spatial correlation at the transmitter-side only, the PEP expression of (14) can be simplified as $\mathbf{R}_{rx}$ corresponds to the identity matrix and thus $P = 1$ and the eigenvalue ξ_1 is equal to one and has a multiplicity of $M_1 = N$. It can be shown that in this case the N-th expansion coefficient $A_{1,N}$ equals one and all other coefficients $A_{1,m}$ $m \neq N$ are equal to zero. Hence, we obtain for the PEP

$$P\left(\mathbf{x} \to \hat{\mathbf{x}}\right) = \mu\left(\frac{\psi}{4\sigma_n^2}\right)^N \sum_{k=0}^{N-1}\binom{N-1+k}{k}\left[1 - \mu\left(\frac{\psi}{4\sigma_n^2}\right)\right]^k \tag{16}$$

with $\mu\left(x\right)$ according to (15).

A. Average BER

Having analytical closed-form expressions for the average PEP, we can derive an accurate approximation for the average BER [10]. Considering only a single vector symbol $\mathbf{x}$, it can easily be seen that its average BER is generally bounded by

$$P_{e,bit}\left(\mathbf{x}\right) \leq \frac{1}{\eta}\sum_{\hat{\mathbf{x}}\neq\mathbf{x}} d\left(\mathbf{x}, \hat{\mathbf{x}}\right) P\left(\mathbf{x} \to \hat{\mathbf{x}}\right), \tag{17}$$

where $d\left(\mathbf{x}, \hat{\mathbf{x}}\right)$ denotes the Hamming distance and indicates the number of differences in the bits associated with the vector symbols $\mathbf{x}$ and $\hat{\mathbf{x}}$, respectively. Averaging $P_{e,bit}\left(\mathbf{x}\right)$ over all 2^η different symbols $\mathbf{x}$ leads to the total bit error probability.

$$P_{e,bit}^{tot} \leq E\left[\frac{1}{\eta}\sum_{\hat{\mathbf{x}}\neq\mathbf{x}} d\left(\mathbf{x}, \hat{\mathbf{x}}\right) P\left(\mathbf{x} \to \hat{\mathbf{x}}\right)\right] = \frac{1}{\eta 2^\eta}\sum_{\mathbf{x}}\sum_{\hat{\mathbf{x}}\neq\mathbf{x}} d\left(\mathbf{x}, \hat{\mathbf{x}}\right) P\left(\mathbf{x} \to \hat{\mathbf{x}}\right). \tag{18}$$

B. Theoretical BER Limits of Spatial Modulation

The asymptotic performance of any space-time code is usually quantified in terms of the corresponding diversity order and coding gain. In this regard, the diversity order specifies how fast the average BER decays with increasing SNR in the high SNR regime whereas the coding gain is reflected by a certain SNR shift. In fact, spatial modulation basically represents just a special space-time code and hence it can be analyzed in exactly the same way. In general, the maximum achievable diversity order is determined by the number of nonzero eigenvalues ψ_i and ξ_j. Assuming a receive correlation matrix $\mathbf{R}_{rx}$ with full rank, the RX diversity is equal to N. However, as can be seen from (9), there is always one nonzero eigenvalue ψ. Therefore, spatial modulation can never achieve a higher diversity order than N.

Considering a system without channel coding, spatial modulation therefore experiences generally a performance penalty compared to most other space-time codes, which are capable of exploiting transmit diversity as well. This also holds for the well-known Alamouti code [11], for example, which transmits the information in two consecutive time intervals over two transmit antennas, thus achieving a diversity order of $2\,N$ resulting in a lower BER in the high SNR regime. However, at low and medium SNR, the BER of spatial modulation can nevertheless be lower due to a higher coding gain.

IV. Optimization of the Prefilter Matrix

As already mentioned before, spatial modulation cannot exploit any transmit diversity and therefore the only way to improve its performance by choosing an appropriate beamforming codebook $\mathbf{P}$ is to optimize the coding gain. Keeping in mind that the eigenvalues ξ_j of $\mathbf{R}_{rx}$ are independent of the prefilter parameters, only ψ may be influenced.

In the following, δ is considered to be the difference between two signal vectors from the Spatial Mapper. In presence of a prefilter $\mathbf{P}$, ψ is given by

$$\psi = \delta^H \mathbf{P}^H \mathbf{R}_{tx} \mathbf{P} \delta. \tag{19}$$

Clearly, minimizing the average BER is equivalent to a joint minimization of all PEPs. As it can be seen from (14) and (15), the PEPs are minimized if their corresponding ψ is maximized. Assuming that all PEPs contribute equally to the average BER, the following optimization criterion is adequate

$$\mathbf{P} = \arg\max_{\mathbf{P}} \left\{ \min_{\delta} \left\{ \psi\left(\delta\right) \right\} \right\}. \tag{20}$$

As the prefilter has to fulfill the power constraint, the optimization in (20) is subjected to $tr\left(\mathbf{P}\mathbf{P}^H\right) = M$.

Please note that the minimum operator in (20) is taken over every signal vector difference, which yields in general $2^\eta\left(2^\eta - 1\right)$ different combinations. The exponentially growing amount of combinations in (20) requires a computer aided evaluation. However, a prefilter sample may be rapidly checked for its performance by calculating its $\psi_{min} = \min_{\delta}\left\{\psi\left(\delta\right)\right\}$. The higher ψ_{min} is, the lower the BER will be. Furthermore, it should be noted that this optimization criterion is invariant with respect to the applied mapping since the impact of $d\left(\mathbf{x}, \hat{\mathbf{x}}\right)$ has been neglected.

V. Simulation Results

In the following, we present selected performance results to illustrate that our analytically calculated average BERs are in good agreement with simulated values and how the performance might be significantly improved by choosing an appropriate codebook matrix $\mathbf{P}$. In this regard, we assume that the receiver performs a ML detection and that for the selection of an appropriate modulation symbol q a standard Gray mapping is used. Besides, spatial correlation at the transmitter and receiver-side is modeled using an exponential correlation model with two independent parameters ρ_{tx} and ρ_{rx}, respectively. Hence, the corresponding correlation matrices $\mathbf{R}_{tx}$ and $\mathbf{R}_{rx}$ are generally given by

$$\left[\mathbf{R}_{tx}\right]_{p,q} = \begin{cases} \rho_{tx}^{p-q} & q \leq p \\ \left[\rho_{tx}^{q-p}\right]^* & q > p \end{cases} \quad p,q = 1,...,M \tag{21}$$

$$\left[\mathbf{R}_{rx}\right]_{p,q} = \begin{cases} \rho_{rx}^{p-q} & q \leq p \\ \left[\rho_{rx}^{q-p}\right]^* & q > p \end{cases} \quad p,q = 1,...,N. \tag{22}$$

Figure 3 shows the average BER of conventional spatial modulation (**P = I**) with $M = 4$ TX and $N = 4$ RX antennas as well as QPSK, resulting in a total spectral efficiency of $4\ \frac{\text{bit}}{\text{s}\cdot\text{Hz}}$. Both simulation and analytical results are plotted for the non-correlated, TX or RX correlated case. First of all, it can be seen that the deviation between the simulated and the analytical results is almost negligible, especially in the high SNR region. Clearly, this holds for all considered correlation coefficients ρ_{tx} and ρ_{rx}. Moreoever, it is obvious that in presence of spatial correlation either at the transmitter- or the receiver-side, the performance degrades considerably compared to the totally uncorrelated case, resulting in an effective SNR loss of more than 6.5 dB in the high SNR region, for example.

Figure 4 shows the BER performance of a MIMO scheme with $M = 2$ TX and $N = 4$ RX antennas. Conventional spatial modulation with QPSK is compared to an Alamouti scheme with 8-PSK, so that the spectral efficiency is equal to $3\ \frac{\text{bit}}{\text{s}\cdot\text{Hz}}$ in both cases. Considering the totally uncorrelated case, spatial modulation clearly exhibits a smaller BER than the Alamouti scheme in the low SNR region. However, with increasing SNR the Alamouti scheme benefits from its higher diversity order, therefore generally leading to a better performance. The intersection of both BER curves will move to higher SNR, when η increases because the Alamouti scheme suffers from the higher modulation order. In case of TX correlation, the Alamouti scheme has a lower BER than conventional spatial modulation, yielding a gain of 4 dB at a BER of 10^{-3}.

Figure 5 shows the BER of a spatial modulation scheme with $M = 2$ TX, $N = 4$ RX antennas and QPSK with and without spatial correlation at the transmitter-side. In order to improve the performance, in this case, the codebook matrix $\mathbf{P}$ has been optimized by means of an exhaustive search and is

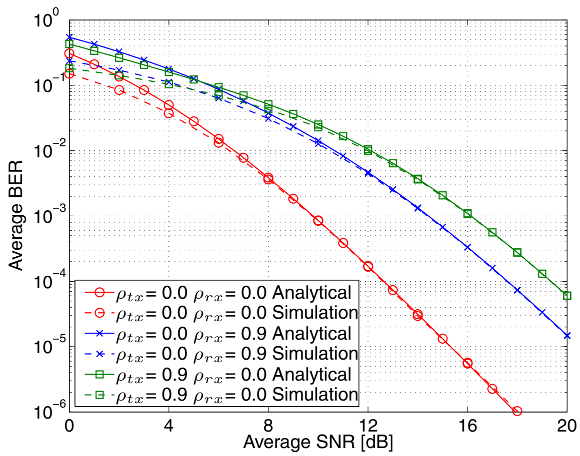

Fig. 3. BER as a function of SNR for conventional spatial modulation with 4 TX, 4 RX antennas and QPSK. Simulation and analytical results are compared.

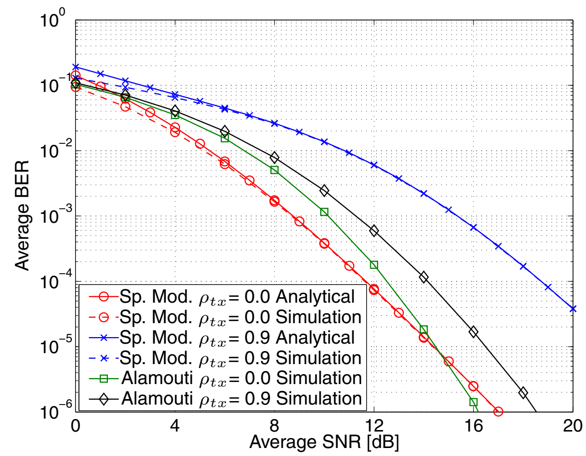

Fig. 4. Comparison between spatial modulation and an Alamouticode, using a MIMO scheme with 2 TX and 4 RX antennas. To achieve $\eta = 3 \frac{\text{bit}}{\text{s}\cdot\text{Hz}}$ spatial modulation uses QPSK and the Alamouti scheme uses 8-PSK.

given by

$$\mathbf{P} = \begin{pmatrix} 0.5 + j0.5 & -0.5 + j0.5 \\ 0.5 - j0.5 & -0.5 + j0.5 \end{pmatrix}. \tag{23}$$

Obviously, without prefilter and spatial correlation, there is a significant loss of 6 dB at BER 10^{-3} compared to the uncorrelated case. With the prefilter of (23), in contrast, this loss is only up to about 2 dB, what illustrates the significant gains that can be gained by means of our generalized spatial modulation scheme.

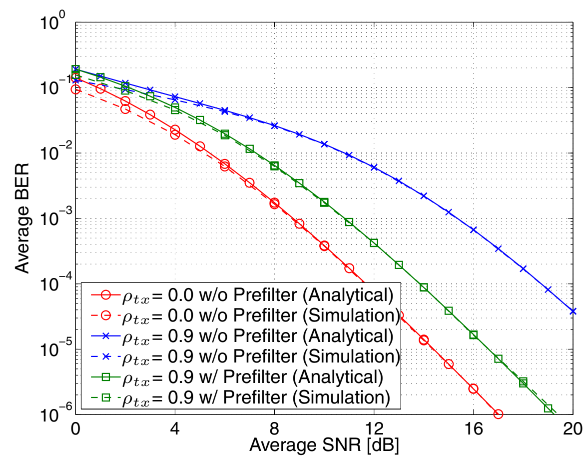

Fig. 5. Performance of spatial modulation with and without prefilter under different TX correlation properties. The MIMO scheme features 2 TX, 4 RX antennas and QPSK symbols. $\rho_{rx} = 0$

VI. Conclusion

We have proposed a generalized spatial modulation scheme, where the bits to be transmitted are encoded through the choice of a certain beamforming vector as well as a particular data symbol. Furthermore, we have performed a theoretical analysis of our approach by deriving a very tight upper bound on the average BER for arbitrary signal point constellations and beamforming codebooks in spatially correlated Rayleigh fading channels. As codebooks provide new degrees of freedom, we have determined a simple optimization criterion for designing an appropriate beamforming codebook, reducing the BER in presence of spatial correlation at the transmitter-side. Finally, simulation results were shown to be in good agreement with our analytically calculated BERs and they illustrated that our generalized approach can yield SNR gains in the order of several dBs compared to conventional spatial modulation.

References

[1] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," *URSI Int. Symp. on Signals, Systems, and Electronics*, pp. 295–300, Sep-Oct 1998.

[2] R. Mesleh, H. Haas, C. W. Ahn, and S. Yun, "Spatial modulation - A new low complexity spectral efficiency enhancing technique," *Int. Conf. on Commun. and Netw. in China, 2006.*, Oct. 2006.

[3] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. on Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, July 2008.

[4] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: Optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545–547, Aug. 2008.

[5] ——, "Generalized space shift keying modulation for mimo channels," *IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun., 2008.*, Sept. 2008.

[6] M. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. Wiley Interscience, 2005.

[7] A. Hedayat, H. Shah, and A. Nosratinia, "Analysis of space-time coding in correlated fading channels," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 6, pp. 2882–2891, Nov. 2005.

[8] G. L. Turin, "The characteristic function of hermitian quadratic forms in complex normal variables," *Biometrika*, vol. 47, no. 1-2, pp. 199–201, 1960.

[9] M.-S. Alouini and A. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. on Commun.*, vol. 47, no. 9, pp. 1324–1334, Sep 1999.

[10] J. G. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2004.

[11] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.