

# Dual-Hop Adaptive Packet Transmission with Regenerative Relaying for Wireless TDD Systems

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**Abstract**—We consider the design and performance analysis of adaptive modulation and coding (AMC) applied to regenerative dual-hop transmission systems, where a source node communicates with a corresponding destination node only indirectly via an intermediate relay station operating in the well-known decode-and-forward mode. Specifically, we consider the general case where the AMC over both hops is performed independently of each other. As such, a buffer is required at the relay station to buffer certain data packets received from the source node before they can be forwarded to the actual destination. The proposed scheme is especially suitable for systems operating in time division duplex (TDD) mode, where channel state information is readily available at the transmitter-side. We analyze the performance of our approach based on a finite-state Markov chain model of our system and we derive exact analytical closed-form expressions for a variety of different performance measures, which are illustrated for various cases by means of selected numerical results.

## I. INTRODUCTION

In wireless multihop systems, the data transmission between a source node and a corresponding destination node is realized on a hop-by-hop basis via one or multiple intermediate relay stations, thereby offering numerous advantages over conventional single-hop systems, such as higher transmission rates, a more robust link performance as well as simpler and more flexible network planning and deployment [1]–[3]. While relayed transmission traditionally has played a crucial role in the area of wireless ad-hoc and sensor networks, it recently has also emerged as a key technology for the future development of cellular infra-structure based networks, such as wireless local area networks, WiMax, or the 3GPP long-term evolution [1], [3], [4]. In such cellular networks, relays might be used to solve coverage problems in a cost-efficient manner and to significantly increase the transmission rates to users located near the cell-edge. Besides, it is possible to increase the system capacity through a spatial reuse of the available resources since both the central base station and the corresponding relay nodes might transmit simultaneously in the same frequency band if they are located far enough apart from each other [3].

While relayed transmission has the potential to significantly improve the average channel quality on the various hops compared to the direct link between source and destination, its ability to mitigate the effects of multipath fading is very limited. For that reason, in practice additionally appropriate link adaptation schemes should be employed, which dynamically adjust various transmission parameters to the current

channel conditions. For conventional single-hop systems, such link adaptation strategies have been extensively studied in the past. In this regard, the focus was mainly put on uncoded adaptive modulation, where the modulation scheme and possibly also the power is dynamically adjusted [5]–[7], as well as adaptive modulation and coding (AMC), where depending on the current channel conditions an appropriate modulation and coding scheme (MCS) is selected [8], [9]. For relay-assisted transmission systems, in contrast, similar investigations have started only recently (see for example [10]–[12]).

In this paper, we address the problem of performing AMC over dual-hop transmission systems with one intermediate relay station operating in the decode-and-forward mode [2]. In particular, we consider the case where the adaptation on both hops is performed independently of each other, so that the relay may have to temporarily buffer data if the relay-to-destination link is not good enough to directly forward all bits received from the source node within a certain time interval. In this regard, we consider to the best of our knowledge for the first time the realistic case with a finite buffer at the relay station. Besides, the AMC is performed based on the instantaneous SNRs on the corresponding hops only, what is particularly suitable for systems operating in time division duplex (TDD) mode, where channel state information (CSI) is readily available at the transmitter side due to channel reciprocity, so that no explicit feedback signaling would be required in that case. However, with this mode of operation, buffer overflows might occur in case that the number of packets transmitted by the source exceeds the number of free slots in the relay buffer. Recently, we have therefore proposed a cross-layer approach in [13], which takes additionally the current buffer filling level at the relay station into account during the MCS selection so that buffer overflows can be completely avoided. However, this approach requires feedback signaling in any case, even for TDD systems. Subsequently, we show that both approaches exhibit almost identical performance, wherefore the required feedback load in case that CSI is available at the transmitter-side anyway is probably not justified in most cases of interest. For that purpose, we carry out a thorough analysis of our dual-hop transmission system by means of a finite-state Markov chain model, based on which we obtain exact analytical closed-form expressions for a variety of different key performance indicators.

The remainder of this paper is organized as follows: In Section II, we outline our system and channel model and we introduce the proposed AMC strategy in detail. In Section III, we construct a finite-state Markov chain model of our system, based on which we analyze the corresponding performance in Section IV. Finally, selected numerical results are presented in Section V, followed by our conclusions in Section VI.

## II. SYSTEM AND CHANNEL MODEL

We consider a dual-hop transmission system, where the data transmission from a source node S to a destination node D is realized via an intermediate relay station R operating in the decode-and-forward mode [2]. In this regard, every end-to-end transmission interval is subdivided into two different time slots of equal length. During the first phase, the source takes a certain number of data packets from its input buffer (which by assumption never runs out of packets), encodes and modulates the corresponding bits and transmits them to the relay station. The relay then tries to demodulate and decode the received signals and puts the successfully recovered packets into a buffer, which can accommodate at most  $L_{\max}$  different packets. Packets which could not be successfully decoded, in contrast, simply are dropped, and their retransmission—if necessary—will have to be taken care of by higher layers. During the second time slot, the relay takes a certain number of packets out of its buffer, encodes and modulates them appropriately and finally transmits them to the destination node D.

We assume that AMC is independently applied to both hops, where the number of packets that might be transmitted within a certain time slot depends on the selected MCS. As a concrete example, we consider the same  $N = 5$  different MCSs as in [14] and as summarized in Table I and we assume that each transmitted packet is encoded separately. As can be seen, the achievable rate with the  $i$ -th mode is greater than or equal to  $i$  times the rate of the first mode. Assuming for simplicity that with the first mode exactly one packet can be transmitted during one time slot, we consequently can transmit a maximum number of  $i$  packets if the  $i$ -th MCS is selected. Furthermore, we assume for simplicity that the buffer in the relay station can store at least as many packets as might be transmitted with the highest-order AMC mode, i.e.,  $L_{\max} \geq N$ .

The MCS selection over both hops is performed solely based on the current channel conditions. This approach is especially suitable for TDD systems, where CSI is readily available at the transmitter-side so that no feedback would be required in that case. In particular, if the SNR  $\gamma_i$  on the

$i$ -th hop ( $i \in \{1, 2\}$ ) falls into a certain region bounded by the switching thresholds  $\vartheta_{i,k}$  and  $\vartheta_{i,k+1}$ , the  $k$ -th AMC mode is selected [5], [8]. The switching thresholds  $\vartheta_{i,k}$  for determining which MCS is supported for a certain SNR  $\gamma_i$  are chosen in such a way that a certain packet error rate (PER) target is not exceeded. Using the standard PER approximation [15]

$$P_{e,i}(\gamma) = \begin{cases} 1, & \text{for } 0 < \gamma < \gamma_{T_i} \\ a_i \exp(-g_i \gamma), & \text{for } \gamma \geq \gamma_{T_i} \end{cases}, \quad (1)$$

where the parameters  $a_i$ ,  $g_i$  as well as  $\gamma_{T_i}$  are given for the considered AMC modes in Table I, we can readily determine the minimum SNR that is required such that with the  $k$ -th MCS a given target PER  $P_{\text{target},i}$  is not exceeded as

$$\vartheta_{i,k} = \frac{1}{g_k} \ln \left( \frac{a_k}{P_{\text{target},i}} \right). \quad (2)$$

In order to facilitate a unified analysis, we always set  $\vartheta_{i,0} = 0$  as well as  $\vartheta_{i,N+1} = \infty$  in the following. If the instantaneous SNR falls into the zero-th bin, i.e., if  $\vartheta_{i,0} \leq \gamma_i < \vartheta_{i,1}$ , this means that even with the most robust MCS the target PER would be exceeded and therefore transmission should be suspended in that case. Finally, we assume that both hops are subject to independent frequency-flat fading, where the channels remain unchanged during one end-to-end transmission interval, but change independently between two such intervals. As a concrete example, we always consider the case where both hops undergo independent but not necessarily identically distributed Nakagami- $m$  fading with fading parameters  $m_1$  and  $m_2$  and average SNR  $\bar{\gamma}_1$  and  $\bar{\gamma}_2$ , respectively. Hence, in that case the probability density function of  $\gamma_i$  is given by

$$p_{\gamma_i}(\gamma) = \frac{m_i^{m_i} \gamma^{m_i-1}}{\bar{\gamma}_i^{m_i} \Gamma(m_i)} \exp\left(-\frac{m_i \gamma}{\bar{\gamma}_i}\right), \quad \gamma \geq 0 \quad (3)$$

with  $\Gamma(\cdot)$  as the well-known Gamma function [16] and the corresponding cumulative distribution function by

$$F_{\gamma_i}(\gamma) = 1 - \frac{\Gamma\left(m_i, \frac{m_i \gamma}{\bar{\gamma}_i}\right)}{\Gamma(m_i)}, \quad \gamma \geq 0 \quad (4)$$

with  $\Gamma(\cdot, \cdot)$  as the upper incomplete Gamma function [16].

## III. FINITE-STATE MARKOV CHAIN MODEL

In the following, we develop a finite-state Markov chain with  $L_{\max} + 1$  states for the performance analysis of the proposed dual-hop transmission systems with AMC, where each state is associated with a certain number of packets in the relay buffer. In particular, if the Markov chain is in the  $i$ -th state ( $0 \leq i \leq L_{\max}$ ), this corresponds to the case where the relay buffer holds  $i$  packets after the transmission from S to R and before the forwarding of packets from R to D. Below, we first of all determine the general transition probability  $p_{i,j}$  that the Markov chain changes from state  $i$  in the current time interval to state  $j$  in the next one. With these transition probabilities, we can then determine in a next step the steady-state probability distribution of our Markov chain, which will eventually serve as the basis for the performance analysis of our system in Section IV.

TABLE I  
AVAILABLE MODULATION AND CODING SCHEMES

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Modulation	BPSK	QPSK	QPSK	16-QAM	16-QAM
Code rate	1/2	1/2	3/4	9/16	3/4
Rate (bps)	0.50	1.00	1.50	2.25	3.00
$a_i$	274.72	90.25	67.61	50.12	53.39
$g_i$	7.99	3.49	1.68	0.66	0.37
$\gamma_{T_i}$ (dB)	-1.533	1.094	3.972	7.702	10.249

### A. Transition Probabilities

The probability  $p_{i,j}$  that the buffer level changes by  $j - i$  packets given that there are currently  $i$  packets in the buffer corresponds to the probability that the relay first of all forwards  $k$  packets ( $0 \leq k \leq \min\{N, i\}$ ) to the destination provided that in total  $i$  packets are available while at the same time after the transmission from the source to the relay exactly  $j - i + k$  are put into the buffer again given that after the forwarding of packets from R to D there are  $L_{\max} - i + k$  free slots for storing additional packets. Exploiting the independence of the two hops and denoting the probabilities of the aforementioned two events by  $q_{k,i}$  and  $r_{j-i+k, L_{\max}-i+k}$ , respectively, we generally can express the transition probabilities  $p_{i,j}$  as

$$p_{i,j} = \sum_{k=0}^{\min\{N,i\}} q_{k,i} r_{j-i+k, L_{\max}-i+k}. \quad (5)$$

If  $k \leq \min\{N, i\} < i$ , i.e., if not all packets that are currently stored in the buffer are forwarded,  $q_{k,i}$  corresponds to the probability that the SNR  $\gamma_2$  on the R-D link falls into the  $k$ -th bin and consequently mode  $k$  is chosen for the transmission. If, in contrast,  $k = i \leq N$ , i.e., if all packets in the buffer are forwarded,  $q_{k,i}$  corresponds to the probability that *at least*  $k$  packets may be transmitted on the R-D link without violating the imposed PER constraint. Hence, we directly obtain

$$q_{k,i} = \begin{cases} F_{\gamma_2}(\vartheta_{2,k+1}) - F_{\gamma_2}(\vartheta_{2,k}), & \text{if } k < i \\ 1 - F_{\gamma_2}(\vartheta_{2,k}), & \text{if } k = i \\ 0, & \text{if } k > i \end{cases}, \quad (6)$$

with  $F_{\gamma_2}(\gamma)$  as the cumulative distribution function of  $\gamma_2$ . The probability  $r_{m,n}$  can be calculated as follows. For  $m < n$ , the probability that  $m$  packets are put into the buffer given that there are  $n$  free slots simply corresponds to the probability that *exactly*  $m$  packets could be successfully decoded whereas for  $m = n$ ,  $r_{m,n}$  is given by the probability that *at least*  $m$  packets could be decoded at the relay station. In case that more packets have been decoded without any errors, the additional ones would have to be discarded. Taking into account that the number of transmitted packets might be larger than the number of successfully decoded packets, we have in general

$$r_{m,n} = \begin{cases} \sum_{\nu=m}^N z_{m,\nu} & \text{if } m < n \\ \sum_{\nu=m}^N \sum_{\eta=m}^{\nu} z_{\eta,\nu} & \text{if } m = n \\ 0 & \text{if } m > n \end{cases}, \quad (7)$$

where  $z_{m,\nu}$  denotes the joint probability that the source transmits  $\nu$  packets out of which exactly  $m$  are successfully recovered at the relay station. The transmission of  $\nu$  packets by the source requires only that the  $\nu$ -th AMC mode is selected or equivalently that  $\vartheta_{1,\nu} \leq \gamma_1 < \vartheta_{1,\nu+1}$ . Besides, it is actually not relevant which  $m$  out of the transmitted  $\nu$  packets are successfully decoded. Consequently, it can easily be seen that the probabilities  $z_{m,\nu}$  can be calculated as

$$z_{m,\nu} = \binom{\nu}{m} \int_{\vartheta_{1,\nu}}^{\vartheta_{1,\nu+1}} (1 - P_{e,\nu}(\gamma))^m \times P_{e,\nu}(\gamma)^{\nu-m} p_{\gamma_1}(\gamma) d\gamma, \quad (8)$$

with  $P_{e,\nu}(\gamma)$  as the PER for SNR  $\gamma$  in case that the  $\nu$ -th AMC mode is selected and  $p_{\gamma_1}(\gamma)$  as the pdf of  $\gamma_1$ . Plugging this expression together with (3) in (8) and capitalizing on [16, eq. (3.381,3)], we obtain the closed-form expression for  $z_{m,\nu}$  for the specific example of Nakagami- $m$  fading as

$$z_{m,\nu} = \sum_{\eta=0}^m \frac{\binom{\nu}{m} \binom{m}{\eta} (-1)^\eta m_1^{m_1} a_\nu^{\eta+\nu-m}}{\left(\frac{m_1}{\bar{\gamma}_1} + g_\nu(\eta + \nu - m)\right)^{m_1} \Gamma(m_1) \bar{\gamma}_1^{m_1}} \times \left[ \Gamma\left(m_1, \left(\frac{m_1}{\bar{\gamma}_1} + g_\nu(\eta + \nu - m)\right) \vartheta_{1,\nu}\right) - \Gamma\left(m_1, \left(\frac{m_1}{\bar{\gamma}_1} + g_\nu(\eta + \nu - m)\right) \vartheta_{1,\nu+1}\right) \right]. \quad (9)$$

Combining (9) with (7), (6) and (5) then directly leads to the desired closed-form expressions for the various state transition probabilities  $p_{i,j}$  of our finite-state Markov chain.

### B. Steady-State Probability Distribution

Having the state transition probabilities  $p_{i,j}$  as determined in the previous section, we can build up a  $(L_{\max} + 1) \times (L_{\max} + 1)$  transition probability matrix  $\mathbf{P}$  such that  $p_{i,j}$  corresponds to the entry in the  $i$ -th row and  $j$ -th column of  $\mathbf{P}$ . The steady-state distribution vector  $\boldsymbol{\pi} = [\pi_0, \pi_1, \dots, \pi_{L_{\max}}]$ , where  $\pi_\nu$  denotes the steady-state probability that the Markov chain is in state  $\nu$ , is then given by the solution of the linear equation system

$$\boldsymbol{\pi} \mathbf{P} = \boldsymbol{\pi}, \quad (10)$$

subject to the additional constraints that  $\pi_i \geq 0 \forall i$  as well as  $\sum_{i=0}^{L_{\max}} \pi_i = 1$ . This solution can be determined by noting that (10) actually is a standard eigenvalue problem with  $\boldsymbol{\pi}$  as a left eigenvector of  $\mathbf{P}$  associated with eigenvalue one and normalized such that all entries sum up to one. So to determine  $\boldsymbol{\pi}$ , we might perform an eigenvalue decomposition of  $\mathbf{P}^T$  and denote the eigenvector associated with eigenvalue one as  $\mathbf{v} = [v_0, v_1, \dots, v_{L_{\max}}]^T$ . Then,  $\boldsymbol{\pi}$  is simply given by

$$\boldsymbol{\pi} = \frac{\mathbf{v}}{\sum_{n=0}^{L_{\max}} v_n}. \quad (11)$$

Based on the steady-state probabilities  $\pi_\nu$  or equivalently the probabilities that the number of packets in the buffer after the transmission from the source to the relay is equal to  $\nu$ , we can easily determine the probabilities  $\pi'_\nu$  that the number of packets in the buffer after the forwarding of packets from the relay to the destination is equal to  $\nu$  as

$$\pi'_\nu = \sum_{i=\nu}^{\min\{L_{\max}, \nu+N\}} \pi_i q_{i-\nu, i}, \quad (12)$$

where  $q_{i-\nu, i}$  was defined in (6).

## IV. PERFORMANCE ANALYSIS

### A. Average Transmission Efficiency

The average transmission efficiency is generally defined as the average number of packets *transmitted* on a certain hop during one end-to-end transmission interval. Let us first of all consider the relay-to-destination link. Based on the mode of

operation, the relay forwards  $k$  packets to the destination if and only if there are more than  $k$  packets in the buffer and  $\vartheta_{2,k} \leq \gamma_2 < \vartheta_{2,k+1}$  or if there are exactly  $k$  packets in the buffer and  $\gamma_2 \geq \vartheta_{2,k}$ . Consequently, the average transmission efficiency  $\eta_{r-d}$  of the R-D link can be readily calculated as

$$\eta_{r-d} = \sum_{i=1}^N i \left( (F_{\gamma_2}(\vartheta_{2,i+1}) - F_{\gamma_2}(\vartheta_{2,i})) \sum_{\nu=i+1}^{L_{\max}} \pi_{\nu} + \pi_i (1 - F_{\gamma_2}(\vartheta_{2,i})) \right). \quad (13)$$

The average transmission efficiency of S-R link, on the other hand, can easily be shown to be given by

$$\eta_{s-r} = \sum_{\nu=1}^N \nu w_{\nu}, \quad (14)$$

where the probabilities  $w_{\nu}$  are given by  $w_{\nu} = F_{\gamma_1}(\vartheta_{1,\nu+1}) - F_{\gamma_1}(\vartheta_{1,\nu})$ . Clearly, (14) simply corresponds to the average transmission efficiency of conventional single-hop systems with AMC [6].

### B. Average Packet Loss Rate

The average packet loss rate generally corresponds to the average number of packets that are either dropped at the relay station due to a buffer overflow or a packet error or dropped at the actual destination due to a packet error over the total average number of packets transmitted by the source. It can readily be shown that this rate can be calculated as

$$P_{\text{loss}} = 1 - \frac{\eta_{r-d} (1 - \overline{\text{PER}}_{r-d})}{\eta_{s-r}}, \quad (15)$$

with  $\eta_{r-d}$  and  $\eta_{s-r}$  according to (13) and (14), respectively, and  $\overline{\text{PER}}_{r-d}$  as the average PER over the R-D link, which can be determined as follows. Based on the mode of operation, if  $i$  packets are transmitted over the R-D link but there are more than  $i$  packets in the buffer, it is obvious that necessarily  $\vartheta_{2,i} \leq \gamma_2 < \vartheta_{2,i+1}$  and consequently that the  $i$ -th AMC mode is selected. However, if  $i$  packets are transmitted and this is at the same time also the total number of packets in the buffer, we just know that  $\gamma_2 \geq \vartheta_{2,i}$  and hence either the  $i$ -th or a higher-order AMC mode might be selected. Having this in mind and noting that the average PER generally corresponds to the average number of erroneously decoded packets over the total number of transmitted packets, we can generally calculate the average PER on the R-D link as

$$\overline{\text{PER}}_{r-d} = \frac{1}{\eta_{r-d}} \sum_{i=1}^N i \left( \int_{\vartheta_{2,i}}^{\vartheta_{2,i+1}} P_{e,i}(\gamma) p_{\gamma_2}(\gamma) d\gamma \sum_{\nu=i+1}^{L_{\max}} \pi_{\nu} + \pi_i \int_{\vartheta_{2,i}}^{\vartheta_{2,\eta+1}} P_{e,\eta}(\gamma) p_{\gamma_2}(\gamma) d\gamma \right), \quad (16)$$

with  $P_{e,i}(\gamma)$  as the PER for SNR  $\gamma$  if the  $i$ -th AMC mode has been selected. Inserting the corresponding expressions according to (1) as well as the pdf of  $\gamma_2$  for the case of

Nakagami- $m$  fading according to (3), we obtain by making use of [16, eq. (3.381,3)] the closed-form result

$$\begin{aligned} \overline{\text{PER}}_{r-d} &= \frac{1}{\eta_{r-d}} \sum_{i=1}^N \sum_{\nu=i+1}^{L_{\max}} \pi_{\nu} \frac{i a_i m_2^{m_2}}{(m_2 + \bar{\gamma}_2 g_i)^{m_2} \Gamma(m_2)} \\ &\times \left[ \Gamma \left( m_2, \left( \frac{m_2}{\bar{\gamma}_2} + g_i \right) \vartheta_{2,i} \right) - \Gamma \left( m_2, \left( \frac{m_2}{\bar{\gamma}_2} + g_i \right) \vartheta_{2,i+1} \right) \right] \\ &+ \pi_i \sum_{\eta=i}^N \frac{i a_{\eta} m_2^{m_2}}{(m_2 + \bar{\gamma}_2 g_{\eta})^{m_2} \Gamma(m_2)} \\ &\times \left[ \Gamma \left( m_2, \left( \frac{m_2}{\bar{\gamma}_2} + g_{\eta} \right) \vartheta_{2,\eta} \right) - \Gamma \left( m_2, \left( \frac{m_2}{\bar{\gamma}_2} + g_{\eta} \right) \vartheta_{2,\eta+1} \right) \right], \end{aligned} \quad (17)$$

with  $\eta_{r-d}$  according to (13) again.

### C. Average Transmission Delay

Another important measure for evaluating the performance of our system is the average transmission delay, which we define as the average delay in end-to-end transmission intervals a packet received by the destination has experienced since its transmission by the source. Since the AMC mode selection is done independently on the S-R and R-D links, the transmission delay is not fixed and dependent on the buffer size. Generally, it corresponds to the average waiting time in the relay buffer plus one additional time interval for the actual transmission itself. In this regard, we can determine the average waiting time by capitalizing on Little's well-known theorem as [17]

$$\Delta_{\text{wait}} = \frac{L'_{\text{avg}}}{\lambda}, \quad (18)$$

where  $\lambda$  denotes the arrival rate of packets that are actually put into the relay buffer, which can be shown to be given by

$$\lambda = \sum_{i=1}^N i \sum_{j=0}^{L_{\max}} r_{i,j} \pi'_{L_{\max}-j}, \quad (19)$$

with  $r_{i,j}$  and  $\pi'_{\nu}$  according to (7) and (12), respectively.  $L'_{\text{avg}}$ , on the other hand, denotes the average buffer level *as seen by an outside observer*, which simply corresponds to the average buffer level after the transmission from the relay to the destination and can hence be readily calculated as

$$L'_{\text{avg}} = \sum_{\nu=1}^{L_{\max}} \nu \pi'_{\nu}. \quad (20)$$

with the probabilities  $\pi'_{\nu}$  according to (12) again.

## V. NUMERICAL RESULTS

In this section, we present numerical results illustrating the performance of the proposed system. All of these results have been verified by means of extensive Monte Carlo simulations, which, however, have not been included here for the sake of clarity. Furthermore, for comparison we present also results for the alternative dual-hop transmission system proposed in [13], where not only the instantaneous channel conditions but also the current buffer filling level is taken into account in the AMC mode selection in order to avoid buffer overflows. In particular,



with this scheme the MCS on the S-R link is reduced accordingly if a buffer overflow might occur otherwise. However, please be reminded that this alternative scheme requires some feedback signaling in any case while this is not necessary with the approach proposed in this paper in presence of a reciprocal channel as given for TDD systems, for example.

Fig. 1 shows the average end-to-end transmission efficiency, which is identical to the transmission efficiency of the R-D link, as a function of the average SNR for different buffer sizes  $L_{\max}$ . Obviously, the performance of the two different AMC selection strategies is almost identical. At a first glance, this might seem to be a bit surprising, but it can be intuitively explained as follows. The probability that with both schemes the same number of packets is put into the relay buffer after a transmission on the S-R link is basically the same. Differences actually might only occur if the MCS that should be used on that link considering the SNR  $\gamma_1$  only allows the transmission of more packets than the relay can additionally store. With the proposed pure SNR-based scheme, all these packets would be transmitted in that case so that it is very likely that the buffer can be fully filled up, even if some of them cannot be successfully decoded. With the buffer-aware mode selection, on the other hand, a more robust MCS would be used in that case, resulting in a significant drop of the corresponding PERs so that packet errors become very unlikely and hence with very high probability the buffer can be fully filled up as well. Consequently, the number of packets in the relay buffer is basically always the same so that also the number of packets forwarded by the relay station is basically identical for both cases, thus eventually leading to the same average transmission efficiency. It can also be seen from Fig. 1 that with increasing buffer size the transmission efficiency can be increased. This is because in that case the probability that the relay runs out of packets and therefore has to forward fewer packets than actually might be transmitted on the R-D link is decreased.

The average packet loss rate is depicted in Fig. 2. Obviously, here we obtain clearly distinct curves for the two different AMC mode selection strategies, what is reasonable since with pure SNR-based mode selection packets actually might be dropped at the relay station due to buffer overflows, thus leading to higher loss rates. However, with increasing buffer size, the difference between the two different approaches is significantly reduced. This is because in that case the probability that a buffer overflow occurs is decreasing. Aside from that, the probability that buffer overflows occur in case of pure SNR-based mode selection also decreases if the average SNR on the R-D link is better than the one of the S-R link as illustrated in Fig. 3. Clearly, if the average SNR on the second hop is just 3 dB above the average SNR of the first hop, the difference between both approaches can be drastically reduced compared to the case with equal SNRs on both links. This is reasonable since in such a case the packets transmitted on the S-R link are very likely immediately forwarded on the R-D link, so that the buffer filling level generally is rather low, thus leading to only few buffer overflows. Please note also that the average packet loss rate is not necessarily always a monotonic

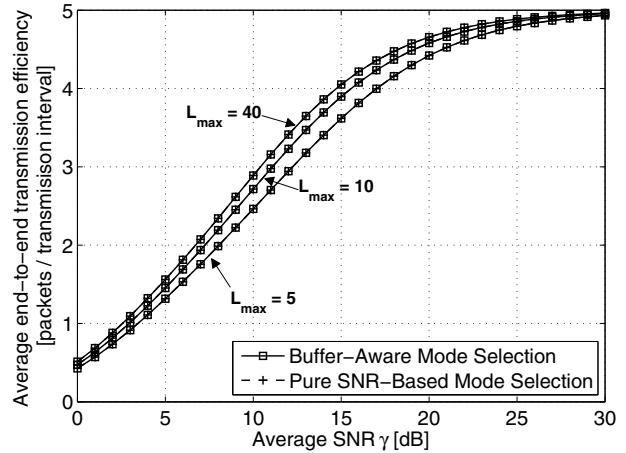


Fig. 1. Average end-to-end transmission efficiency in packets per transmission interval for various buffer sizes as a function of the average SNR  $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$  with Rayleigh-fading on both hops and  $P_{\text{target},1} = P_{\text{target},2} = 0.1$ .

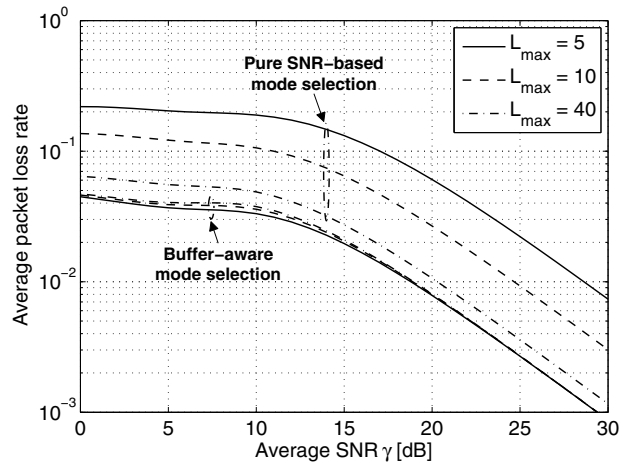


Fig. 2. Average packet loss rate as a function of the average SNR  $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$  with Rayleigh-fading on both hops and  $P_{\text{target},1} = P_{\text{target},2} = 0.1$ .

function of the average SNR since we have only a finite set of MCSs. Hence, if the average SNR increases so that more often a higher-order MCS is chosen, the average PER first of all may increase as well. This is reflected by the increase of the average packet loss rate in case of pure SNR-based mode selection and for  $\Delta_{\text{snr},r-d} = 1$  or 2 dB in Fig. 3, for example.

Finally, Fig. 4 depicts the end-to-end transmission delay for different buffer sizes. Clearly, also here both AMC mode selection strategies exhibit almost identical performances, what can be explained in exactly the same way as for the average transmission efficiency before. Besides, with increasing SNR the delay first of all is significantly reduced because the number of packets transmitted during one time interval generally increases and therefore packets in the relay buffer are faster forwarded on the R-D link. However, at a certain point the curves flatten out, where the limiting value depends on the buffer size. This is because in the high SNR regime

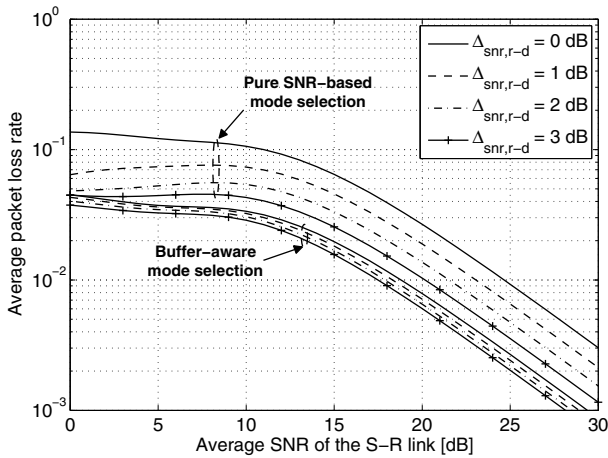


Fig. 3. Average packet loss rate versus the average SNR  $\bar{\gamma}_1$  of the first hop with Rayleigh-fading on both hops,  $L_{\max} = 10$ , and  $P_{\text{target},1} = P_{\text{target},2} = 0.1$ . The average SNR of the second hop is always given by  $\bar{\gamma}_2 = \bar{\gamma}_1 + \Delta_{\text{snr},r-d}$ .

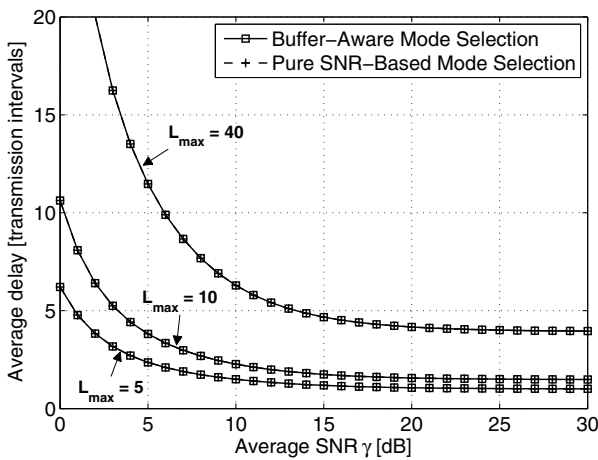


Fig. 4. Average end-to-end transmission delay for various buffer sizes as a function of the average SNR  $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$  with Rayleigh-fading on both hops and  $P_{\text{target},1} = P_{\text{target},2} = 0.1$ .

there is generally a non-zero probability that more than  $N$  packets are accumulated in the relay buffer. Hence, it is not always possible to immediately forward all packets received from the source in the same transmission interval towards the destination, even if on the R-D link almost always the highest AMC mode is selected. As a consequence, the corresponding waiting time is directly increased. With larger buffer sizes the probability that packets are accumulated in the buffer is increasing as well and therefore the observed delay floor is an increasing value of the buffer size.

## VI. CONCLUSION

We have proposed and analyzed the performance of a dual-hop adaptive packet transmission system with regenerative relaying, where the AMC mode selection is performed solely based on the current channel conditions, thus making it particularly attractive for TDD systems, where CSI is readily avail-

able at the transmitter-side. In order to model the dynamics of the relay buffer, we have established a finite-state Markov chain model of our system, based on which we have derived analytical closed-form expressions for a variety of different performance indicators. Furthermore, we have compared the performance of the proposed scheme to an alternative approach which takes also the current buffer filling level at the relay station into account during the AMC mode selection in order to completely avoid possible buffer overflows. Interestingly, it turned out that the performance of both schemes is basically identical, except for the average packet loss rate, which is generally higher with the proposed pure SNR-based mode selection. However, if the buffer is sufficiently big or the average SNR on the R-D link better than the one on the S-R link, also here the differences become almost negligible.

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