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Achieving Exponential Diversity in Wireless Multihop Systems with Regenerative Relays

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***Abstract*—We present and analyze two different power allocation strategies for considerably improving the average bit error rate (BER) performance of wireless multihop systems with an arbitrary number of regenerative relay stations. In both cases, the power allocation is done based on instantaneous channel state information subject to an overall sum power constraint as well as possible additional per-node power constraints and it is optimal in terms of minimizing an upper bound on the average end-to-end BER. It is shown that for a general class of fading distributions with both schemes exponential diversity can be achieved and we express the corresponding upper bounds on the average BER in closed-form for the important case of Nakagami-m fading on all hops. Finally, the performance of our approach is illustrated and evaluated by means of both numerical and simulation results.**

I. Introduction

In wireless communications, the average signal power generally drops rapidly with increasing distance between transmitter and receiver due to the detrimental effects of both path loss and shadow fading. In order to achieve nevertheless a good performance over rather long distances, theoretically the transmit power could be increased, but this is usually only possible to some extent due to governmental regulations and/or technical limitations and since this might increase the interference level experienced by other users. An alternative approach, which has attracted a lot of research attention during the past few years, is to subdivide the distance between the source and the destination node into multiple shorter hops by making use of intermediate relay stations, which process the signals received from their preceding node in an appropriate way before forwarding them to the next node. The average signal-to-noise ratio (SNR) on each hop is then typically much higher than the SNR on the direct link between source and destination, wherefore the performance might be significantly improved this way. By doing so, it is possible to extend the radio range and data rate coverage of existing cellular networks in a cost-efficient way, but clearly relayed transmission also represents a key enabling technology for wireless ad-hoc and sensor networks and it is likely to become an integral part of a variety of future wireless communication systems [1], [2].

Multihop systems generally provide many degrees of freedom for performing the resource allocation on the various hops and significant performance gains might be attained by doing this properly. A particularly important question is how to optimally adjust the transmit power of the source node and the relay stations based on the prevalent channel conditions and subject to various power constraints. Power allocation strategies aiming at minimizing the end-to-end outage probability have been derived in [3]–[5], for example, whereas in [6], [7] the goal was to maximize the mutual information between source and destination. Only recently, also the more practically oriented problem of minimizing the end-to-end error rate has been addressed in [8], [9], but in both contributions the authors assumed that the total power in the system is always constant, what is generally not optimal, of course.

In this paper, we present two power allocation strategies aiming at minimizing the average end-to-end bit error rate (BER) of wireless multihop systems with regenerative relays, which always perform hard decisions on the received signals before forwarding them on the next hop. In this regard, we do not request as in [8], [9] that the total power in the system is always constant, but instead we rather take potential total and individual *average* power constraints into account. The basic idea is similar to a recent contribution by Sharma *et al.* [10], who derived BER-optimal power allocation schemes for conventional single-hop systems, which were shown to achieve exponential diversity in presence of Rayleigh-fading. In fact, their result for single-antenna transmission turns out to be a special case of our approach, which, however, is applicable to an arbitrary number of hops. Besides, we prove that exponential diversity is not only achieved in presence of Rayleigh-fading, but also for a rather general class of fading distributions, including the common Nakagami-m distribution.

The remainder of this paper is organized as follows: In Section II, we outline our system model whereas the actual power allocation strategies are derived and analyzed in Section III. Afterwards, some performance results are presented in Section IV, followed by concluding remarks in Section V.

II. System Model

We consider a multihop communication system with N hops consisting of a source node, a destination node, and $N-1$ regenerative relay stations. Every end-to-end transmission is subdivided into N different time intervals of equal length. During the first phase, the source node transmits a certain number of bits to the first relay station, which thereupon tries to detect these bits and forwards the corresponding estimates to the next node. This process is then repeated on a hop-by-hop basis until finally the actual destination node is reached. In this regard, we assume that each node processes always only the signals received from its immediate predecessor node, what significantly reduces the complexity and power consumption

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compared to a system where every node combines always all received signals in an appropriate way [9]. Besides, the channels on all hops are assumed to be subject to independent but not necessarily identically distributed frequency-flat fading and all signals are perturbed by additive white Gaussian noise (AWGN) with zero mean and variance σ^2, which—without loss of generality—is assumed to be the same on all hops. In the discrete-time equivalent baseband domain, the input-output relationship of the k-th hop hence can be written as

$$y_k = \sqrt{P_k}\, h_k\, s_k + n_k, \tag{1}$$

where s_k denotes the symbol transmitted by the k-th node, y_k the signal received by the $(k+1)$-th node, n_k the AWGN term and h_k and P_k the corresponding channel coefficient and transmit power level, respectively.

In the following, we want to improve the end-to-end BER of our system by adjusting the transmit powers P_k appropriately based on perfect channel state information (csi). In this regard, we denote for notational convenience the average power of a transmitting node by P_0, which is generally given by $P_0 = \frac{1}{N}\sum_{k=1}^{N} P_k$[1]. Hence, every P_k can be related to P_0 by means of a power allocation coefficient p_k as $P_k = p_k\, P_0$ and we can define a reference SNR $\bar{\gamma}$ as $\bar{\gamma} = P_0/\sigma^2$. The instantaneous SNR on the k-th hop consequently can be expressed as

$$\gamma_k = |h_k|^2\, \frac{P_k}{\sigma^2} = \xi_k\, p_k\, \bar{\gamma}, \tag{2}$$

where we have introduced the short-hand notation $\xi_k = |h_k|^2$.

III. Power Allocation Strategies

As already mentioned before, we aim at minimizing the average end-to-end BER by means of proper power allocation based on perfect csi of all hops. In this regard, we focus on the important case where the bits to be transmitted are modulated using binary phase shift keying (BPSK), but the results derived for this case can easily be extended to other modulation schemes as well. For BPSK transmission, the instantaneous end-to-end BER generally can be expressed as [9], [11]

$$P_e = \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} \sum_{\substack{\mathbb{A}\cup\bar{\mathbb{A}}=\{1,\ldots,N\} \\ |\mathbb{A}|=2k-1}} \prod_{m\in\mathbb{A}} P_{e,m} \prod_{n\in\bar{\mathbb{A}}} \left(1 - P_{e,n}\right), \tag{3}$$

where the summation has to be taken over all possibilities for partitioning the set of hop indices $\{1,\ldots,N\}$ into two disjoint subsets $\mathbb{A}$ and $\bar{\mathbb{A}}$ such that the cardinality of $\mathbb{A}$ is odd and where $P_{e,k}$ denotes the instantaneous BER on the k-th hop, which is well-known to be given by

$$P_{e,k} = \tfrac{1}{2}\,\mathrm{erfc}\sqrt{\gamma_k} = \tfrac{1}{2}\,\mathrm{erfc}\sqrt{p_k\, \xi_k\, \bar{\gamma}}. \tag{4}$$

A direct minimization of (3) by means of proper power allocation, however, unfortunately seems to be very involved. For that reason, we rather consider a simpler upper bound on the end-to-end BER. For that purpose, we first of all note that for most cases of practical interest, the BERs on the individual hops should be small enough such that the probability that multiple errors occur on different hops during one end-to-end transmission becomes virtually negligible compared to the error probability of one hop itself [9]. Hence, we generally can upper-bound the end-to-end BER by the sum of the individual per-hop BERs. Furthermore, by applying the Chernoff-bound to (4), we can upper-bound the BERs on the individual hops, so that we finally obtain for the end-to-end BER

$$P_e \le \frac{1}{2}\sum_{k=1}^{N} \exp\left(-p_k\, \xi_k\, \bar{\gamma}\right), \tag{5}$$

which will serve as the basis for our further analysis. In this regard, we consider two different cases, which differ in whether or not a transmission takes always place or whether transmission is suspended in some situations.

A. Unconditional Transmission

In the first case, we assume that every node always transmits, independent of the actual channel conditions and independent of the allocated power. This implies that—similarly to [10]—a node would even then transmit if the allocated power equals zero[2]. Besides, we first of all do not impose any constraints on the available csi, so that each power allocation coefficient p_ν might be a function of all channel coefficients ξ_i. However, as will be seen later, for the optimal power allocation strategy global perfect csi is actually not necessary. In general, our optimization problem can be formulated as

$$\begin{aligned} &\{p_1, p_2, \ldots, p_N\}_{\text{opt}} \\ &\quad = \arg \min_{\substack{\{p_1,\ldots,p_N\} \\ p_i \ge 0\ \forall\ i}} \frac{1}{2}\,\mathbb{E}\left[\sum_{i=1}^{N} \exp\left(-p_i(\xi_1,\ldots,\xi_N)\,\xi_i\,\bar{\gamma}\right)\right], \end{aligned} \tag{6}$$

$$\text{subject to} \quad \sum_{i=1}^{N} \mathbb{E}\left[p_i(\xi_1,\ldots,\xi_N)\right] = N \tag{7}$$

$$\mathbb{E}\left[p_i(\xi_1,\ldots,\xi_N)\right] \le \alpha_i\ \forall\ i, \tag{8}$$

where (7) denotes a total sum power constraint, which assures that the total power in the system is fixed, and (8) denotes additional individual average power constraints, which assure that the average transmit power of the i-th node during the phases when the i-th node is active does not exceed a certain value α_i. In this regard, we always assume without loss of generality that $\sum_{i=1}^{N}\alpha_i \ge N$. Otherwise, the sum power constraint actually would be ineffective, but the case where $\sum_{i=1}^{N}\alpha_i < N$ can easily be traced back to the case explicitly considered in the sequel by replacing the original (ineffective) sum power constraint P_0 by $P_0' = \frac{1}{N}\sum_{i=1}^{N}\alpha_i\, P_0$. Under this assumption, the solution of our optimization problem according to (6) is given by the following theorem:

Theorem 1: The optimal power allocation strategy for minimizing the upper bound on the average end-to-end BER

[1]Please note that the actual average power of an arbitrary node would be $\frac{1}{N}\, P_0$ since each node transmits only during every N-th time interval.

[2]In fact, no real transmission takes place in this case, of course, but the node would assume that it has transmitted the corresponding bits anyway.

according to (6) subject to (7) and (8) is given by

$$p_{\nu,\mathrm{opt}} = \begin{cases} \frac{1}{\xi_\nu\,\bar\gamma}\ln\left(\frac{\xi_\nu\,\bar\gamma}{\max\{\lambda,\rho_\nu\}}\right) & \text{if } \xi_\nu \geq \frac{\max\{\lambda,\rho_\nu\}}{\bar\gamma} \\ 0 & \text{otherwise} \end{cases}, \quad (9)$$

where the parameters λ and ρ_ν are implicitly given by

$$\int_{\rho_\nu/\bar\gamma}^{\infty} \frac{1}{\xi_\nu\,\bar\gamma}\ln\left(\frac{\xi_\nu\,\bar\gamma}{\rho_\nu}\right) p_{\xi_\nu}(\xi_\nu)\,d\xi_\nu = \alpha_\nu \quad (10)$$

$$\sum_{\nu=1}^{N}\int_{\frac{\max\{\lambda,\rho_\nu\}}{\bar\gamma}}^{\infty} \frac{1}{\xi_\nu\,\bar\gamma}\ln\left(\frac{\xi_\nu\,\bar\gamma}{\max\{\lambda,\rho_\nu\}}\right) p_{\xi_\nu}(\xi_\nu)\,d\xi_\nu = N. \quad (11)$$

Proof: Clearly, the considered problem is convex and might be solved by means of Lagrange multipliers, where the corresponding Lagrangian can be written as

$$\begin{aligned} &\mathcal{L}(p_1,\ldots,p_N,\lambda,\mu_1,\ldots,\mu_N) \\ &= \sum_{i=1}^{N}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{N-\text{fold}} \exp\left(-p_i(\xi_1,\ldots,\xi_N)\,\xi_i\,\bar\gamma\right)\prod_{k=1}^{N} p_{\xi_k}(\xi_k)\,d\xi_k \\ &+\sum_{i=1}^{N}\mu_i\left[\underbrace{\int_0^\infty\cdots\int_0^\infty}_{N-\text{fold}} p_i(\xi_1,\ldots,\xi_N)\prod_{k=1}^{N} p_{\xi_k}(\xi_k)\,d\xi_k - \alpha_i\right] \\ &+\lambda\left[\sum_{i=1}^{N}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{N-\text{fold}} p_i(\xi_1,\ldots,\xi_N)\prod_{k=1}^{N} p_{\xi_k}(\xi_k)d\xi_k - N\right] \end{aligned} \quad (12)$$

with λ and $\mu_1,\ldots,\mu_N$ as Lagrange multipliers. Making this expression stationary with respect to p_ν, we find

$$\xi_\nu\,\bar\gamma\,\exp\left(-p_\nu\,\xi_\nu\,\bar\gamma\right) = \lambda + \mu_\nu, \quad \nu = 1,\ldots,N. \quad (13)$$

Taking additionally the non-negativity constraint into account and rearranging the terms a bit, we obtain

$$p_{\nu,\mathrm{opt}} = \begin{cases} \frac{1}{\xi_\nu\,\bar\gamma}\ln\left(\frac{\xi_\nu\,\bar\gamma}{\lambda+\mu_\nu}\right) & \text{if } \xi_\nu \geq \frac{\lambda+\mu_\nu}{\bar\gamma} \\ 0 & \text{otherwise} \end{cases}, \quad (14)$$

where the parameters λ and μ_ν can be determined from the sum and individual power constraints according to (7) and (8), respectively. Clearly, the parameters μ_ν have to be adjusted such that the individual average power constraints are satisfied whereas λ has to be chosen such that the sum power constraint is fulfilled. From the complementary slackness condition of the well-known Karush-Kuhn-Tucker optimality criteria as well as the fact that the average power that is allocated to a certain node is a strictly decreasing function of $\lambda + \mu_\nu$ (what can easily be seen from (14)), we can deduce that the optimal μ_ν can be expressed in terms of λ as $\mu_\nu = \max\{0, \rho_\nu - \lambda\}$, where ρ_ν is the value of μ that we would obtain if the total power constraint would be ineffective, i.e., for $\lambda = 0$, which is implicitly given by (10). The parameter λ itself can then be determined from the sum power constraint according to (11), what consequently concludes the proof. ■

Intuitively, the power allocation strategy according to Theorem 1 can be understood as an iterative process as follows: We start by allocating zero power to all nodes ($\lambda \to \infty$). Then, we gradually increase the general power level by decreasing λ. Once the power that would be allocated to a certain node for a given λ exceeds the corresponding individual power constraint α_ν, it is fixed to its maximum power and subsequently only the power of the remaining nodes is further increased subject to a modified total power constraint, which equals P_0 minus the average power already allocated to all clipped nodes. This process is then continued until the λ, for which the overall sum power constraint is met, has been found.

For general fading distributions, both λ and ρ_ν unfortunately cannot be given in closed-form. However, it is always possible to determine both of them efficiently numerically, what is shortly illustrated for λ, assuming that the ρ_ν have already been determined. Introducing a function

$$g(\lambda) = \sum_{\nu=1}^{N}\int_{\frac{\max\{\lambda,\rho_\nu\}}{\bar\gamma}}^{\infty} \ln\left(\frac{\xi_\nu\,\bar\gamma}{\max\{\lambda,\rho_\nu\}}\right)\frac{p_{\xi_\nu}(\xi_\nu)}{\xi_\nu\,\bar\gamma}d\xi_\nu - N, \quad (15)$$

the problem of finding the optimal λ reduces to the standard root-finding problem $g(\lambda) = 0$. In this regard, we note that $g(\lambda)$ is a continuous function of λ and that $g(\lambda = 0) = \sum_{\nu=1}^{N}\alpha_\nu - N \geq 0$ as well as $\lim_{\lambda\to\infty} g(\lambda) = -N$. Furthermore, it can easily be checked that $g(\lambda)$ is a strictly decreasing function of λ. Hence, by exploiting these properties, a highly accurate numerical solution of (7) can always be easily determined.

When using the power allocation strategy according to (9), the instantaneous end-to-end BER can be readily bounded as

$$P_e \leq \frac{1}{2}\sum_{k=1}^{N}\min\left\{1, \frac{\max\{\lambda,\rho_k\}}{\xi_k\,\bar\gamma}\right\}. \quad (16)$$

Hence, an upper bound on the *average* BER can be obtained by integrating (16) over the distributions of the various ξ_k as

$$\bar P_e \leq \frac{1}{2}\sum_{k=1}^{N}\left[\int_{\zeta_k}^{\infty}\frac{\zeta_k}{\xi_k}\,p_{\xi_k}(\xi_k)\,d\xi_k + F_{\xi_k}(\zeta_k)\right], \quad (17)$$

with the short-hand notation $\zeta_k = \max\{\lambda,\rho_k\}/\bar\gamma$. For the important case of independent but not necessarily identically distributed Nakagami-m fading on all hops, we have

$$F_{\xi_k}(\xi_k) = \frac{1}{\Gamma(m_k)}\,\gamma\left(m_k, \frac{m_k}{\Omega_k}\,\xi_k\right) \quad (18)$$

$$p_{\xi_k}(\xi_k) = \frac{m_k^{m_k}}{\Omega_k^{m_k}\,\Gamma(m_k)}\,\xi_k^{m_k-1}\exp\left(-\frac{m_k}{\Omega_k}\,\xi_k\right), \quad (19)$$

where m_k and Ω_k denote the fading parameter and average power gain of the k-th hop, respectively, and with $\gamma(\cdot,\cdot)$ as the lower incomplete gamma function [12]. Plugging these expressions in (17) and solving the contained integrals using [12, eq. (3.381,3)], we can bound $\bar P_e$ in closed-form as

$$\begin{aligned} \bar P_e \leq \sum_{k=1}^{N}\frac{1}{2\,\Gamma(m_k)}&\left[\Gamma\left(m_k - 1, \frac{m_k\,\max\{\lambda,\rho_k\}}{\Omega_k\,\bar\gamma}\right)\right. \\ &\left.\times\frac{\max\{\lambda,\rho_k\}\,m_k}{\Omega_k\,\bar\gamma} + \gamma\left(m_k, \frac{m_k\,\max\{\lambda,\rho_k\}}{\Omega_k\,\bar\gamma}\right)\right], \end{aligned} \quad (20)$$

with $\Gamma(\cdot,\cdot)$ as the upper incomplete gamma function [12].

Theorem 2: The power allocation strategy according to (9) – (11) achieves exponential diversity.

Proof: First of all, we note from (17) that

$$\bar{P}_e \leq \frac{1}{2}\sum_{k=1}^{N}\left[\frac{\max\{\lambda,\rho_k\}}{\bar{\gamma}}\left[\int_0^\infty \frac{1}{\xi_k}\,p_{\xi_k}(\xi_k)\,d\xi_k - \int_0^{\frac{\max\{\lambda,\rho_k\}}{\bar{\gamma}}} \frac{1}{\xi_k}\,p_{\xi_k}(\xi_k)\,d\xi_k\right] + F_{\xi_k}\left(\frac{\max\{\lambda,\rho_k\}}{\bar{\gamma}}\right)\right] \quad (21)$$

$$\leq \frac{1}{2}\sum_{k=1}^{N}\left[F_{\xi_k}\left(\frac{\max\{\lambda,\rho_k\}}{\bar{\gamma}}\right) + \frac{\max\{\lambda,\rho_k\}}{\bar{\gamma}}\left[\mathbb{E}\left[\frac{1}{\xi_k}\right] - \int_0^{\max\{\lambda,\rho_k\}/\bar{\gamma}} \frac{\bar{\gamma}}{\max\{\lambda,\rho_k\}}\,p_{\xi_k}(\xi_k)\,d\xi_k\right]\right] \quad (22)$$

$$\leq \frac{1}{2}\,\frac{\max\{\lambda,\max_k\rho_k\}}{\bar{\gamma}}\,\Xi, \quad (23)$$

where we have introduced for brevity the short-hand notation

$$\Xi = \sum_{k=1}^{N}\mathbb{E}\left[\frac{1}{\xi_k}\right]. \quad (24)$$

In this regard, we have to distinguish two different cases:
Case 1: $\max\{\lambda,\max_k\rho_k\} = \lambda$
In this case, none of the individual average power constraints is effective and consequently it can easily be concluded from (11) that λ is then implicitly given by

$$\sum_{i=1}^{N}\int_{\lambda/\bar{\gamma}}^{\infty}\frac{1}{\xi_i\,\bar{\gamma}}\,\ln\left(\frac{\xi_i\,\bar{\gamma}}{\lambda}\right)\,p_{\xi_i}(\xi_i)\,d\xi_i = N. \quad (25)$$

For the Rayleigh-fading case, it can easily be shown similarly to the approach used in [10] that $\lambda/\bar{\gamma}$ decays exponentially with $\bar{\gamma}$ and together with (23) that consequently exponential diversity is achieved. Due to space constraints, however, we do not explicitly present the corresponding proof here, but instead we briefly outline the proof for another broad class of fading distributions, namely for all distributions for which $\mathbb{E}\left[1/\xi_k\right]$ exists. Please note that this holds for the important case of Nakagami-m fading with $m > 1$, for example, but also for many other fading distributions. Rewriting (25) as

$$\sum_{i=1}^{N}\left[\int_{\lambda/\bar{\gamma}}^{\infty}\frac{\ln(\xi_i)}{\xi_i\,N}\,p_{\xi_i}(\xi_i)\,d\xi_i + \int_{\lambda/\bar{\gamma}}^{\infty}\frac{\ln(\bar{\gamma}/\lambda)}{\xi_i\,N}p_{\xi_i}(\xi_i)\,d\xi_i\right] = \bar{\gamma} \quad (26)$$

and exploiting then that generally $\ln(x)/x \leq 1$ as well as $\lambda \geq 0$, we find after some basic manipulations

$$N\,\bar{\gamma} \leq N + \ln\left(\frac{\bar{\gamma}}{\lambda}\right)\sum_{i=1}^{N}\mathbb{E}\left[\frac{1}{\xi_i}\right]. \quad (27)$$

From this expression, it then directly follows that generally $\lambda/\bar{\gamma} \leq \exp\left(-N(\bar{\gamma}-1)/\Xi\right)$, with Ξ according to (24), and with (23) we hence find that

$$\bar{P}_e \leq \frac{1}{2}\,\exp\left(-\frac{N\,(\bar{\gamma}-1)}{\Xi}\right)\,\Xi, \quad (28)$$

what proves that exponential diversity is achieved.
Case 2: $\max\{\lambda,\max_k\rho_k\} = \max_k\rho_k$
In this case, it directly follows from (23) that the end-to-end performance is dominated by the hop with the maximal value for ρ_k. Denoting for notational convenience $\eta = \arg\max_k\rho_k$, $\rho_\eta = \max_k\rho_k$ can be determined from the corresponding individual average power constraint according to (10) as

$$\int_{\rho_\eta/\bar{\gamma}}^{\infty}\frac{1}{\xi_\eta\,\bar{\gamma}}\,\ln\left(\frac{\xi_\eta\,\bar{\gamma}}{\rho_\eta}\right)\,p_{\xi_\eta}(\xi_\eta)\,d\xi_\eta = \alpha_\eta. \quad (29)$$

By means of exactly the same considerations as before it then can be shown that (a detailed derivation is omitted here due to space constraints) $\alpha_\eta\,\bar{\gamma} \leq 1 + \ln\left(\bar{\gamma}/\rho_\eta\right)\,\mathbb{E}\left[1/\xi_\eta\right]$, from which follows together with (23) that in this case

$$\bar{P}_e \leq \frac{1}{2}\,\exp\left(-\frac{\alpha_\eta\,\bar{\gamma}-1}{\mathbb{E}\left[\frac{1}{\xi_\eta}\right]}\right)\,\mathbb{E}\left[\frac{1}{\xi_\eta}\right], \quad (30)$$

so that obviously also exponential diversity can be achieved. Hence, putting both cases together, it is evident that our power allocation strategy always leads to exponential diversity. ■

Interestingly, it can be seen that for the calculation of the optimal power allocation coefficients $p_{\nu,\mathrm{opt}}$ according to (9) only the channels to the respective next hop have to be known instantaneously whereas the other parameters, namely λ and ρ_ν depend only on the channel distributions of the various hops and hence change only at a rather slow pace. This is a very nice property since consequently no global instantaneous csi is necessary, thus limiting the feedback requirements.

B. Conditional Transmission

In the previous scheme, we assumed that all nodes always transmit, even if the power allocated to one node is zero. In such a case, however, the BER on the corresponding link would be equal to one half, thus leading to a random decision between zero and one. Consequently, all other nodes of the end-to-end transmission chain basically would waste power for forwarding a totally unreliable bit. In order to overcome this drawback, we modify the power allocation introduced in the previous section such that data transmission is suspended if at least one node would get zero power. By doing so, the large per-hop BERs of one half can be avoided and at the same time the performance in case that a transmission takes place can be improved since more power is available for these cases then. However, it should be noted that with this approach it is necessary that all nodes are aware of whether a certain node would get zero power according to the previous scheme or not, thus resulting in a higher signaling load between the various nodes. Besides, by suspending transmission in some cases, the spectral efficiency is decreased and at this point it is not obvious whether the expected performance gain in terms of a lower BER justifies this loss in spectral efficiency or not.

For simplicity and due to space constraints, we restrict in the following to considering only a total sum power constraint, but additional individual average power constraints might be easily taken into account as well, similarly to the approach presented before. As already mentioned, the approach with conditional transmission actually is almost the same as the one with unconditional transmission treated before, except that all nodes suspend transmission if at least one node would get zero power. Hence, this strategy can be readily expressed as

$$p_{\nu,\text{cond}} = \begin{cases} \frac{1}{\xi_\nu \bar{\gamma}} \ln\left(\frac{\xi_\nu \bar{\gamma}}{\lambda}\right) & \text{if } \min_i \xi_i \geq \frac{\lambda}{\bar{\gamma}} \\ 0 & \text{otherwise} \end{cases}, \quad (31)$$

where λ is given by the modified sum power constraint

$$\sum_{i=1}^{N} \int_{\frac{\lambda}{\bar{\gamma}}}^{\infty} \ln\left(\frac{\xi_i \bar{\gamma}}{\lambda}\right) \frac{p_{\xi_i}(\xi_i)}{\xi_i \bar{\gamma}} d\xi_i \prod_{\substack{k=1 \\ k\neq i}}^{N} \left(1 - F_{\xi_k}\left(\frac{\lambda}{\bar{\gamma}}\right)\right) = N, \quad (32)$$

which takes into account that a transmission takes only place if all nodes transmit at non-zero power. As before, λ generally cannot be calculated analytically in closed-form, but it can always be efficiently determined numerically.

Plugging (31) in (5) and integrating over the distributions of all ξ_k, we can upper-bound the average end-to-end BER by

$$\bar{P}_e \leq \frac{\lambda}{2\bar{\gamma}} \sum_{k=1}^{N} \int_{\lambda/\bar{\gamma}}^{\infty} \frac{1}{\xi_k} p_{\xi_k}(\xi_k) d\xi_k \left(1 - F_{\xi_k}\left(\frac{\lambda}{\bar{\gamma}}\right)\right)^{-1}. \quad (33)$$

For the special but important case of Nakagami-m fading again, we obtain by inserting (18) and (19) in (33) and making use of [12, eq. (3.381,3)] the closed-form expression

$$\bar{P}_e \leq \frac{\lambda}{2\bar{\gamma}} \sum_{k=1}^{N} \frac{m_k}{\Omega_k} \frac{\Gamma\left(m_k - 1, \frac{m_k \lambda}{\Omega_k \bar{\gamma}}\right)}{\Gamma\left(m_k, \frac{m_k \lambda}{\Omega_k \bar{\gamma}}\right)}, \quad (34)$$

with $\Gamma(\cdot,\cdot)$ as the upper incomplete gamma function again.

Theorem 3: The power allocation strategy according to (31) and (32) achieves exponential diversity.

Proof: The proof is actually quite similar to the proof of Theorem 1, wherefore we only outline the major steps in the following, without going into every single detail. As before, we focus in this regard on the broad class of channel distributions for which $\mathbb{E}[1/\xi_i]$ exists, noting that the proof for the Rayleigh-fading case might be done in a similar way exploiting some of the properties presented in [10].

Using that $1 - F_{\xi_k}(\xi_0) \leq 1$, we see that as for the first case of the proof for the previously considered scheme, (27) holds here as well, wherefore as before $\lambda/\bar{\gamma} \leq \exp(-N(\bar{\gamma}-1)/\Xi)$, with Ξ according to (24). Besides, we exploit that generally

$$\int_{a}^{\infty} \frac{1}{\xi_k} p_{\xi_k}(\xi_k)\, d\xi_k \leq \mathbb{E}\left[\frac{1}{\xi_k}\right] (1 - F_{\xi_k}(a)), \quad a \geq 0. \quad (35)$$

This can easily be seen by defining $g(a) = \int_a^\infty \frac{1}{\xi_k} p_{\xi_k}(\xi_k)\, d\xi_k - \mathbb{E}\left[\frac{1}{\xi_k}\right] (1 - F_{\xi_k}(a))$, which clearly is a continuous function of a and which satisfies $g(a = 0) = \lim_{a\to\infty} g(a) = 0$. Furthermore, we find $\frac{\partial}{\partial a} g(a) = \left[\mathbb{E}\left[\frac{1}{\xi_k}\right] - \frac{1}{a}\right] p_{\xi_k}(a)$, which has exactly one root at $a = \frac{1}{\mathbb{E}[1/\xi_k]}$, i.e., $g(a)$ has only one stationary point. Finally, it can easily be checked that $\frac{\partial}{\partial a} g(a)\big|_{a=0} < 0$. Putting all these facts together, we can conclude that $g(a) \leq 0 \ \forall\ a > 0$ and hence that (35) always holds. Applying this result to (33), we finally obtain $\bar{P}_e \leq \frac{1}{2}\exp(-N(\bar{\gamma}-1)/\Xi)$, which is exactly the same upper bound as the one for the previously considered scheme according to (30) and which hence proves that also here exponential diversity can be achieved. ■

IV. Performance Results

Fig. 1 depicts the average end-to-end BER for a dual-hop system with identically distributed Nakagami-m fading on both hops, considering only a sum power constraint as well as different fading parameters. Clearly, in all cases exponential diversity is achieved, where the first scheme with unconditional transmission always leads to a slightly worse performance than the other one. However, with increasing fading parameters this difference becomes almost negligible and for m = 10 the corresponding curves actually cannot be distinguished anymore. This is because in this case the probability that with the first scheme zero power would be allocated to a certain node and hence the probability that with the second scheme transmission is suspended is approaching zero, hence leading to exactly the same performance in both cases. Hence, we conclude that the considerably higher feedback requirements and coordination effort of the conditional transmission scheme presumably do not really justify the only marginal reduction of the average BER in most cases.

Fig. 2 illustrates how individual average power constraints affect the average BER performance in case of unconditional transmission over two hops. In this regard, we assume Nakagami-m fading again, where on the first hop m_1 = 1 and on the second hop m_2 = 10. Besides, the average power of the relay node is unconstraint whereas the average power constraint α_1 is varied. Clearly, without any individual power constraints, the best performance can be achieved whereas the performance degrades the more restrictive the individual power constraint α_1. For comparison, we have also included the average BER for the case where both the source and the relay node transmit with constant power, which clearly leads to a significantly worse performance compared to the optimized power allocation with $\alpha_1 \geq 2$, particularly at high SNRs due to the non-exponential diversity order in that case.

Finally, Fig. 3 shows the *average* power that is allocated to the source and the relay node (when the nodes are active) of a dual-hop system with unconditional transmission in Nakagami-m fading with potentially different fading parameters on both hops. In this regard, m_2 always equals one whereas m_1 is varied. For high average SNRs, obviously always more power is allocated to the hop with the worse channel conditions, i.e., the smaller fading parameter, where the power imbalance is increasing with increasing imbalance of the fading parameters. This is basically what one would expect since in multihop systems the end-to-end BER performance is always dominated by the worst link [9]. However, for low average SNRs, it is surprisingly the other way around. This is because the power allocation threshold $\lambda/\bar{\gamma}$ is always the same for both hops and if the average SNR is low

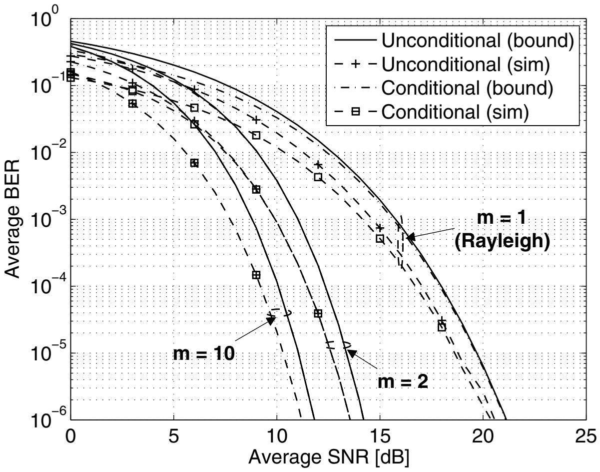

Fig. 1. Average end-to-end BER as a function of the average SNR $\bar{\gamma}$ for dual-hop transmission in Nakagami-m fading with $m_1 = m_2 = m$ and unity average power gains. Individual average power constraints are not considered.

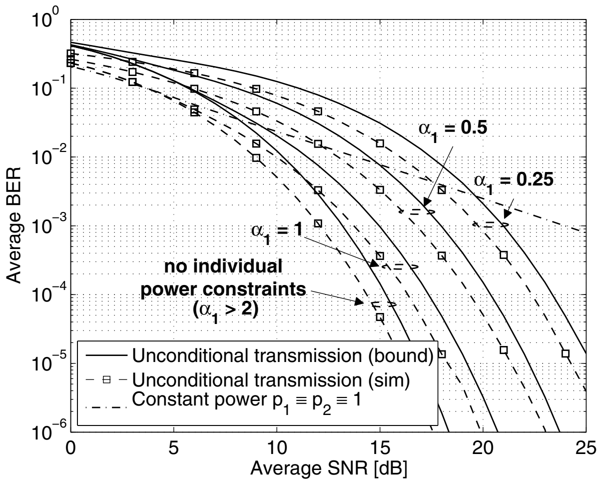

Fig. 2. Impact of an individual average power constraint α_1 for the source node on the average end-to-end BER of a dual-hop system in Nakagami-m fading with $m_1 = 1$, $m_2 = 10$ and unity average power gains.

enough, transmission is suspended in most cases whereas at time instants where transmission is not suspended, the channel conditions of the more severe fading hops generally are better on average than those of the other hop, so that in this case the latter one actually represents the bottleneck and therefore gets more power on average.

V. Conclusion

We have derived and analyzed two different power allocation strategies for wireless multihop system with an arbitrary number of regenerative relay stations, aiming at minimizing the average end-to-end BER subject to various power constraints. The first scheme is optimal in terms of minimizing an upper bound on the average end-to-end BER, assuming that every node always transmits, independent of the actual channel conditions. The second scheme then further improves

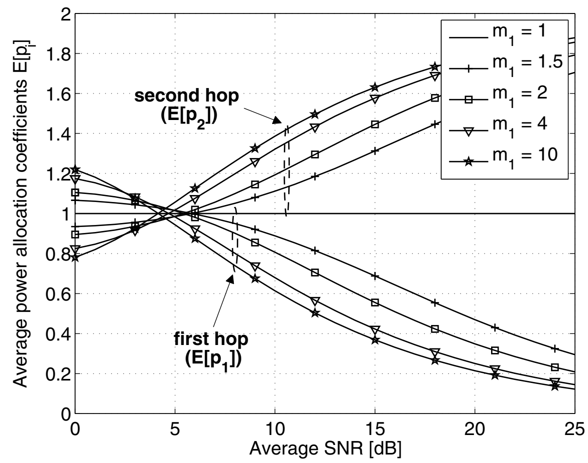

Fig. 3. Average power coefficients versus average SNR for dual-hop transmission in Nakagami-m fading with $m_2 = 1$, unity average power gains and various values for m_1. Individual average power gains are not considered.

the BER performance of the first approach by suspending transmission in case that at least one channel is so poor that the corresponding bit error rate would be one half. Both schemes were proven to achieve exponential diversity, and as one result it turned out that the BER reduction that can be achieved by suspending transmission in some cases probably does not justify the associated increased signaling load.

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