Outage-Optimal Transmit Antenna Selection for Cooperative Decode-and-Forward Systems

Andreas Müller and Joachim Speidel
Institute of Telecommunications, University of Stuttgart, Germany
E-mail: {andreas.mueller, joachim.speidel}@inue.uni-stuttgart.de

***Abstract*—We propose and analyze three different transmit antenna selection schemes for cooperative diversity systems with regenerative relays. Our goal is to minimize the information outage probability by properly selecting one out of several available antenna elements at the source node. This problem is generally more involved than transmit antenna selection for conventional single-hop systems since the selection of a certain antenna element affects always both the source-to-relay as well as the source-to-destination link. The various schemes that we present are based upon different kinds of channel state information and we analyze all of them in terms of the achievable outage probability as well as the asymptotic behavior in the high SNR regime.**

I. Introduction

In recent years, cooperative diversity systems have attracted a considerable amount of research attention due to their potential to obtain spatial diversity gain and hence to improve the performance of wireless communication systems by establishing virtual antenna arrays made up by the actual source node as well as one or multiple relay stations [1]–[3]. These relay stations might be either part of a fixed network infrastructure or they might correspond to other collaborating users nearby. Any relay node generally processes the signals received from the source node in an appropriate way before forwarding them to the actual destination, which then can combine these signals with the signal directly received from the source, thereby significantly improving the performance in most cases. In this regard, a variety of different cooperation protocols have been proposed in literature so far, the most prominent ones of them being the well-known amplify-and-forward and decode-and-forward approaches [1].

As a matter of course, the benefits of cooperative diversity might be combined with the benefits offered by conventional multiple-input multiple-output (MIMO) systems if one or multiple nodes of a cooperative diversity system are equipped with multiple antenna elements [4], [5]. However, this would clearly increase the costs of such nodes, in particular due to the increased number of required radio-frequency (RF) chains as well as the increased signal processing complexity. A promising approach to partially alleviate these drawbacks while keeping many of the benefits offered by multiple-antenna nodes—which is well-known from conventional MIMO systems—is to perform intelligent antenna selection [6]. This way, the number of required RF chains corresponds only to the number of active antenna elements while in most cases the same diversity order can be achieved as with a full complexity system, where always all available antenna elements are used.

In this paper, we propose and analyze various strategies for selecting an appropriate transmit antenna at the source node of a cooperative diversity system with a regenerative decode-and-forward relay. The different schemes are based upon different kinds of channel state information (csi) and they are optimal in terms of minimizing the outage probability between source and destination conditioned on the available csi. We derive generic expressions for the corresponding outage performance for arbitrary channel distributions and we explicitly present closed-form expressions for the important case of Rayleigh fading on all links. Furthermore, we investigate the asymptotic behavior of all schemes in the high signal-to-noise ratio (SNR) regime and we quantify the achievable diversity order.

The remainder of this paper is organized as follows: In Section II, we outline our system model. The different antenna selection strategies are introduced in Section III whereas the corresponding outage probabilities are derived in Section IV. Afterwards, we perform a high SNR analysis in Section V, followed by some numerical results in Section VI and finally some concluding remarks in Section VII.

II. System Model

We consider a cooperative diversity system, where a source node communicates with a destination node with the support of an intermediate regenerative relay station. Every end-to-end transmission is subdivided into two different phases of equal length. During the first phase, the source broadcasts a message, which is received by both the relay station and the destination. The relay station then tries to completely decode the message and only if the message was successfully decoded, it reencodes it again with the same code and forwards it in the second phase to the actual destination. If the decoding procedure at the relay fails, the relay station remains silent during the second phase. In case that the relay has forwarded the message, the destination combines the corresponding received signal with the signal directly received from the source using maximum ratio combining and then performs the actual decoding.

We assume that the source node is equipped with N antenna elements, but before every transmission only one of them is selected based on feedback information received from the destination and relay node, respectively. Both the relay station and the destination node might be equipped with multiple antennas as well, but in fact this of no relevance for our further analysis since we always consider the instantaneous signal-to-noise ratios (SNRs) of the source-to-relay and source-to-

978-1-4244-2517-4/09/$20.00 ©2009 IEEE

destination links after appropriate combining of all available received signals. In this regard, we denote the SNR of the source-to-destination link if the i-th transmit antenna has been selected by $\gamma_{S,i}$ and the corresponding SNR of the source-to-relay link by $\gamma_{R,i}$, where all $\gamma_{S,i}$ and $\gamma_{R,i}$ are assumed to be independent of each other. Besides, the effective SNR of the relay-to-destination link is always denoted by γ_D in the following. All channels are assumed to be subject to frequency-flat fading and to remain constant during the transmission of one codeword. Furthermore, the relay station is assumed to have perfect csi of the source-to-relay link and the destination of the relay-to-destination and source-to-destination links, respectively.

III. Antenna Selection Strategies

The transmit antenna selection at the source node is done in such a way that the conditional probability that the mutual information between source and destination falls below a given information rate R is minimized, given the csi fed back by the relay and destination node, respectively. Using the results presented in [7]–[9], it can easily be shown that the mutual information between source and destination in case that the i-th antenna element at the source is selected is given by

$$I_i = \frac{1}{2}\log_2\left(1 + \max\{\gamma_{S,i}, \min\{\gamma_{R,i}, \gamma_{S,i} + \gamma_D\}\}\right). \quad (1)$$

Hence, the corresponding (information) outage probability $P_{\text{out},i}$ = Prob$[I_i < R]$ can be readily expressed as

$$P_{\text{out},i} = \text{Prob}\left[\max\{\gamma_{S,i}, \min\{\gamma_{R,i}, \gamma_{S,i} + \gamma_D\}\} \leq \gamma_T\right], \quad (2)$$

where we have introduced for brevity the short-hand notation

$$\gamma_T = 2^{2R} - 1. \quad (3)$$

Clearly, an outage occurs iff the source-to-destination link is in outage, i.e., if $\gamma_{S,i} < \gamma_T$, while at the same time the source-to-relay or the combined source-to-destination and relay-to-destination link are in outage as well, i.e., if $\gamma_{R,i} < \gamma_T$ or $\gamma_{S,i}+\gamma_D < \gamma_T$. Mathematically, the optimal antenna selection strategy consequently can be formulated as

$$k_{\text{opt}} = \arg\min_{i=1,\ldots,N} P_{\text{out},i}(\gamma_T|\text{csi}), \quad (4)$$

where k_{opt} denotes the index of the transmit antenna to be selected and $P_{\text{out},i}(\gamma_T|\text{csi})$ the outage probability in case that the i-th antenna element is selected conditioned on the available csi. In this regard, we consider three different kinds of csi in the following, which differ in the amount of required feedback information as well as the achievable performance.

Scheme 1: In the first case, the destination checks whether or not there is a transmit antenna at the source with which the source-to-destination link would be not in outage and if so, it feeds the corresponding antenna index back to the source. Likewise, the relay checks whether or not there is a transmit antenna at the source with which the source-to-relay link would not be in outage and feeds back the corresponding index to the source if this is the case. The source then basically always selects the antenna requested by the destination and only in case that the destination has signaled an outage with all possible transmit antennas it selects the antenna requested by the relay. If also the source-to-relay link would be in outage for all possible transmit antennas, the source does not transmit anything since an outage would occur anyway.

Scheme 2: The second scheme is similar to the first one with the only difference that the relay node is assumed to be not only aware of the all possible SNRs $\gamma_{R,i}$ between the source and itself, but also of the distributions of all $\gamma_{S,i}$ between source and destination. Since these distributions usually change at a rather slow pace, they have to be signaled to the relay only rather infrequently via a low-rate feedback channel. However, being aware of this information, the performance can be improved because if there are multiple antenna elements with which the source-to-relay link would not be in outage, the relay now can always request the one out of them for which the expected SNR on the source-to-destination link is maximal. Please note that this scheme obviously reduces to the first one in case that all $\gamma_{S,i}$ are identically distributed.

Scheme 3: For our last approach, we assume that the source has perfect csi of all possible SNRs $\gamma_{S,i}$ and $\gamma_{R,i}$, respectively. As before, the source then first of all checks whether there is an antenna for which the source-to-destination link would not be in outage and only if this is not the case, it considers all antennas for which the source-to-relay link would not be in outage and then selects the one out of them for which the corresponding $\gamma_{S,i}$ becomes maximal.

IV. Outage Probability Analysis

With each of the previously introduced schemes, an outage occurs in any case if with all possible antenna elements at the source both the source-to-destination and the source-to-relay link would in outage. Besides, with all considered strategies no outage occurs if there is at least one transmit antenna with which the source-to-destination link would not be in outage. The actual difference between the different schemes becomes only apparent in case that with all transmit antennas the source-to-destination link would be in outage while there are at least two antennas for which the source-to-relay link would not be in outage. In this case, the selection of one out of these antenna elements can be improved if better csi is available. Generally, the actual outage probability with either of the three approaches can be written in a unified way as

$$\begin{aligned} P_{\text{out},\nu} = \text{Prob}[\max_i \gamma_{S,i} < \gamma_T] \Big[& \text{Prob}[\max_i \gamma_{R,i} < \gamma_T] \\ & + \sum_{\substack{\mathcal{A}\cup\bar{\mathcal{A}}=\{1,\ldots,N\} \\ |\mathcal{A}|\geq 1}} \prod_{i\in\mathcal{A}} \text{Prob}[\gamma_{R,i} \geq \gamma_T] \\ & \times \prod_{i\in\bar{\mathcal{A}}} \text{Prob}[\gamma_{R,i} < \gamma_T]\, P_\nu(\mathcal{A}, \gamma_T) \Big], \quad \nu \in \{1,2,3\} \quad (5) \end{aligned}$$

where the summation has to be taken over all possibilities for partitioning the set of antenna indices $\{1,\ldots,N\}$ into two disjoint subsets $\mathcal{A}$ and $\bar{\mathcal{A}}$ with the cardinality of $\mathcal{A}$ being at least one. The actual difference between the distinct schemes is reflected by different expressions for the term $P_\nu(\mathcal{A},\gamma_T)$, which is given for the various approaches by

$$P_1(\mathcal{A},\gamma_T) = \text{Prob}[\gamma_D + \underset{i\in\mathcal{A}}{\text{rand}}\,\gamma_{S,i} < \gamma_T |\, \underset{i\in\mathcal{A}}{\text{rand}}\,\gamma_{S,i} < \gamma_T] \quad (6)$$

$$P_2(\mathcal{A},\gamma_T) = \text{Prob}[\gamma_D + \gamma_{S,\mu(\mathcal{A})} < \gamma_T | \gamma_{S,\mu(\mathcal{A})} < \gamma_T] \quad (7)$$

$$P_3(\mathcal{A},\gamma_T) = \text{Prob}[\gamma_D + \max_{i\in\mathcal{A}}\gamma_{S,i} < \gamma_T |\, \max_{i\in\mathcal{A}}\gamma_{S,i} < \gamma_T]. \quad (8)$$

Here, $\text{rand}_{i\in\mathcal{A}}$ means that a random index out of $\mathcal{A}$ is selected whereas $\mu(\mathcal{A})$ denotes the index out of $\mathcal{A}$ for which we can expect the largest SNR on the source-to-destination link, i.e.,

$$\mu(\mathcal{A}) = \arg\max_{i\in\mathcal{A}} E[\gamma_{S,i}|\gamma_{S,i} < \gamma_T]. \quad (9)$$

In this regard, we take the expectation conditioned on $\gamma_{S,i} < \gamma_T$ since the source follows the request of the relay only if the source-to-destination link would be in outage for all possible transmit antennas. In general, $\mu(\mathcal{A})$ can be calculated as

$$\mu(\mathcal{A}) = \arg\max_{i\in\mathcal{A}} \frac{1}{F_{\gamma_{S,i}}(\gamma_T)} \int_0^{\gamma_T} x\, p_{\gamma_{S,i}}(x)\, dx, \quad (10)$$

with $F_{\gamma_{S,i}}(x)$ and $p_{\gamma_{S,i}}(x)$ as the cumulative distribution function (cdf) and probability density function (pdf) of $\gamma_{S,i}$, respectively. Using these functions as well as the cdf $F_{\gamma_{R,i}}(\gamma)$ of $\gamma_{R,i}$, (5) can be rewritten as

$$P_{\text{out},\,\nu} = \prod_{i=1}^{N} F_{\gamma_{S,i}}(\gamma_T) \left[\prod_{i=1}^{N} F_{\gamma_{R,i}}(\gamma_T) + \sum_{\substack{\mathcal{A}\cup\bar{\mathcal{A}}=\{1,\dots,N\}\\ |\mathcal{A}|\geq 1}} \times \left(\prod_{i\in\mathcal{A}}(1-F_{\gamma_{R,i}}(\gamma_T)) \prod_{i\in\bar{\mathcal{A}}} F_{\gamma_{R,i}}(\gamma_T)\, P_\nu(\mathcal{A},\gamma_T)\right)\right]. \quad (11)$$

Besides, exploiting that for an arbitrary random variable X with cdf $F_X(x)$, which is independent of γ_D, we have

$$\text{Prob}[\gamma_D + X < \gamma_T | X < \gamma_T] = \int_0^{\gamma_T} \frac{F_X(\gamma_T - x)}{F_X(\gamma_T)}\, p_{\gamma_D}(x)\, dx, \quad (12)$$

with $p_{\gamma_D}(\gamma)$ as the pdf of γ_D, the probabilities $P_\nu(\mathcal{A},\gamma_T)$ according to (6)–(8) can easily be shown to be given by

$$P_1(\mathcal{A},\gamma_T) = \frac{1}{|\mathcal{A}|}\sum_{i\in\mathcal{A}} \int_0^{\gamma_T} \frac{F_{\gamma_{S,i}}(\gamma_T - x)}{F_{\gamma_{S,i}}(\gamma_T)} p_{\gamma_D}(x)\, dx \quad (13)$$

$$P_2(\mathcal{A},\gamma_T) = \int_0^{\gamma_T} \frac{F_{\gamma_{S,\mu(\mathcal{A})}}(\gamma_T - x)}{F_{\gamma_{S,\mu(\mathcal{A})}}(\gamma_T)} p_{\gamma_D}(x)\, dx \quad (14)$$

$$P_3(\mathcal{A},\gamma_T) = \int_0^{\gamma_T} \frac{\prod_{i\in\mathcal{A}} F_{\gamma_{S,i}}(\gamma_T - x)}{\prod_{i\in\mathcal{A}} F_{\gamma_{S,i}}(\gamma_T)} p_{\gamma_D}(x)\, dx. \quad (15)$$

These integrals can be solved analytically in closed-form for a wide variety of different fading distributions, including the frequently considered Rayleigh or Nakagami-m fading cases, for example. However, since the general expressions quickly become rather lengthy, we only focus on one selected example in the following, for which we explicitly present closed-form results. Particularly, for the important case where $F_{\gamma_{S,i}}(\gamma) = F_{\gamma_S}(\gamma)\ \forall\, i$ and $F_{\gamma_{R,i}} = F_{\gamma_R}(\gamma)\ \forall\, i$, i.e., in case that all $\gamma_{S,i}$ as well as all $\gamma_{R,i}$ are identically distributed, it is actually not relevant anymore which indices belong to the set $\mathcal{A}$ and which ones to $\bar{\mathcal{A}}$, but the only thing that matters are the cardinalities of these two sets. Therefore, it can easily be shown that (11) simplifies in this case to

$$P^{\text{iid}}_{\text{out},\,\nu} = (F_{\gamma_S}(\gamma_T))^N \left[(F_{\gamma_R}(\gamma_T))^N + \sum_{n=1}^{N} (F_{\gamma_R}(\gamma_T))^{N-n} \times \binom{N}{n} (1 - F_{\gamma_R}(\gamma_T))^n\, P^{\text{iid}}_\nu(n,\gamma_T)\right], \quad (16)$$

where the individual probabilities $P^{\text{iid}}_\nu(n,\gamma_T)$ are given by

$$P^{\text{iid}}_1(n,\gamma_T) = P^{\text{iid}}_2(n,\gamma_T) = \frac{1}{F_{\gamma_S}(\gamma_T)} \int_0^{\gamma_T} F_{\gamma_S}(\gamma_T - x)\, p_{\gamma_D}(x)\, dx \quad (17)$$

$$P^{\text{iid}}_3(n,\gamma_T) = \frac{1}{(F_{\gamma_S}(\gamma_T))^n} \int_0^{\gamma_T} (F_{\gamma_S}(\gamma_T - x))^n\, p_{\gamma_D}(x)\, dx \quad (18)$$

As already mentioned before, if all source-to-destination links are identically distributed, the first and the second scheme become identical, wherefore $P^{\text{iid}}_1(n,\gamma_T) = P^{\text{iid}}_2(n,\gamma_T)$.

If we assume in addition Rayleigh fading on all hops as well as only one antenna element at both relay and destination, i.e., if $F_{\gamma_S}(\gamma) = 1 - e^{-\gamma/\bar{\gamma}_S}$ and $F_{\gamma_R}(\gamma) = 1 - e^{-\gamma/\bar{\gamma}_R}$, we obtain

$$P^{\text{Ray}}_{\text{out},\,\nu} = \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}_S}}\right)^N \left[\left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}_R}}\right)^N + \sum_{n=1}^{N}\binom{N}{n} \times \left(e^{-\frac{\gamma_T}{\bar{\gamma}_R}}\right)^n \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}_R}}\right)^{N-n} P^{\text{Ray}}_\nu(n,\gamma_T)\right], \quad (19)$$

where the scheme-dependent probabilities $P^{\text{Ray}}_\nu(n,\gamma_T)$ can be calculated based on (13) – (15) analytically in closed-form as

$$P^{\text{Ray}}_1(n,\gamma_T) = P^{\text{Ray}}_2(n,\gamma_T) = \bar{\gamma}_D \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}_D}}\right) - \frac{e^{-\frac{\gamma_T}{\bar{\gamma}_S}}}{\frac{1}{\bar{\gamma}_S} - \frac{1}{\bar{\gamma}_D}} \left(e^{\gamma_T\left(\frac{1}{\bar{\gamma}_S} - \frac{1}{\bar{\gamma}_D}\right)} - 1\right) \quad (20)$$

$$P^{\text{Ray}}_3(n,\gamma_T) = \frac{1}{\bar{\gamma}_D\left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}_S}}\right)^n} \sum_{k=0}^{n}\binom{n}{k}(-1)^k \times \frac{e^{-k\frac{\gamma_T}{\bar{\gamma}_S}}}{\frac{k}{\bar{\gamma}_S} - \frac{1}{\bar{\gamma}_D}} \left(e^{\gamma_T\left(\frac{k}{\bar{\gamma}_S} - \frac{1}{\bar{\gamma}_D}\right)} - 1\right). \quad (21)$$

V. Asymptotic Behavior and Diversity Analysis

In the following, we analyze the asymptotic behavior of our schemes in the high SNR regime. In this regard, we consider virtually arbitrary fading distributions again, assuming only that the pdf of each SNR $\gamma_k \in \{\gamma_{S,i}, \gamma_{R,i}, \gamma_3\}$ can be accurately approximated in the high SNR regime by

$$p_{\gamma_k}(\gamma) \approx \frac{\alpha_k}{\bar{\gamma}_k^{\delta_k}}\, \gamma^{\delta_k - 1} + o\left(\frac{1}{\bar{\gamma}_k^{\delta_k+1}}\right), \quad (22)$$

where $\bar{\gamma}_k$ denotes the corresponding average SNR, δ_k the corresponding diversity order, α_k a constant factor and $o(\cdot)$ higher order terms of $\frac{1}{\bar{\gamma}_k}$, which become negligible as $\bar{\gamma}_k \to \infty$. Please note that this approximation is valid for virtually all

SNR distributions of practical interest [10]. The corresponding cdf can then easily be shown to be approximately given by

$$F_{\gamma_k}(\gamma) \approx \frac{\alpha_k}{\delta_k} \left(\frac{\gamma}{\bar{\gamma}_k}\right)^{\delta_k} + o\left(\frac{1}{\bar{\gamma}_k^{\delta_k+1}}\right). \quad (23)$$

In the following, we introduce a reference SNR $\bar{\gamma}$, out of which the average SNRs of all links can be easily determined as $\bar{\gamma}_{S,i} = \zeta_{S,i}\,\bar{\gamma}$, $\bar{\gamma}_{R,i} = \zeta_{R,i}\,\bar{\gamma}$, and $\bar{\gamma}_D = \zeta_D\,\bar{\gamma}$ with $\zeta_{S,i}$, $\zeta_{R,i}$ and ζ_D as constant coefficients. The high SNR asymptotes of the outage probability can then be defined as $P_{\text{high},\nu} = P_{\text{out},\nu}|_{\bar{\gamma}\to\infty}$. For analytical traceability, we furthermore assume that all diversity orders $\delta_{S,i}$, $\delta_{R,i}$, and δ_D are integer values[1]. Plugging the approximations according to (22) and (23) in (11) and considering only the dominant terms, we obtain

$$P_{\text{out},\nu}|_{\bar{\gamma}\to\infty} \approx \frac{1}{\bar{\gamma}^{\delta_{S,\text{tot}}}} \prod_{i=1}^{N} \frac{\alpha_{S,i}}{\delta_{S,i}} \left(\frac{\gamma_T}{\zeta_{S,i}}\right)^{\delta_{S,i}} \left[\frac{1}{\bar{\gamma}^{\delta_{R,\text{tot}}}} \prod_{i=1}^{N} \frac{\alpha_{R,i}}{\delta_{R,i}} \left(\frac{\gamma_T}{\zeta_{R,i}}\right)^{\delta_{R,i}} + \sum_{\substack{\mathcal{A}\cup\bar{\mathcal{A}}=\{1,\ldots,N\}\\ |\mathcal{A}|\geq 1}} \prod_{i\in\bar{\mathcal{A}}} \frac{\alpha_{R,i}}{\delta_{R,i}} \left(\frac{\gamma_T}{\zeta_{R,i}\bar{\gamma}}\right)^{\delta_{R,i}} \Upsilon_\nu(\mathcal{A},\gamma_T)\right] \quad (24)$$

where $\Upsilon_\nu(\mathcal{A},\gamma_T) = P_{\text{cond},\nu}(\mathcal{A},\gamma_T)|_{\bar{\gamma}\to\infty}$ and with

$$\delta_{K,tot} = \sum_{i=1}^{N} \delta_{K,i}, \qquad K \in \{S; R\}. \quad (25)$$

Based on (13) as well as (22) and (23) again, we find

$$\Upsilon_1(\mathcal{A},\gamma_T) = \frac{1}{|\mathcal{A}|} \sum_{i\in\mathcal{A}} \int_0^{\gamma_T} \frac{(\gamma_T - x)^{\delta_{S,i}}\,\alpha_D}{\gamma_T^{\delta_{S,i}}(\zeta_D\,\bar{\gamma})^{\delta_D}}\, x^{\delta_D - 1}\, dx. \quad (26)$$

Expanding $(\gamma_T - x)^{\delta_{S,i}}$ into a finite sum by making use of the binomial theorem and solving the contained integral, we get

$$\Upsilon_1(\mathcal{A},\gamma_T) = \frac{1}{|\mathcal{A}|} \sum_{i\in\mathcal{A}} \sum_{k=0}^{\delta_{S,i}} \binom{\delta_{S,i}}{k} \frac{(-1)^k\,\alpha_D}{k+\delta_D} \left(\frac{\gamma_T}{\zeta_D\,\bar{\gamma}}\right)^{\delta_D}. \quad (27)$$

Likewise, it can easily be shown that (a detailed derivation is omitted here due to space constraints)

$$\Upsilon_2(\mathcal{A},\gamma_T) = \sum_{k=0}^{\delta_{S,\mu(\mathcal{A})}} \frac{(-1)^k\,\alpha_D}{\delta_D + k} \left(\frac{\gamma_T}{\zeta_D\,\bar{\gamma}}\right)^{\delta_D} \binom{\delta_{S,\mu(\mathcal{A})}}{k} \quad (28)$$

$$\Upsilon_3(\mathcal{A},\gamma_T) = \sum_{k=0}^{\rho(\mathcal{A})} \frac{(-1)^k\,\alpha_D}{k+\delta_D} \binom{\rho(\mathcal{A})}{k} \left(\frac{\gamma_T}{\zeta_D\,\bar{\gamma}}\right)^{\delta_D}, \quad (29)$$

where we have introduced for brevity the short-hand notation

$$\rho(\mathcal{A}) = \sum_{i\in\mathcal{A}} \delta_{S,i}. \quad (30)$$

Clearly, for all considered schemes we have $\Upsilon_\nu(\mathcal{A},\gamma_T) \sim \frac{1}{\bar{\gamma}^{\delta_D}}$ as $\bar{\gamma}\to\infty$, independently of $\mathcal{A}$. Hence, we can conclude from (24) that in the high SNR regime it is sufficient to consider only the set $\mathcal{A} = \{1,\ldots,N\}$ since for any other $\mathcal{A}$ the term $\Upsilon_\nu(\mathcal{A},\gamma_T)$ would be multiplied with additional $1/\bar{\gamma}$ terms. Besides, we find with (22) and (23) for $\mu(\mathcal{A})$ in (29) that

$$\mu(\mathcal{A})|_{\bar{\gamma}\to\infty} = \arg\max_i \delta_{S,i}. \quad (31)$$

Putting everything together, we hence finally get as a general expression for the asymptotic outage probability

$$P_{\text{high},\nu} = \frac{1}{\bar{\gamma}^{\delta_{S,\text{tot}}}} \prod_{i=1}^{N} \frac{\alpha_{S,i}}{\delta_{S,i}} \left(\frac{\gamma_T}{\zeta_{S,i}}\right)^{\delta_{S,i}} \times \left[\frac{1}{\bar{\gamma}^{\delta_{R,\text{tot}}}} \prod_{i=1}^{N} \frac{\alpha_{R,i}}{\delta_{R,i}} \left(\frac{\gamma_T}{\zeta_{R,i}}\right)^{\delta_{R,i}} + \frac{1}{\bar{\gamma}^{\delta_D}} \vartheta_\nu(\gamma_T)\right], \quad (32)$$

where we have for the individual schemes

$$\vartheta_1(\gamma_T) = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=0}^{\delta_{S,i}} \binom{\delta_{S,i}}{k} (-1)^k \frac{\alpha_D}{k+\delta_D} \left(\frac{\gamma_T}{\zeta_D}\right)^{\delta_D} \quad (33)$$

$$\vartheta_2(\gamma_T) = \sum_{k=0}^{\max_i \delta_{S,i}} \frac{\alpha_D}{k+\delta_D} \binom{\max_i \delta_{S,i}}{k} (-1)^k \left(\frac{\gamma_T}{\zeta_D}\right)^{\delta_D} \quad (34)$$

$$\vartheta_3(\gamma_T) = \sum_{k=0}^{\delta_{S,\text{tot}}} \frac{\alpha_D}{k+\delta_D} \binom{\delta_{S,\text{tot}}}{k} (-1)^k \left(\frac{\gamma_T}{\zeta_D}\right)^{\delta_D}. \quad (35)$$

From (32), it can easily be seen that $P_{\text{high},\nu} \sim \frac{1}{\bar{\gamma}^{\delta_e}}$ with δ_e as the end-to-end diversity order, which is clearly given by

$$\delta_e = \sum_{i=1}^{N} \delta_{S,i} + \min\left\{\sum_{i=1}^{N} \delta_{R,i}, \delta_D\right\}. \quad (36)$$

Hence, if $\sum_{i=1}^{N} \delta_{R,i} < \delta_D$, the second term in the square brackets in (32) can be reasonably neglected in the high SNR regime whereas for $\sum_{i=1}^{N} \delta_{R,i} > \delta_D$, the first term becomes negligible. Only if $\sum_{i=1}^{N} \delta_{R,i} = \delta_D$, both terms have to be considered. From these considerations, it directly follows that in the first case all three schemes eventually lead to the same asymptotic outage performance in the high SNR regime.

Based on (32), we can also quantify the effective SNR gain in the high SNR regime of one scheme over the other. In this regard, the SNR gain/loss of the k-th scheme compared to the j-th one can easily be shown to be given by [11]

$$\Delta_{k,j} = \sqrt[\delta_e]{\frac{P_{\text{high, j}}}{P_{\text{high, k}}}}, \quad (37)$$

which simplifies in the special case where $\sum_{i=1}^{N} \delta_{R,i} > \delta_D$ to $\Delta_{k,j} = \sqrt[\delta_e]{\frac{\vartheta_j(\gamma_T)}{\vartheta_k(\gamma_T)}}$, with $\vartheta_\nu(\gamma_T)$ according to (33) – (35).

VI. Numerical Results

Fig. 1 depicts the outage probability as a function of the average SNR $\bar{\gamma}$ for various numbers of transmit antennas with $\zeta_{S,i} = \zeta_S = 1\ \forall\ i$ as well as $\zeta_{R,i} = \zeta_R = \zeta_D = 16\ \forall\ i$. This corresponds to a scenario where the source-to-relay and relay-to-destination links have always a 12 dB higher average SNR than the source-to-destination link. Obviously, by

[1]The corresponding results for non-integer diversity orders can easily be approximated based on the results provided here by means of interpolation.

performing transmit antenna selection the performance can be significantly improved compared to the case with one antenna at the source only. Besides, it can be seen that the high SNR asymptotes are really tight in the high SNR regime and that our theoretical results are in perfect agreement with simulated values. Furthermore, we note that for the considered scenario the performance of scheme two is identical to the one of scheme one since all source-to-destination links are identically distributed. In general, it can also be seen that scheme three always slightly outperforms the other ones, where the gain is an increasing function of the number of transmit antennas.

Fig. 2 illustrates the effective SNR gains of schemes two and three over scheme one in the high SNR regime for two transmit antennas and Nakagami-m fading on all links. For Nakagami-m fading, it can easily be shown that δ_k and α_k according to (22) are generally given by $\delta_k = m_k$ and $\alpha_k = m_k^{m_k}/\Gamma(m_k)$, where m_k denotes the Nakagami-m fading parameter and $\Gamma(\cdot)$ the gamma function. In the considered example, we set the fading parameter m_R of both source-to-relay links to four, the fading parameter $m_{S,1}$ of the link between the first source antenna and the destination to one, and we vary the fading parameter $m_{S,2}$ between the second transmit antenna and the destination as well as the fading parameter m_D of the relay-to-destination link. Clearly, the gain of scheme three is always higher than the one of scheme two, but with increasing $m_{S,2}$ both curves converge. This is reasonable because in this case the link corresponding to $m_{S,2}$ is basically always better than the one corresponding to $m_{S,1}$, so that with both schemes always the second transmit antenna would be the preferred choice. Besides, it can also be seen that for $m_{S,2} = 1$, scheme two does not provide any gain since both source-to-destination links are identically distributed in this case whereas scheme one still can achievable a considerable improvement.

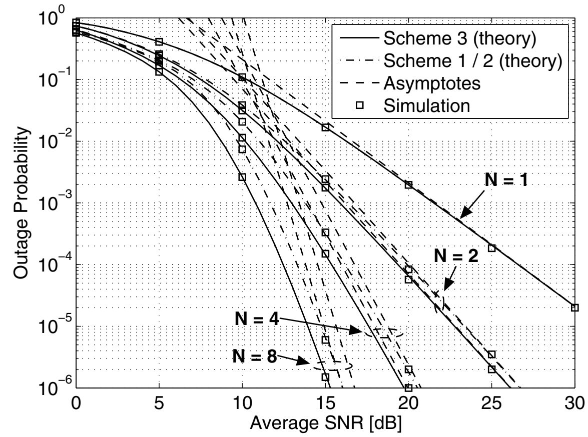

Fig. 1. Outage probability versus the average SNR $\bar{\gamma}$ for R = 2 bit/s/Hz, Rayleigh fading on all hops as well as ζ_S = 1, $\zeta_R = \zeta_D$ = 16, and different numbers N of available antenna elements at the source node.

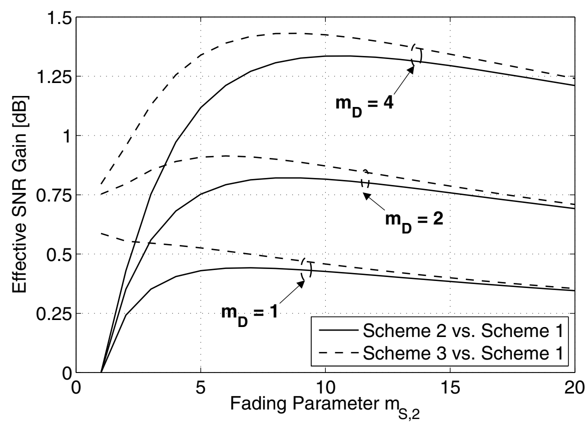

Fig. 2. Effective SNR gain in the high SNR regime of schemes 2 and 3 over scheme 1 for two transmit antennas, ζ_S = 1, $\zeta_R = \zeta_D$ = 16 and Nakagami-m fading with m_R = 4, $m_{S,1}$ = 1, and various values for $m_{S,2}$ and m_D.

VII. Conclusion

We have considered the problem of selecting an appropriate transmit antenna at the source node of cooperative diversity systems with regenerative relays. In this regard, we have proposed three different schemes based on different kinds of csi and we have derived generic expressions for the corresponding outage probabilities, which were given in closed-form for the important case of Rayleigh fading on all hops. Furthermore, we have derived the respective high SNR asymptotes and it was shown that all schemes achieve the same diversity order. Finally, simulation results were shown to be in perfect agreement with our theoretically calculated values, thus verifying the accuracy of our analysis.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[2] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74 – 80, Oct. 2004.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity – part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927 – 1938, Nov. 2003.

[4] Y. Fan and J. Thompson, "MIMO configurations for relay channels: theory and practice," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1774 – 1786, May 2007.

[5] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29 – 43, Jan. 2005.

[6] A. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microwave Mag.*, vol. 5, no. 1, pp. 46 – 56, Mar. 2004.

[7] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700 – 702, Aug. 2005.

[8] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 813 – 815, Dec. 2006.

[9] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834 – 836, Dec. 2006.

[10] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389 – 1398, Aug. 2003.

[11] A. Müller and J. Speidel, "Orthogonal STBC in general Nakagami-m fading channels: BER analysis and optimal power allocation," in *Proc. IEEE Veh. Technol. Conf.*, Apr. 2007.