Dual-Hop Adaptive Packet Transmission Systems with Regenerative Relaying

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***Abstract*—We propose and analyze two different approaches for performing adaptive modulation and coding (AMC) in dual-hop transmission systems, where a source node S communicates with a certain destination node D via an intermediate decode-and-forwarding relay station R. In this regard, we consider the most general case, where the AMC on the S-R and R-D links might be performed independently of each other, thus requiring a potential buffering of data packets at the relay station before they can be forwarded to the destination D. With our first approach, AMC is performed solely based on the instantaneous channel realizations, without considering the current buffer level at the relay station, what consequently might result in packet losses due to potential buffer overflows. Our second approach eliminates this problem by taking the current buffer level into account in the AMC transmission over the first hop and reducing the transmission rate accordingly if the buffer would overflow otherwise. Both schemes are analyzed analytically based on a finite-state Markov chain model of our system. Building upon this model, we derive analytical closed-form expressions for a variety of different key performance indicators, which are illustrated for various practical cases by means of selected numerical results.**

***Index Terms*—Adaptive modulation and coding, decode-and-forward, multihop transmission, regenerative relaying.**

I. Introduction

WIRELESS relay-assisted communication systems make use of one or multiple intermediate relay stations for forwarding data from a source node to the corresponding destination node, thereby offering numerous advantages over conventional single-hop systems, such as higher transmission rates, a more robust link performance as well as simpler and more flexible network planning and deployment [1]–[5]. While relayed transmission traditionally has played a crucial role in the area of wireless ad-hoc and sensor networks, where in fact each node itself might act as relay node for other nodes in order to achieve a high connectivity, it recently has also emerged as a key technology for the future development of cellular infra-structure based networks, such as WiMax or the 3GPP long-term evolution (LTE) [4]–[7]. In such cellular networks, relays might be readily used to solve common coverage problems in a cost-efficient manner and to significantly increase the transmission rates to users located near the cell-edge.

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In recent years, a large number of relaying schemes has been proposed and analyzed in literature, with the well-known decode-and-forward (DF) and amplify-and-forward (AF) protocols as the presumably most prominent ones among them [2], [8]–[11]. In general, DF protocols allow for a more flexible and adaptive handling and transmission than AF at the expense of a slightly increased complexity. While relayed transmission has the potential to significantly improve the average channel quality, its ability to mitigate the effects of multipath fading is, however, very limited. For that reason, practical multihop systems should additionally employ appropriate link adaptation schemes, which dynamically adjust various transmission parameters such as the modulation scheme, code rate, or transmission power to the prevalent channel conditions. Link adaptation strategies for conventional single-hop systems have been extensively studied during the past decade. In this regard, the focus was mainly put on uncoded adaptive modulation, where the modulation scheme and possibly also the power is dynamically adjusted [12]–[14], as well as adaptive modulation and coding (AMC), where depending on the current channel conditions an appropriate modulation and coding scheme (MCS) is selected [15]–[18]. Only very recently, also the design and analysis of corresponding schemes for relay-assisted communication systems have experienced an increasing amount of research attention. In [19], for example, the authors propose and analyze an adaptive modulation scheme for cooperative AF systems with an arbitrary number of parallel relay stations transmitting on orthogonal channels. In [20], a similar system is considered, but with incremental opportunistic AF relaying, where a relay participates only in the transmission if the SNR on the direct link is so poor that even the most robust modulation scheme would not be supported, in which case then, however, only the best out of the available relay stations is selected. A first elementary study of AMC schemes combined with an appropriate automatic repeat request (ARQ) protocol for different DF and coded cooperation systems has been conducted in [21]. Recently, the performance of cooperative DF systems with relay selection and adaptive modulation on the relay-to-destination link only combined with an accordingly adjusted transmission time interval for that link has been investigated in [22]. Finally, the combination of AMC for three-node DF systems in conjunction with incremental ARQ, where the relay only forwards the message transmitted from the source if the destination is not able to decode it correctly, has been addressed in [23].

In this paper, we address the important problem of performing AMC over dual-hop transmission systems with one intermediate regenerative relay station. Basically, there are two different options for implementing AMC in such a scenario. On the one hand, both hops might always use the same transmission scheme during one end-to-end transmission. In this case, the relay always immediately forwards all bits received from the source to the actual destination, without the need for an exhaustive buffering of data at the relay station. A first analysis of this approach recently has been presented in [24] for the case of uncoded adaptive modulation and with different kinds of feedback information. Alternatively, the constraint that both hops should always adopt the same transmission scheme can be relaxed so that the adaptation on both hops is basically performed independently of each other. Since this approach is less restrictive, one might intuitively expect that it should lead to a better performance. At the same time, however, independent AMC on both hops also mandates the relay to temporarily buffer data if the relay-to-destination link is not good enough to directly forward all bits received from the source node in a certain time interval. Clearly, this results in a direct increase and variability of the end-to-end transmission delay and tightens the memory requirements of the relay station.

In the paper at hand, we carry out a thorough design and analysis of two different strategies for performing independent AMC over dual-hop systems. To the best of our knowledge, despite its obvious relevance for practical systems, such independent AMC on the different hops combined with a finite buffer at the relay station has never been considered in literature before. Note that depending on how the AMC is realized over the first hop, buffer overflows might or might not occur at the relay station. With our first strategy, AMC over the first hop is performed solely based on the instantaneous channel realizations, without considering how many additional packets the relay station might store, what consequently might result in packet losses due to potential buffer overflows. Our second strategy completely eliminates the risk of buffer overflows by taking the current buffer level into account in the AMC transmission over the first hop and reducing the transmission rate accordingly if the buffer would overflow otherwise. In the following, we develop a finite-state Markov chain model for the analysis of both strategies over general fading channels, based on which we obtain exact analytical expressions for a wide variety of key performance indicators, including the average transmission efficiency, the average packet loss rate as well as the average transmission delay. Our results can be readily used for investigating the effects of different buffer sizes and different sets of MCSs on the system performance and hence our analysis represents a useful framework for the design and evaluation of dual-hop AMC systems.

The remainder of this paper is organized as follows: In Section II, we outline our system and channel model and we introduce the two different AMC strategies considered in this paper. In Section III, we construct a finite-state Markov chain model of our system and we determine the corresponding state transition as well as steady-state probabilities. Based on these results, various key performance indicators are derived analytically in closed-form in Section IV, followed by selected numerical results in Section V and finally some concluding remarks in Section VI.

TABLE I
AVAILABLE MODULATION AND CODING SCHEMES

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**
Modulation	BPSK	QPSK	QPSK	16-QAM	16-QAM
Code rate	1/2	1/2	3/4	9/16	3/4
Rate (bps)	0.50	1.00	1.50	2.25	3.00
a_k	274.72	90.25	67.61	50.12	53.39
g_k	7.99	3.49	1.68	0.66	0.37
γ_{T_k} (dB)	-1.533	1.094	3.972	7.702	10.249

II. SYSTEM AND CHANNEL MODEL

We consider a dual-hop transmission system as depicted in Fig. 1, where the data transmission from a source node S to a destination node D is realized via an intermediate regenerative relay station R operating in the decode-and-forward mode [2]. In this regard, every end-to-end transmission interval between source and destination is subdivided into two different time slots of equal length T_F. During the first time slot, the source node takes a number of data packets of fixed size from its input buffer—which by assumption never runs out of data—encodes and modulates the corresponding bits and then transmits them to the relay station. Thereupon, the relay tries to demodulate and decode the received signals and puts the successfully recovered packets[1] into a buffer, which can accommodate at most L_{max} different data units. Packets which could not be successfully decoded, in contrast, simply are dropped, and their retransmission, if necessary, will have to be taken care of by higher layers. During the second time slot, the relay then takes a certain number of packets out of its buffer, encodes and modulates them appropriately and finally transmits them to the actual destination node D.

We assume that AMC is independently applied to both hops, so that the number of packets that might be transmitted within a certain time slot might be different for both hops and depends on the selected MCSs. The transmit power of both source and relay, on the other hand, is kept constant, noting that further performance improvements could be obtained by dynamically adjusting the power levels as well, what, however, is left for further studies. As a concrete example, we consider $N = 5$ different MCSs as summarized in Table I [25] and we assume that each transmitted packet is encoded separately. As can be seen, the achievable rate with the k-th mode is greater than or equal to k times the rate of the first mode. Assuming for simplicity that with the first mode exactly one packet can be transmitted during one time slot of length T_F, we consequently can transmit a maximum number of k packets if the k-th MCS is selected. An example for a possible time flow under this assumption is depicted in Fig. 2. Furthermore, we assume for notational convenience that the buffer in the relay station can store at least as many packets as might be transmitted with the highest-order AMC mode, i.e., $L_{\text{max}} \geq N$.

We consider two different schemes for the MCS selection over the first hop, which differ in whether or not the current buffer filling level at the relay station is taken into account. Similar to conventional AMC approaches [12], [15], the MCS selection with the first scheme is solely based on the current

[1]Please note that the relay station might be able to detect whether a certain packet has been successfully decoded or not by performing a cyclic redundancy check, for example.

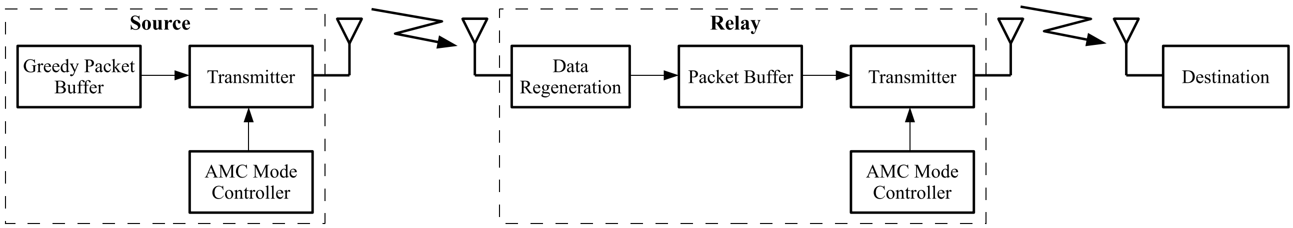

Fig. 1. Considered dual-hop transmission system with adaptive modulation and coding.

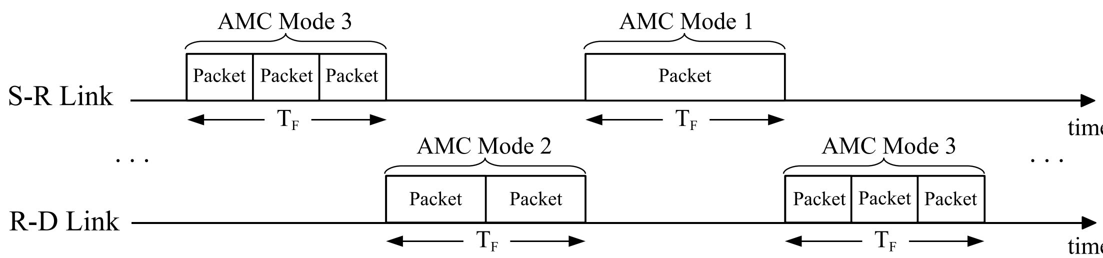

Fig. 2. Basic frame structure and exemplary time flow of the considered adaptive dual-hop transmission system.

channel conditions, i.e., if the SNR[2] γ_n on the n-th hop ($n \in \{1, 2\}$) falls into a certain region bounded by the switching thresholds $\vartheta_{n,k}$ and $\vartheta_{n,k+1}$, the k-th AMC mode is selected. With this scheme, however, it obviously might happen that the source node transmits more packets than the buffer at the relay station can additionally accommodate, resulting in a potential buffer overflow. In order to avoid this problem, the second proposed scheme takes the current buffer filling level into account during the MCS selection for the first hop. Specifically, after having determined the MCS that could be supported by the quality of the first hop, the relay checks whether the number of packets that would be transmitted with this MCS exceeds the number of free slots in its buffer. If this is the case, a lower-order AMC mode is selected so that all transmitted packets eventually might be buffered by the relay station.

Please note that both approaches might be relevant in practice. The first pure SNR-based scheme could be readily applied to a system with a reciprocal channel, where the source does not require any feedback from the relay station to perform the actual AMC on the source-to-relay link. This holds for example for almost any time-division duplex system, such as WiMax or LTE-TDD. For the second scheme, which takes the current buffer level at the relay station into account, the relay always has to feed back some information to the source node for proper operation, even if the source were aware of the current quality of the first hop due to channel reciprocity. Therefore, this scheme is more suitable for frequency-division duplex systems, such as LTE-FDD, for example. For the sake of clarity, we assume that the MCS selection over the second hop is purely based on the prevailing channel conditions of that hop, as described above.

The AMC switching thresholds $\vartheta_{n,k}$ for determining which MCS is supported for a certain SNR γ_n are generally chosen in such a way that a certain packet error rate (PER) target is not exceeded. Using the standard PER approximation [26]

$$\mathrm{P}_{e,k}(\gamma) = \begin{cases} 1, & \text{for } 0 < \gamma < \gamma_{T_k} \\ a_k \exp(-g_k\,\gamma), & \text{for } \gamma \geq \gamma_{T_k} \end{cases}, \quad (1)$$

where the constants a_k and g_k as well as the threshold value γ_{T_k} are given for the considered AMC modes in Table I, we can readily determine the minimum SNR that is required such that with the k-th MCS a given target PER $P_{\text{target,n}}$ is not exceeded as

$$\vartheta_{n,k} = \frac{1}{g_k} \ln\left(\frac{a_k}{P_{\text{target,n}}}\right). \quad (2)$$

In order to facilitate a unified analysis, we always set $\vartheta_{n,0} = 0$ as well as $\vartheta_{n,N+1} = \infty$ in the following. If the instantaneous SNR falls into the zero-th bin, i.e., if $\vartheta_{n,0} \leq \gamma_n < \vartheta_{n,1}$, this means that even with the most robust MCS the target PER would be exceeded and therefore transmission should be suspended in that case. Furthermore, we assume that packets which could not be successfully decoded on either link are simply dropped, leaving their retransmission—if necessary—to upper layers, what, however, is not explicitly considered here. Finally, we assume that both hops are subject to independent frequency-flat fading, where the channels remain unchanged during one end-to-end transmission interval, but change independently between two such intervals. As a concrete example, we always consider the case where both hops undergo independent but not necessarily identically distributed Nakagami-m fading with fading parameters m_1 and m_2 and average SNR $\bar{\gamma}_1$ and $\bar{\gamma}_2$, respectively. In this case, the probability density function of the SNR γ_n is given by

$$p_{\gamma_n}(\gamma) = \frac{m_n^{m_n}\,\gamma^{m_n-1}}{\bar{\gamma}_n^{m_n}\,\Gamma(m_n)} \exp\left(-\frac{m_n\,\gamma}{\bar{\gamma}_n}\right), \qquad \gamma \geq 0 \quad (3)$$

with $\Gamma(\cdot)$ as the Gamma function and the corresponding

[2]In this work, the SNR is always defined as the received signal power per packet over the average noise power per packet.

cumulative distribution function by

$$F_{\gamma_n}(\gamma) = 1 - \frac{\Gamma\left(m_n, \frac{m_n\,\gamma}{\bar{\gamma}_n}\right)}{\Gamma(m_n)}, \qquad \gamma \geq 0 \tag{4}$$

with $\Gamma(\cdot,\cdot)$ as the upper incomplete Gamma function [27].

III. Finite-State Markov Chain Model

We develop a finite-state Markov chain with $L_{\max}+1$ states for the performance analysis of the AMC based dual-hop transmission system under consideration, where each state is associated with a certain number of packets in the relay buffer. In particular, if the Markov chain is in the i-th state ($0 \leq i \leq L_{\max}$), this corresponds to the case where the relay buffer currently holds i packets. In this regard, we basically have two different options for particularizing our model by considering the current buffer levels either before the transmission from S to R or after the S-to-R transmission. Both approaches eventually should lead to the same performance results and in the following we therefore consider without loss of generality always the buffer levels after the transmission from S to R and before the forwarding of packets from R to D, what seems to lead to a slightly simpler analysis than the alternative modeling approach.

A. Markov Chain Transition Probabilities

In general, the buffer level at the relay station changes from i packets in the current time interval to j packets in the next one if the relay first of all forwards k packets ($0 \leq k \leq \min\{N,i\}$) to the destination while at the same time after the transmission from the source to the relay exactly $j-i+k$ packets are put into the buffer again. Denoting the probability that k packets are forwarded by the relay station given that in total i packets are available in its buffer by $q_{k,i}$ and likewise the probability that after the transmission from S to R exactly u packets are stored in the relay buffer provided that it has capacity for storing w additional packets by $r_{u,w}$, we generally can express the transition probability from state i to state j, $p_{i,j}$, as

$$p_{i,j} = \sum_{k=0}^{\min\{N,i\}} q_{k,i}\; r_{j-i+k,L_{\max}-i+k}. \tag{5}$$

Clearly, the probability $q_{k,i}$ is for both strategies the same due to their common MCS selection methods over the second hop. In particular, if $k \leq \min\{N,i\} < i$, i.e., if not all packets that are currently stored in the buffer are forwarded, $q_{k,i}$ simply corresponds to the probability that the instantaneous SNR γ_2 on the R-D link falls into the k-th bin and consequently mode k is chosen for the transmission. If, in contrast, $k = i \leq N$, i.e., if all packets in the buffer are forwarded, $q_{k,i}$ corresponds to the probability that the R-D link is good enough so that *at least* k packets might be transmitted without violating the imposed PER constraint. Hence, we directly obtain

$$q_{k,i} = \begin{cases} F_{\gamma_2}(\vartheta_{2,k+1}) - F_{\gamma_2}(\vartheta_{2,k}), & \text{if } k < i \\ 1 - F_{\gamma_2}(\vartheta_{2,k}), & \text{if } k = i \\ 0, & \text{if } k > i \end{cases}, \tag{6}$$

with $F_{\gamma_2}(\gamma)$ as the cumulative distribution function of γ_2. For the considered example with Nakagami-m fading on both hops, we obtain by plugging (4) in (6)

$$q_{k,i} = \begin{cases} \frac{1}{\Gamma(m_2)}\left[\Gamma\left(m_2, \frac{m_2\,\vartheta_{2,k}}{\bar{\gamma}_2}\right) - \Gamma\left(m_2, \frac{m_2\,\vartheta_{2,k+1}}{\bar{\gamma}_2}\right)\right], & \text{if } k < i \\ \frac{1}{\Gamma(m_2)}\Gamma\left(m_2, \frac{m_2\,\vartheta_{2,k}}{\bar{\gamma}_2}\right), & \text{if } k = i \\ 0, & \text{if } k > i \end{cases}. \tag{7}$$

The probability $r_{u,w}$ that after the transmission from S to R exactly u packets are put into the buffer provided that it can accommodate at most w more packets, on the other hand, is different for the two different AMC approaches since the number of packets transmitted by the source may or may not be influenced by the current buffer level at the relay station. For that reason, we separately consider these two cases in the following.

1) Approach I: Pure SNR-Based Mode Selection: If the AMC mode selection on the source-to-relay link is solely done based on the current SNR γ_1 without taking the current buffer level into account, it might happen that actually more packets are transmitted than the number of free slots available in the relay buffer, thus leading to a potential buffer overflow. For $u < w$, the probability $r_{u,w}$ that u packets are put into the buffer given that there are w available slots simply corresponds to the probability that *exactly* u packets could be successfully decoded whereas for $u = w$, $r_{u,w}$ is given by the probability that *at least* u packets could be decoded at the relay station. In case that actually more packets have been decoded without any errors, the additional ones would have to be discarded. Taking into account that the number of transmitted packets might be larger than the number of successfully decoded packets, we have in general

$$r_{u,w} = \begin{cases} \sum_{k=u}^{N} z_{u,k} & \text{if } u < w \\ \sum_{k=u}^{N}\sum_{\eta=u}^{k} z_{\eta,k} & \text{if } u = w \\ 0 & \text{if } u > w \end{cases}, \tag{8}$$

where $z_{u,k}$ denotes the joint probability that the source transmits k packets out of which exactly u are successfully recovered at the relay station. The transmission of k packets by the source requires only that the k-th AMC mode is selected or equivalently that $\vartheta_{1,k} \leq \gamma_1 < \vartheta_{1,k+1}$. Besides, it is actually not relevant which u out of the transmitted k packets are successfully decoded. Consequently, it can easily be seen that the probabilities $z_{u,k}$ can be calculated as

$$z_{u,k} = \binom{k}{u} \int_{\vartheta_{1,k}}^{\vartheta_{1,k+1}} \left(1 - P_{e,k}(\gamma)\right)^u P_{e,k}(\gamma)^{k-u}\, p_{\gamma_1}(\gamma)\, d\gamma, \tag{9}$$

with $P_{e,k}(\gamma)$ as the PER for SNR γ in case that the k-th AMC mode is selected and $p_{\gamma_1}(\gamma)$ as the pdf of γ_1. With the PER expressions according to (1) and by making use of the binomial theorem, it can be shown that

$$\left(1 - P_{e,k}(\gamma)\right)^u P_{e,k}(\gamma)^{k-u} = \sum_{\eta=0}^{u} \binom{u}{\eta} (-1)^\eta\, a_k^{\eta+k-u}\, \exp\left(-g_k\,\gamma\,(\eta+k-u)\right). \tag{10}$$

Plugging this expression together with (3) in (9) and capitalizing on the integration result given in [27, eq. (3.381,3)], we obtain a closed-form expression for $z_{u,k}$ for the specific example of Nakagami-m fading as

$$z_{u,k} = \sum_{\eta=0}^{m} \frac{\binom{k}{u}\binom{u}{\eta}(-1)^\eta m_1^{m_1} a_k^{\eta+k-u}}{\left(\frac{m_1}{\bar{\gamma}_1}+g_k(\eta+k-u)\right)^{m_1}\Gamma(m_1)\,\bar{\gamma}_1^{m_1}} \times\left[\Gamma\left(m_1,\left(\frac{m_1}{\bar{\gamma}_1}+g_k(\eta+k-u)\right)\vartheta_{1,k}\right) -\Gamma\left(m_1,\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)\vartheta_{1,k+1}\right)\right]. \quad (11)$$

Combining (11) with (8), (7), and (5) then directly leads to the desired closed-form expressions for the various state transition probabilities $p_{i,j}$ for the case of SNR-based mode selection.

2) Approach II: Buffer-Aware Mode Selection: With our second approach, a buffer overflow at the relay station never can happen since the number of packets transmitted by the source always corresponds to the minimum out of the number that might be transmitted considering only the instantaneous SNR γ_1 on the S-R link and the number of available free slots in the relay buffer. In consequence, the probability $r_{u,w}$ that u packets are put into the buffer always is given by the probability that *exactly* u packets are correctly decoded, but the number of packets that may be transmitted by the source now is dependent on w. In general, we readily obtain

$$r_{u,w} = \sum_{k=u}^{\phi_w} t_{u,k,w}, \quad (12)$$

where we have introduced for brevity the short-hand notation

$$\phi_w = \min\{N, w\} \quad (13)$$

and where $t_{u,k,w}$ denotes the joint probability that the source transmits k packets given that there are w available slots in the relay buffer while the relay successfully decodes exactly u out of these k packets. Clearly, for $k < w$, the transmission of k packets requires that $\vartheta_{1,k} \leq \gamma_1 < \vartheta_{1,k+1}$ whereas for $k = w$ we necessarily have $\gamma_1 \geq \vartheta_{1,k}$ since even if the instantaneous channel would be good enough to support a higher AMC mode, still at most only k packets can be transmitted to avoid a potential buffer overflow. Consequently, we obtain for $t_{u,k,w}$ the general expression

$$t_{u,k,w} = \begin{cases} \binom{k}{u}\int_{\vartheta_{1,k}}^{\vartheta_{1,k+1}}(1-P_{e,k}(\gamma))^u \times P_{e,k}(\gamma)^{k-u}\,p_{\gamma_1}(\gamma)\,d\gamma, & k < \phi_w \\ \binom{\phi_w}{u}\int_{\vartheta_{1,k}}^{\infty}(1-P_{e,k}(\gamma))^u \times P_{e,k}(\gamma)^{k-u}\,p_{\gamma_1}(\gamma)\,d\gamma, & k = \phi_w \\ 0 & k > \phi_w \end{cases} \quad (14)$$

As before, we can make use of the relationship according to (10), insert the Nakagami-m fading pdf as given by (3) and invoke the integration result provided in [27, eq. (3.381,3)], thus finally leading to the closed-form expression

$$t_{u,k,w} = \begin{cases} \frac{m_1^{m_1}}{\bar{\gamma}_1^{m_1}\Gamma(m_1)}\binom{k}{u}\sum_{\eta=0}^{u}\frac{\binom{u}{\eta}(-1)^\eta a_k^{\eta+k-u}}{\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)^{m_1}} \times\left[\Gamma\left(m_1,\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)\vartheta_{1,k}\right) -\Gamma\left(m_1,\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)\vartheta_{1,k+1}\right)\right], & k < \phi_n \\ \frac{m_1^{m_1}}{\bar{\gamma}_1^{m_1}\Gamma(m_1)}\sum_{\eta=0}^{u}\binom{k}{u}\binom{u}{\eta}\frac{(-1)^\eta a_k^{\eta+k-u}}{\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)^{m_1}} \times\Gamma\left(m_1,\left(\frac{m_1}{\bar{\gamma}_1}+g_k\,(\eta+k-u)\right)\vartheta_{1,k}\right), & k = \phi_w \\ 0, & k > \phi_n \end{cases} \quad (15)$$

Combining (15) with (12), (7) and (5), we then get the desired closed-form expressions for the state transition probabilities of our Markov chain in case of buffer-aware AMC mode selection.

B. Steady-State Probability Distribution

Having the state transition probabilities $p_{i,j}$ for both AMC selection schemes as determined in the previous section, we can build up a $(L_{\max}+1)\times(L_{\max}+1)$ transition probability matrix $\mathbf{P}$ such that $p_{i,j}$ corresponds to the entry in the i-th row and j-th column of $\mathbf{P}$. The steady-state distribution vector $\boldsymbol{\pi} = [\pi_0, \pi_1, \ldots, \pi_{L_{\max}}]$, where π_ν denotes the steady-state probability that the Markov chain is in state ν, is then given by the solution of the linear equation system

$$\boldsymbol{\pi}\,\mathbf{P} = \boldsymbol{\pi}, \quad (16)$$

subject to the additional constraints that $\pi_i \geq 0\ \forall\ i$ as well as $\sum_{i=0}^{L_{\max}} \pi_i = 1$. Please note that since our Markov chain is both aperiodic and irreducible (as can easily be verified), always a *unique* solution of (16) exists. This solution can be determined by noting that (16) actually is a standard eigenvalue problem with $\boldsymbol{\pi}$ as a left eigenvector of $\mathbf{P}$ associated with eigenvalue one and normalized such that all entries sum up to one. So to determine $\boldsymbol{\pi}$, we might perform an eigenvalue decomposition of $\mathbf{P}^T$ and denote the eigenvector associated with eigenvalue one as $\mathbf{v} = [v_0, v_1, \ldots, v_{L_{\max}}]^T$. Then, $\boldsymbol{\pi}$ is simply given by

$$\boldsymbol{\pi} = \frac{\mathbf{v}}{\sum_{n=0}^{L_{\max}} v_n}. \quad (17)$$

Alternatively, $\boldsymbol{\pi}$ might be determined in an iterative fashion by exploiting that $\lim_{n\to\infty}\mathbf{P}^n = \mathbf{1}\,\boldsymbol{\pi}$, where $\mathbf{1}$ denotes a $(L_{\max}+1)$-dimensional column vector for which all entries are equal to one.

Based on the steady-state probabilities π_ν or equivalently the probabilities that the number of packets in the buffer after the transmission from the source to the relay is equal to ν, we now can also easily determine the steady-state probabilities π'_ν that the number of packets in the buffer after the forwarding of packets from the relay to the destination is equal to ν. This is the case if before the transmission on the R-D link i packets were in the buffer ($\nu \leq i \leq \min\{L_{\max}, \nu+N\}$) and then $i-\nu$ packets have been forwarded. Hence, we can directly determine π'_ν as

$$\pi'_\nu = \sum_{i=\nu}^{\min\{L_{\max},\nu+N\}} \pi_i\, q_{i-\nu,i}, \quad (18)$$

where $q_{i-\nu,i}$ according to (6) denotes the probability that the relay forwards $i-\nu$ packets provided that there are i packets in the buffer.

IV. Performance Analysis

In the following, we derive analytical closed-form expressions for a variety of different key performance indicators. For that purpose, we capitalize on the steady-state probabilities π_ν and π'_ν that have been derived for the two different AMC selection strategies in the previous section.

A. Average Buffer Filling Level

The average buffer filling level corresponds to the average number of packets stored in the buffer. It can be used to determine whether the dimensioning of the buffer and the adjustment of the PER targets for the two different hops is appropriate or not. In this regard, we have to distinguish again the situation right after the transmission from S to R and the one after the transmission from R to D. For the first case, we readily obtain

$$L_{\text{avg}} = \sum_{\nu=1}^{L_{\max}} \nu\, \pi_\nu \tag{19}$$

where depending on the considered AMC selection strategy the steady-state probability π_ν has to be calculated appropriately. Likewise, we obtain for the average buffer filling level after the forwarding of packets from the relay to the destination

$$L'_{\text{avg}} = \sum_{\nu=1}^{L_{\max}} \nu\, \pi'_\nu, \tag{20}$$

with π'_ν according to (18).

B. Average Transmission Efficiency

The average transmission efficiency is defined as the average number of packets *transmitted* during one end-to-end transmission interval. Hence, we obviously have to distinguish between the average transmission efficiency of the source-to-relay and the one of the relay-to-destination link since the source and the relay do generally not transmit the same number of packets on average. This is because the relay might discard packets if either a buffer overflow occurs or if they could not have been successfully decoded. The actual *end-to-end* average transmission efficiency corresponds then simply to the average transmission efficiency of the relay-to-destination link.

Consider first of all the relay-to-destination link. Based on the mode of operation, the relay forwards k packets to the destination if and only if there are more than k packets in the buffer and $\vartheta_{2,k} \le \gamma_2 < \vartheta_{2,k+1}$ or if there are exactly k packets in the buffer and $\gamma_2 \ge \vartheta_{2,k}$. Consequently, the average transmission efficiency $\eta_{\text{r-d}}$ of the R-D link can be readily calculated as

$$\begin{aligned}\eta_{\text{r-d}} \;=\; \sum_{k=1}^{N} k \Bigg(&\left(F_{\gamma_2}(\vartheta_{2,k+1}) - F_{\gamma_2}(\vartheta_{2,k})\right) \sum_{\nu=k+1}^{L_{\max}} \pi_\nu \\ &+\pi_k \left(1 - F_{\gamma_2}(\vartheta_{2,k})\right)\Bigg). \end{aligned} \tag{21}$$

Please note that this result is valid for both AMC selection strategies. If we consider the average transmission efficiency of the source-to-destination link, however, we have to distinguish between the two different AMC selection strategies.

1) Approach I: Pure SNR-Based Mode Selection: If the AMC mode selection for the S-R link is solely based on the SNR γ_1, the source always transmits exactly k packets if $\vartheta_{1,k} \le \gamma_1 < \vartheta_{2,k}$. Hence the corresponding average transmission efficiency can simply be calculated as

$$\eta_{\text{s-r}} = \sum_{k=1}^{N} k\, w_k, \tag{22}$$

where the probability w_k is given by

$$w_k = F_{\gamma_1}(\vartheta_{1,k+1}) - F_{\gamma_1}(\vartheta_{1,k}). \tag{23}$$

Considering the example of Nakagami-m fading again, we obtain by plugging (4) into (23)

$$w_k = \frac{1}{\Gamma(m_1)} \left[\Gamma\left(m_1, \frac{m_1}{\bar{\gamma}_1}\,\vartheta_{1,k}\right) - \Gamma\left(m_1, \frac{m_1}{\bar{\gamma}_1}\,\vartheta_{1,k+1}\right)\right]. \tag{24}$$

2) Approach II: Buffer-Aware Mode Selection: If the current buffer level is taken into account, the source never transmits more packets than the buffer can accommodate. In this regard, the buffer levels *after* the transmission from the relay to destination are relevant, which are characterized by the probabilities π'_ν according to (18). In particular, the source transmits exactly k packets if $\vartheta_{1,k} \le \gamma_1 < \vartheta_{1,k+1}$ and the buffer can store more than k additional packets or if the buffer can store exactly k more packets and $\gamma_1 \ge \vartheta_{1,k}$. Mathematically, the average transmission efficiency on the S-R link consequently can be calculated as

$$\eta_{\text{s-r}} = \sum_{k=1}^{N} k \sum_{\nu=0}^{L_{\max}} w_{k,\nu}\, \pi'_{L_{\max}-\nu}, \tag{25}$$

with $w_{k,\nu}$ as the probability that the source transmits k packets provided that the buffer at the relay station can accommodate at most ν packets, which can be shown to be given by

$$w_{k,\nu} = \begin{cases} F_{\gamma_1}(\vartheta_{1,k+1}) - F_{\gamma_1}(\vartheta_{1,k}), & \text{if } k < \phi_\nu \\ 1 - F_{\gamma_1}(\vartheta_{1,k}), & \text{if } k = \phi_\nu \\ 0, & \text{if } k > \phi_\nu \end{cases} \tag{26}$$

with ϕ_ν according to (13).

C. Average Packet Error Ratio

We now consider the average packet error performance of our system. Similar to the average transmission efficiency, the average packet error ratio generally is different for the S-R and the R-D links. Therefore, they are treated separately in the following. Besides, we also have to distinguish the two considered MCS selection strategies for the source-to-relay link again.

For the R-D link, both strategies always select the AMC mode that is supported by the current SNR γ_2, independent of how many packets are available in the buffer. Hence, if k packets are transmitted but there are more than k packets in the buffer, we necessarily have $\vartheta_{2,k} \le \gamma_2 < \vartheta_{2,k+1}$ and consequently the k-th AMC mode is selected. However, if k packets are transmitted and this is at the same time also the total number of packets in the buffer, we just know that $\gamma_2 \geq \vartheta_{2,k}$ and as such either the k-th or a higher-order AMC mode might be selected. Having this in mind and noting that the average PER generally corresponds to the average number of erroneously decoded packets over the total number of transmitted packets, we can calculate the average PER on the R-D link as

$$\overline{\text{PER}}_{\text{r-d}} = \frac{1}{\eta_{\text{r-d}}} \sum_{k=1}^{N} k \left(\int_{\vartheta_{2,k}}^{\vartheta_{2,k+1}} P_{e,k}(\gamma)\, p_{\gamma_2}(\gamma)\, d\gamma \sum_{\nu=k+1}^{L_{\max}} \pi_\nu + \pi_k \sum_{\eta=k}^{N} \int_{\vartheta_{2,\eta}}^{\vartheta_{2,\eta+1}} P_{e,\eta}(\gamma)\, p_{\gamma_2}(\gamma)\, d\gamma \right), \tag{27}$$

with $P_{e,k}(\gamma)$ as the PER for SNR γ if the k-th AMC mode has been selected. Inserting the corresponding expressions according to (1) as well as the pdf of γ_2 for Nakagami-m fading according to (3), we obtain by making use of [27, eq. (3.381,3)] the closed-form result

$$\begin{aligned}\overline{\text{PER}}_{\text{r-d}} = &\frac{1}{\eta_{\text{r-d}}} \sum_{k=1}^{N} \sum_{\nu=k+1}^{L_{\max}} \frac{\pi_\nu\, k\, a_k\, m_2^{m_2}}{(m_2 + \bar{\gamma}_2\, g_k)^{m_2}\, \Gamma(m_2)} \\ &\times \left[\Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_k\right) \vartheta_{2,k}\right) - \Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_k\right) \vartheta_{2,k+1}\right)\right] \\ &+ \pi_k \sum_{\eta=k}^{N} \frac{k\, a_\eta\, m_2^{m_2}}{(m_2 + \bar{\gamma}_2\, g_\eta)^{m_2}\, \Gamma(m_2)} \\ &\times \left[\Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_\eta\right) \vartheta_{2,\eta}\right) - \Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_\eta\right) \vartheta_{2,\eta+1}\right)\right], \end{aligned} \tag{28}$$

with $\eta_{\text{r-d}}$ according to (21).

Next, we study the PER performance for the S-R link for the two MCS selection strategies under consideration separately in the following.

1) Approach I: Pure SNR-Based Mode Selection: If the pure SNR-based MCS mode selection strategy is adopted, we can obtain the PER just as in case of AMC for conventional single-hop systems [13] as

$$\overline{\text{PER}}_{\text{s-r}} = \frac{1}{\eta_{\text{s-r}}} \sum_{k=1}^{N} k \int_{\vartheta_{1,k}}^{\vartheta_{1,k+1}} P_{e,k}(\gamma)\, p_{\gamma_1}(\gamma)\, d\gamma, \tag{29}$$

which can be calculated for the case with Nakagami-m fading again in closed-form as

$$\begin{aligned}\overline{\text{PER}}_{\text{s-r}} = &\sum_{k=1}^{N} \frac{k\, a_k\, m_2^{m_2}}{\eta_{\text{s-r}}\, (m_2 + \bar{\gamma}_2\, g_k)^{m_2}\, \Gamma(m_2)} \\ &\times \left[\Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_k\right) \vartheta_{2,k}\right) - \Gamma\left(m_2, \left(\tfrac{m_2}{\bar{\gamma}_2} + g_k\right) \vartheta_{2,k+1}\right)\right], \end{aligned} \tag{30}$$

where $\eta_{\text{s-r}}$ is given by (22). However, please note that $\overline{\text{PER}}_{\text{s-r}}$ does not include packets that have actually been successfully decoded at the relay station, but then have to be discarded due to a buffer overflow. For quantifying the joint impact of both packet errors and buffer overflows at the relay station, we therefore define the drop probability $P_{\text{drop, relay}}$ as the average number of packets dropped by the relay station due to any of these two reasons over the average number of packets transmitted by the source. Clearly, this probability is just one minus the average number of packets put into the relay buffer over the average number of packets transmitted by the source. In this regard, we have already determined in (8) the probability $r_{u,w}$ that u packets are put into buffer given that there are w free slots in the buffer and consequently we can directly derive a corresponding expression for $P_{\text{drop, relay}}$ based on $r_{u,w}$ as

$$P_{\text{drop, relay}} = 1 - \frac{1}{\eta_{\text{s-r}}} \sum_{u=1}^{N} u \sum_{w=0}^{L_{\max}} r_{u,w}\, \pi'_{L_{\max}-w}, \tag{31}$$

with π'_μ as the steady-state probabilities after the transmission from R to D according to (18).

2) Approach II: Buffer-Aware Mode Selection: With the buffer-aware mode selection strategy, the selected AMC mode for the S-R link always corresponds to the minimum out of the AMC mode that can be supported by the SNR γ_1 and the AMC mode that can transmit exactly as many packets as the relay buffer can accommodate. Hence, the k-th mode is chosen if either more than k slots in the buffer are free and $\vartheta_{1,k} \leq \gamma_1 < \vartheta_{1,k+1}$ or if exactly k slots in the buffer are free and $\gamma_1 \geq \vartheta_{1,k}$. Consequently, the average PER on this hop can be readily determined as

$$\overline{\text{PER}}_{\text{s-r}} = \frac{1}{\eta_{\text{s-r}}} \sum_{k=1}^{N} k \left(\int_{\vartheta_{1,k}}^{\vartheta_{1,k+1}} P_{e,k}(\gamma)\, p_{\gamma_1}(\gamma)\, d\gamma \sum_{\nu=0}^{L_{\max}-k-1} \pi'_\nu + \pi'_{L_{\max}-k} \int_{\vartheta_{1,k}}^{\infty} P_{e,k}(\gamma)\, p_{\gamma_1}(\gamma)\, d\gamma \right). \tag{32}$$

Considering as a concrete example again Nakagami-m fading as well as the PER expressions according to (1), we obtain the closed-form expression

$$\begin{aligned}\overline{\text{PER}}_{\text{s-r}} \;=\; &\frac{1}{\eta_{\text{s-r}}} \sum_{k=1}^{N} \frac{k\, a_k\, m_1^{m_1}}{(m_1 + \bar{\gamma}_1\, g_k)^{m_1}\, \Gamma(m_1)} \\ &\times \left[\pi'_{L_{\max}-k}\, \Gamma\left(m_1, \left(\tfrac{m_1}{\bar{\gamma}_1} g_k\right) \vartheta_{1,k}\right) \right. \\ &+ \left(\sum_{\nu=0}^{L_{\max}-k-1} \pi'_\nu \right) \left[\Gamma\left(m_1, \left(\tfrac{m_1}{\bar{\gamma}_1} + g_k\right) \vartheta_{1,k}\right) \right. \\ &\left.\left. - \Gamma\left(m_1, \left(\tfrac{m_1}{\bar{\gamma}_1} + g_k\right) \vartheta_{1,k+1}\right)\right]\right] \end{aligned} \tag{33}$$

with $\eta_{\text{s-r}}$ according to (25).

D. Average Delay

Another important measure for characterizing our system performance is the average transmission delay, which we define as the average delay in end-to-end transmission intervals a packet received by the destination has experienced since its transmission by the source. Generally, it corresponds to the average waiting time in the relay buffer plus one additional time interval for the actual packet transmission itself. In this regard, we can determine the average waiting time by capitalizing on Little's well-known theorem as [28]

$$\Delta_{\text{wait}} = \frac{L'_{\text{avg}}}{\lambda}, \tag{34}$$

where λ denotes the rate of successfully decoded packets arriving at the relay station, i.e., the arrival rate of packets that are actually put into the relay buffer, and L'_{avg} denotes the average buffer level *as seen by an outside observer*, which simply corresponds to the average buffer level after the transmission from the relay to the destination, as given by (20)[3]. For the average arrival rate λ, we have to distinguish the two different AMC selection strategies, since the number of packets put into the buffer generally might be different for both approaches.

1) Approach I: Pure SNR-Based Mode Selection: For the pure SNR-based approach, the average number of packets put into the relay buffer during one transmission interval corresponding to the arrival rate λ has already implicitly been determined in (31) and is given by

$$\lambda = \sum_{u=1}^{N} u \sum_{w=0}^{L_{\max}} r_{u,w}\, \pi'_{L_{\max}-w}, \tag{35}$$

with $r_{u,w}$ according to (8). Hence, the average transmission delay can readily be expressed by combining (35) with (20) and (34) as

$$\Delta_{\text{delay}} = 1 + \frac{\sum_{\nu=1}^{L_{\max}} \nu\, \pi'_\nu}{\sum_{u=1}^{N} u \sum_{w=0}^{L_{\max}} r_{u,w}\, \pi'_{L_{\max}-w}}. \tag{36}$$

2) Approach II: Buffer-Aware Mode Selection: With the buffer-aware AMC mode selection, the average number of packets put into the buffer within one time interval corresponds to the average number of successfully decoded packets at the relay station, which is given by

$$\lambda = \eta_{\text{s-r}} \left(1 - \overline{\text{PER}}_{\text{s-r}}\right), \tag{37}$$

with $\eta_{\text{s-r}}$ according to (25) and $\overline{\text{PER}}_{\text{s-r}}$ according to (29). Hence, the actual average end-to-end transmission delay generally is given by

$$\Delta_{\text{delay}} = 1 + \frac{\sum_{\nu=1}^{L_{\max}} \nu\, \pi'_\nu}{\eta_{\text{s-r}} \left(1 - \overline{\text{PER}}_{\text{s-r}}\right)}. \tag{38}$$

E. Average Packet Loss Rate

Finally, we consider as another important performance the average packet loss rate, which is defined as the average number of packets transmitted by the source that are either dropped at the relay station due to a buffer overflow or a packet error or at the actual destination due to a packet error over the total average number of packets transmitted by the source. It can easily be seen that this rate generally can be determined for both AMC selection strategies as

$$P_{\text{loss}} = 1 - \frac{\eta_{\text{r-d}} \left(1 - \overline{\text{PER}}_{\text{r-d}}\right)}{\eta_{\text{s-r}}}, \tag{39}$$

with $\eta_{\text{r-d}}$ and $\overline{\text{PER}}_{\text{r-d}}$ according to (21) and (27) and $\eta_{\text{s-r}}$ according to (22) for pure SNR-based AMC mode selection and (25) for the buffer-aware approach.

V. Numerical Results

To illustrate the performance of the dual-hop adaptive relaying transmission system, we present in the following selected numerical results, which have been obtained by evaluating the analytical expressions derived in the previous section using Matlab. Interestingly, it turned out that for all considered scenarios and the AMC modes according to Table I the differences between the steady-state probability distributions for the two different AMC mode selection schemes were basically negligible. This is shown for one particular case in Fig. 3, where the steady-state probabilities of both approaches are juxtaposed. At a first glance, it might seem a little bit surprising that with both schemes almost the same steady-state probabilities are obtained, but in fact this can be intuitively explained as follows: On the relay-to-destination link, the number of transmitted packets for a certain SNR γ_2 is always the same in both cases. On the source-to-relay link, in contrast, the number of transmitted packets is only different for the two approaches if the maximum number of packets that might be transmitted such that the PER target $P_{\text{target},1}$ is not exceeded given that the SNR γ_1 is larger than the number of packets the relay buffer can additionally store. With pure SNR-based mode selection, in this case actually more packets are transmitted than the buffer can store so that even if some of them cannot be successfully decoded, it is most likely to fully fill the buffer. With the buffer-aware approach, on the other hand, the number of transmitted packets would be adjusted to the number of packets the buffer can store, but the chosen AMC mode would be lower than the one that actually would match to the current channel conditions. In consequence, the corresponding PER drops significantly and therefore with a very high probability the buffer can be completely filled up in this case as well. Hence, the number of packets that are stored in the relay buffer during one transmission interval is almost always the same, thus leading to virtually the same steady-state distribution.

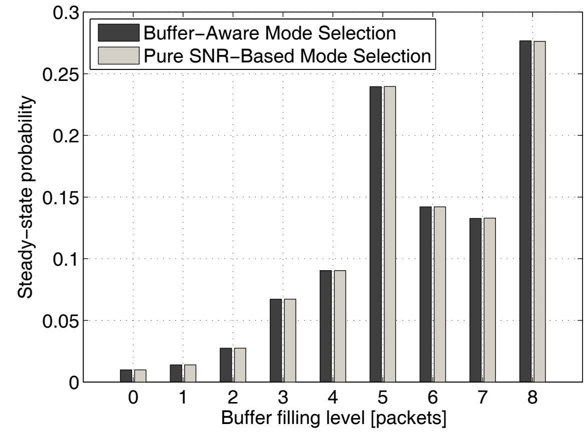

Fig. 3. Steady-state probabilities for the different buffer filling levels after the transmission from S to R for $\bar{\gamma}_1 = \bar{\gamma}_2 = 15\,\text{dB}$, Rayleigh-fading on both hops, $L_{\max} = 8$, and $P_{\text{target},1} = P_{\text{target},2} = 0.1$.

A direct consequence of the basically negligible differences in the steady-state probability distribution is that most other key performance indicators are virtually the same for both mode selection strategies as well. This holds particularly for the average end-to-end transmission efficiency, which is identical to the transmission efficiency $\eta_{\text{r-d}}$ of the R-D link. As an example, Fig. 4 shows the average end-to-end transmission

[3]Please note that we assume that packets successfully transmitted on the S-R link are put into the relay buffer only at the very end of the first transmission phase whereas the packets to be transmitted on the R-D link are taken out of the buffer at the very beginning of the second transmission phase.

efficiency as a function of the average SNR for different buffer sizes $L_{\max}$. Clearly, it can be seen that the curves corresponding to the two different AMC selection schemes actually cannot be visually distinguished and hence with both approaches approximately the same performance is achieved. Besides, it becomes obvious that with increasing buffer size the transmission efficiency can be slightly increased, what is due to the fact that in this case the probability that the relay runs out of packets and therefore has to forward less packets than actually might be transmitted on the R-D link is decreased.

The average end-to-end transmission delay of the packets received by the destination versus the average SNR $\gamma = \bar{\gamma}_1 = \bar{\gamma}_2$ is depicted in Fig. 5 for different buffer sizes. Obviously, both mode selection strategies again lead to nearly the same results. Furthermore, as can be seen, with increasing SNR the transmission delay can be significantly reduced because the number of packets transmitted during one time interval generally increases and therefore packets in the relay buffer are served faster. However, at a certain point the curves flatten out, where the limiting value obviously is dependent on the buffer size. This is due to the fact that in the high SNR regime there is generally a non-zero probability that more than N packets are accumulated in the relay buffer. This is illustrated in Fig. 6, where we depict the cumulative distribution function of the number of packets in the relay buffer after the transmission from the source to the relay for the case that the buffer can store at most $L_{\max} = 10$ packets. For sufficiently high average SNRs, in most cases the highest-order AMC mode is chosen, so that mostly N packets are put into the relay buffer after the transmission from S to R while at the same time usually N packets are forwarded again during the second time slot. However, if it happens only once that on the R-D link a lower-order MCS has to be chosen, the number of packets in the relay buffer quickly exceeds N and is only reduced again to exactly N if the S-R link is once also getting very poor. Since this happens in the high SNR regime only very rarely as well, a situation with more than N packets in the relay buffer represents a stable state. For that reason, it is not always possible to immediately forward all packets received from the source in the same transmission interval towards the destination, even if almost always the highest AMC mode is selected on the R-D link, thus leading to a transmission delay larger than one. We also notice that the delay floor is an increasing function of the buffer size. Intuitively, with larger buffer sizes the probability that packets are accumulated in the buffer is increasing as well.

The average packet loss rate for both AMC mode selection strategies is considered in Fig. 7. Unlike for the average transmission efficiency and the average delay performance, we obtain clearly two distinct sets of curves for the two different strategies. Note that with the buffer-aware scheme, packets are only dropped due to decoding errors whereas with the pure SNR-based approach packets might be also discarded at the relay station due to a buffer overflow. As can be seen, the packet loss rate is as expected generally smaller for the buffer-aware approach and in this case the impact of the buffer size is very small. In fact, small buffer sizes lead to a slightly smaller loss rate because in this case more often

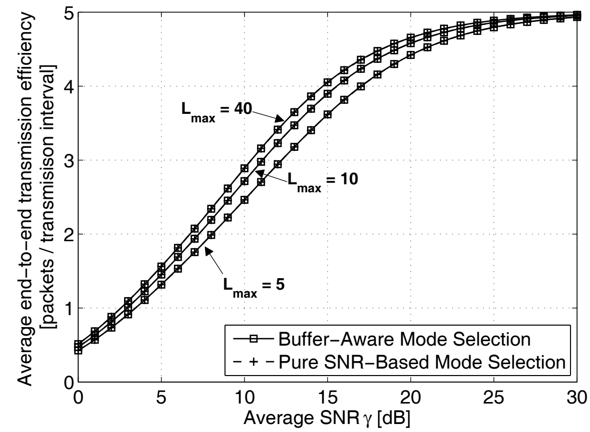

Fig. 4. Average end-to-end transmission efficiency in packets per transmission interval for various buffer sizes as a function of the average SNR $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$ with Rayleigh-fading on both hops and $P_{\text{target},1} = P_{\text{target},2} = 0.1$.

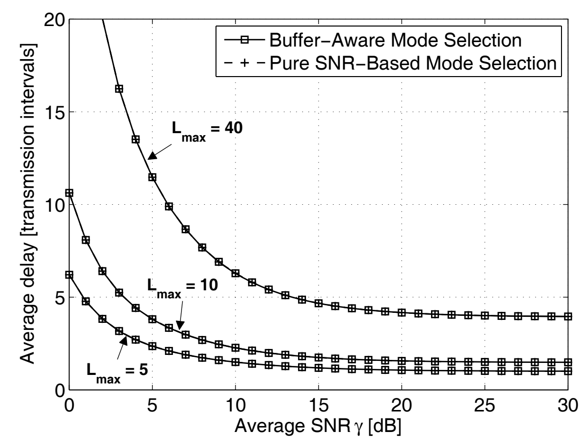

Fig. 5. Average end-to-end transmission delay for various buffer sizes as a function of the average SNR $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$ with Rayleigh-fading on both hops and $P_{\text{target},1} = P_{\text{target},2} = 0.1$.

a more robust AMC mode is chosen than supported by the corresponding SNRs, thus leading to a reduction of the packet loss rate. For the pure SNR-based approach, it is exactly the other way around. Here, larger buffer sizes are advantageous since the main contribution to the packet loss rate comes from packets discarded at the relay station due to a buffer overflow and by increasing the buffer size, the probability that such an overflow occurs can be significantly reduced. Considering that the average transmission efficiency is virtually the same for both AMC selection schemes while at the same time the average packet loss rate is smaller for the buffer-aware approach, it is quite evident that the buffer-aware approach is superior to the the pure-SNR based strategy, especially for FDD systems where certain feedback is mandatory in any case.

VI. Conclusion

We have proposed and thoroughly analyzed two different approaches for performing adaptive modulation and coding in dual-hop transmission systems with one regenerative decode-and-forward relay. With the first approach, an appropriate modulation and coding scheme is selected solely based on the

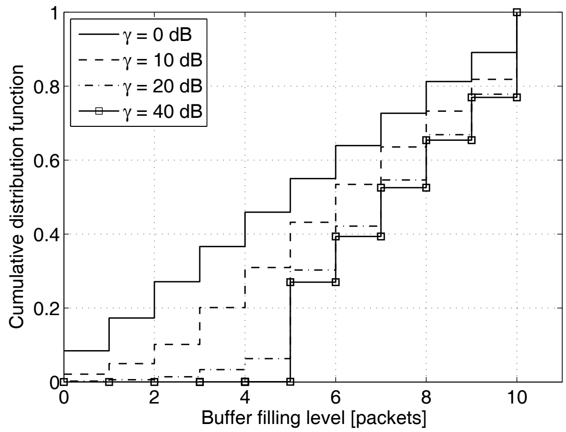

Fig. 6. Cumulative distribution function of the buffer filling level after the transmission from S to R for different average SNRs $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$ as well as Rayleigh-fading on both hops, $L_{\max} = 10$, and $P_{\text{target},1} = P_{\text{target},2} = 0.1$.

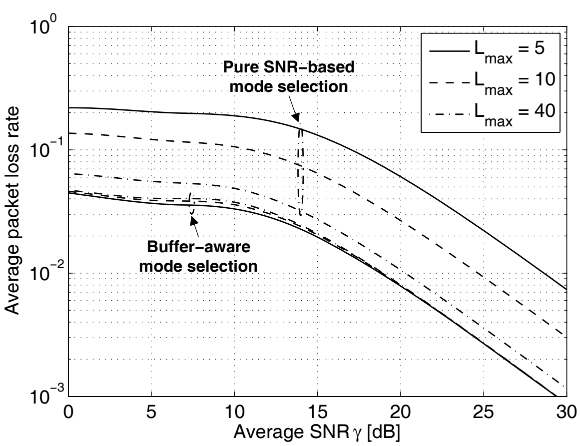

Fig. 7. Average overall packet loss rate for both considered mode selection strategies as a function of the average SNR $\bar{\gamma}_1 = \bar{\gamma}_2 = \gamma$ with Rayleigh-fading on both hops and $P_{\text{target},1} = P_{\text{target},2} = 0.1$.

current channel conditions whereas with the second one also the buffer filling level at the relay station is taken into account, thereby avoiding possible buffer overflows and hence generally reducing the number of packet losses. The performance of both schemes has been investigated by means a finite-state Markov model of our system, based on which we have derived analytical closed-form expressions for a variety of different performance indicators such as the transmission efficiency or the end-to-end transmission delay. Interestingly, it turned out that the distribution of the number of packets in the relay buffer is for both mode selection strategies almost the same, which in turn leads to a nearly identical system transmission efficiency and delay performance. On the other hand, the buffer-aware strategy exhibits a significant advantage regarding the average packet loss rate by eliminating possible buffer overflows at the relay station and as such it constitutes a viable implementation approach for adaptive relayed transmission systems in practice. As an extension of this work, we are currently investigating the performance of AMC for dual-hop systems combined with various ARQ protocols.

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