Joint Base Station Association and Power Allocation for Uplink Sum-Rate Maximization

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***Abstract*—In this paper, the problem of sum-rate maximization with Quality of Service (QoS) for a multi-cell multi-user uplink is addressed. The problem is formulated as a Mixed Integer Nonlinear Programming (MINLP) problem with non-convex feasible region and hence is difficult to solve. A primal-dual infeasible-interior-point method (IIPM) is applied to jointly optimize resources. With this method, simultaneously optimizing the Base Station Association (BSA) and Power Allocation (PA) is possible. Further to reduce the size, the problem is decomposed into two subproblems. The NP-hard Integer Programming (IP) BSA subproblem from the decomposition is solved by two different methods. One method uses the IIPM and other uses a Semidefinite Programming formulation. The PA subproblem is solved iteratively by IIPM. Simulation results converge to the optimum obtained by an existing exhaustive search. Apart from the sum-rate objective, the IIPM is applicable to broad class of utility functions and objectives and it also eliminates the requirement of an initial primal feasible point to begin the algorithm.**

***Index Terms*—Base station association, power allocation, mixed integer nonlinear program, primal-dual infeasible-interior-point method.**

I. Introduction

Spectrum scarcity, underutilization of network resources, increase in the demand for wireless services make the process of interference management and resource allocation crucial. With the User Equipments (UEs) operating on the same radio resource, joint optimization of the network degrees-of-freedom e.g., PA, BSA and beamforming should be considered to realize the potential advantages. Also the broadcast nature of the shared wireless channel causes considerable interference among the network entities thereby deteriorating the performance. Controlling the multi-user interference (MUI) and providing a reliable QoS while maximizing the spectral efficiency is important from the network operator's point of view. Due to the MUI, achieving the required QoS for a UE is coupled with the other UE powers. Utility based resource allocation is a good approach to deal with QoS issue. Resource allocation and management is even more challenging in Heterogeneous Networks (HetNets) since the access points are plug-and-play type and the network topology is unplanned.

The problem of uplink sum-rate maximization with QoS for a single cell is well understood and several iterative algorithms exist e.g., [1–4]. Additional constraints like subcarrier assignment and user fairness is addressed in them. In some cases, if different BSs in a multi-cell case are identified as different subcarriers then the above algorithms can be extended with a modification. Else the problem has to be evaluated for all possible BSA combinations which is practically not viable. For a multi-cell case of HetNets the problem was solved via difference-of-convex (DC) functions programming approach in [5]. However DC programming requires the problem to have a special structure, making it difficult to be extended for other utility functions. In [6] throughput maximization with Femtocell compensation has been solved as a multiobjective problem by Branch and Bound (BB) method. The enumeration in BB may lead to an exponential increase in computational complexity.

In this paper a primal-dual infeasible-interior-point method [7, 8] is applied to solve the problem of sum-rate maximization with QoS. Compared to the other methods the advantage of this method is that it can be applied to a broad class of utility functions and objectives. It also eliminates the requirement of an initial primal feasible point to begin the algorithm. Choosing as initial point for as algorithm may at times be as hard as solving the original problem. IIPM to find the local optimum solves only a series of linear equations. To reduce the size, the problem is decomposed into two subproblems. One subproblem each for BSA and PA. In this paper the NP-hard IP subproblem of BSA is efficiently solved using two different methods. One is based on iterative IIPM while the other on Semidefinite Programming (SDP) formulation. The actual problem is converted to a Nonlinear Programming (NLP) problem by adding new constraints. The same IIPM is used for PA. Further, the IIPM simultaneously solves PA and BSA subproblems in a single stage without separating them.

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Section II explains the system and problem formulation. Section III-A explains the primal-dual IIPM framework. This framework is applied for PA in Section III-B, for BSA in Section III-C and for simultaneous PA and BSA in Section III-F. In Section III-D the BSA subproblem is solved using a convex formulation.

II. System and Problem Formulation

N single transmit antenna UEs (UE_j, $j = 1, 2, ..., N$) communicate with one of the M BSs (BS_i, $i = 1, 2, ..., M$) each equipped with N_r receive antenna elements. With uncorrelated symbols and noise, the Signal to Interference Noise Ratio (SINR) γ_{ij} is given by

$$\gamma_{ij} = \frac{p_j \mathbf{u}_{ij}^H \mathbf{h}_{ij} \mathbf{h}_{ij}^H \mathbf{u}_{ij}}{\sum_{k \neq j} p_k \mathbf{u}_{ij}^H \mathbf{h}_{ik} \mathbf{h}_{ik}^H \mathbf{u}_{ij} + \sigma_i^2}. \tag{1}$$

p_j is the transmit power of UE_j. $\mathbf{h}_{ij}$ is the $N_r \times 1$ flat fading channel between the UE_j and BS_i. Elements of $\mathbf{h}_{ij}$ are independent and identically distributed (i.i.d.) circular symmetric complex Gaussian random variables with zero mean and unit variance. Additive White Gaussian Noise (AWGN) at BS_i has zero mean and variance σ_i^2. $\mathbf{u}_{ij}$ is the $N_r \times 1$ receive beamforming vector with the l_2-norm $\|\mathbf{u}_{ij}\|_2 = 1$. Linear Minimum Mean Square Error (MMSE) beamforming vectors [9] are considered and are given by

$$\mathbf{u}_{ij} = \left(\sigma_i^2 \mathbf{I}_{N_r} + \sum_{k=1, k \neq j}^{N} p_k \mathbf{h}_{ik} \mathbf{h}_{ik}^H \right)^{-1} \mathbf{h}_{ij}, \tag{2}$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix, $(\cdot)^H$ is the Hermitian and $(\cdot)^{-1}$ is the matrix inverse. $\mathbf{U} = [\mathbf{u}_{11}, \mathbf{u}_{21}, ..., \mathbf{u}_{MN}]$ is the overall MMSE matrix. Rate r_{ij} is given by

$$r_{ij} = \log_2(1 + \gamma_{ij}). \tag{3}$$

The association of UE_j with BS_i is given by an integer variable α_{ij}. If UE_j is associated with BS_i $\alpha_{ij} = 1$ else $\alpha_{ij} = 0$ i.e.,

$$\alpha_{ij} \in \{0, 1\}, \quad \forall i, \forall j. \tag{4}$$

Though it is a multi-BS reception system, macrodiversity is not considered. Each UE is associated with only one BS. It is expressed as

$$\sum_{i=1}^{M} \alpha_{ij} = 1, \quad \forall j. \tag{5}$$

The considered QoS metric is the UE rate and the constraint is given as

$$\sum_{i=1}^{M} \alpha_{ij} r_{ij} \geq r_{th}, \quad \forall j, \tag{6}$$

where r_{th} is the threshold rate. If (6) is infeasible then either some UEs are dropped or the threshold is lowered. In [5] this was tackled by an l_1-norm heuristic. With P_{max} as the maximum transmit power the bounded UE power is

$$p_j \in [0, P_{max}], \quad \forall j. \tag{7}$$

For UE_j, $\boldsymbol{\alpha}_j = [\alpha_{1j}, \alpha_{2j}, ..., \alpha_{Mj}]^T$ is the BSA vector, $\mathbf{r}_j = [r_{1j}.r_{2j}, ..., r_{Mj}]^T$ is the rate vector. Overall BSA and power vectors are $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^T, \boldsymbol{\alpha}_2^T, ..., \boldsymbol{\alpha}_N^T]^T$ and $\mathbf{p} = [p_1, p_2, ..., p_N]^T$ respectively. $(\cdot)^T$ is the transpose.

The MINLP optimization problem of sum-rate maximization with QoS is given by

$$\text{P1:} \quad \underset{\mathbf{p}, \boldsymbol{\alpha}}{\text{maximize}} \quad \sum_{i=1}^{M} \sum_{j=1}^{N} \alpha_{ij} r_{ij} \tag{8a}$$

$$\text{s.t.} \quad (4), (5), (6), (7). \tag{8b}$$

The objective in (8a) is the sum-rate expression. To reduce the problem size of P1, the integer and continuous variables are separated leading to a two stage problem formulation. For a given BSA, P1 becomes an NLP optimization problem (P1_a) in continuous variable $\mathbf{p}$.

$$\text{P1}_a\text{:} \quad \underset{\mathbf{p}}{\text{maximize}} \quad \sum_{i=1}^{M} \sum_{j=1}^{N} \alpha_{ij} r_{ij} \tag{9a}$$

$$\text{s.t.} \quad (6), (7). \tag{9b}$$

For a given PA, P1 becomes a 0-1 linear IP optimization problem (P1_b) in $\boldsymbol{\alpha}$.

$$\text{P1}_b\text{:} \quad \underset{\boldsymbol{\alpha}}{\text{maximize}} \quad \sum_{i=1}^{M} \sum_{j=1}^{N} \alpha_{ij} r_{ij} \tag{10a}$$

$$\text{s.t.} \quad (4), (5), (6). \tag{10b}$$

Constraints w.r.t., $\mathbf{U}$ are not explicitly mentioned in (8) and (9) since $\mathbf{U}$ is updated by (2) whenever there is a change in PA.

III. Problem Solving

A. Algorithm Framework

The IIPM from [7, 8] is considered. It solves a constrained NLP optimization problem as a series of linear equations. Consider an optimization problem P2 in N_1 nonnegative primal variables $\mathbf{z}$, with objective $f_0(\mathbf{z})$. N_2 constraints of N UEs are rearranged in vector form as $\mathbf{c}(\mathbf{z}) = [\mathbf{c}_1^T(\mathbf{z}), \mathbf{c}_2^T(\mathbf{z}), ..., \mathbf{c}_N^T(\mathbf{z})]^T$. It is written as

$$\text{P2:} \quad \underset{\mathbf{z}}{\text{minimize}} \quad f_0(\mathbf{z}) \tag{11a}$$

$$\text{s.t.} \quad \mathbf{c}(\mathbf{z}) - \mathbf{s} = 0, \tag{11b}$$

where $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, ..., \mathbf{s}_N^T]^T$ is the slack variable vector. The inequality and equality constraints in $\mathbf{c}(\mathbf{z})$ are arranged such that $\mathbf{s}$ is nonnegative. A Lagrangian is formed for P2 and is given as

$$L(\mathbf{z},\mathbf{s},\boldsymbol{\lambda}) = f_0(\mathbf{z}) - \boldsymbol{\lambda}^T(\mathbf{c}(\mathbf{z}) - \mathbf{s}). \tag{12}$$

$\boldsymbol{\lambda}$ is the nonnegative vector of Lagrangian variables corresponding to the dual feasibility. Along with the non-negativity of $\mathbf{s}$ and $\boldsymbol{\lambda}$, the Karash-Kuhn-Tucker (KKT) conditions in the vector form are

$$\mathbf{F} = \begin{bmatrix} \nabla_{(\mathbf{z},\mathbf{s},\boldsymbol{\lambda})} L(\mathbf{z},\mathbf{s},\boldsymbol{\lambda}) \\ \mathbf{c}(\mathbf{z}) - \mathbf{s} \\ \mathbf{D}_{\boldsymbol{\lambda}}\mathbf{D}_{\mathbf{s}}\mathbf{1}_{N_2} \end{bmatrix} = 0. \tag{13}$$

The first equation set in $\mathbf{F}$ is the first order derivative condition w.r.t., $\mathbf{z}$, $\mathbf{s}$, $\boldsymbol{\lambda}$. $\nabla(\cdot)$ is the gradient. The second equation set is the primal feasibility condition and the third equation is the complementary slackness condition. $\mathbf{1}_{N_2}$ is the $N_2 \times 1$ vector with all elements equal to unity. $\mathbf{D_s}$, $\mathbf{D}_{\boldsymbol{\lambda}}$ are the diagonal matrices with $\mathbf{s}$ and $\boldsymbol{\lambda}$ in their diagonal respectively.

With t as the iteration index and given initial values of $\mathbf{z}$, $\mathbf{s}$, $\boldsymbol{\lambda}$ the following steps constitute the algorithm. Search directions $[\Delta_{\mathbf{z}}^T, \Delta_{\mathbf{s}}^T, \Delta_{\boldsymbol{\lambda}}^T]^T$ called the Newton directions for the next iteration are evaluated as

$$\mathbf{J}_t[\Delta_{\mathbf{z}_{t+1}}^T, \Delta_{\mathbf{s}_{t+1}}^T, \Delta_{\boldsymbol{\lambda}_{t+1}}^T]^T = \mathbf{F}_t + [\mathbf{0}_{N_1}^T, -\mu_t \mathbf{1}_{N_2}^T, \mathbf{0}_{N_2}^T]^T \tag{14}$$

where $\mathbf{J}$ is the Jacobian of $\mathbf{F}$ w.r.t., $[\mathbf{z}^T, \mathbf{s}^T, \boldsymbol{\lambda}^T]^T$, $\mu_t = \frac{\boldsymbol{\lambda}_t^T \mathbf{s}_t}{N_2}$ is a complementarity measure. The system of equations in (14) form a perturbed KKT system. The variables are updated as

$$[\mathbf{s}_{t+1}^T, \mathbf{z}_{t+1}^T]^T = \left[\mathbf{s}_t^T, \mathbf{z}_t^T\right]^T + \delta_{\mathbf{s}_{t+1}}[\Delta_{\mathbf{s}_{t+1}}^T, \Delta_{\mathbf{z}_{t+1}}^T]^T, \tag{15a}$$
$$\boldsymbol{\lambda}_{t+1} = \boldsymbol{\lambda}_t + \delta_{\boldsymbol{\lambda}_{t+1}} \Delta_{\boldsymbol{\lambda}_{t+1}}^T, \tag{15b}$$

where the step sizes $\delta_{\mathbf{s}_{t+1}}$ and $\delta_{\boldsymbol{\lambda}_{t+1}}$ are a solution to

$$\max\left\{\delta_{\mathbf{s}_{t+1}} | \delta_{\mathbf{s}_{t+1}} \in (0,1], \mathbf{s}_{t+1} \geq (1-\tau)\mathbf{s}_t\right\}, \tag{16a}$$
$$\max\left\{\delta_{\boldsymbol{\lambda}_{t+1}} | \delta_{\boldsymbol{\lambda}_{t+1}} \in (0,1], \boldsymbol{\lambda}_{t+1} \geq (1-\tau)\boldsymbol{\lambda}_t\right\}, \tag{16b}$$

and $\tau \in (0,1)$. These iterations called Newton iterations are repeated till the required convergence is met. The described sequence of steps are given in Algorithm-1.

The problem is assumed feasible. The involved functions are continuous and continuously differentiable. At optimal values $(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ the conditions of Strict Complementarity and Linear Independence Constraint Qualification (LICQ) should hold. The Lagrangian bound $L(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ is equal to the primal optimum $f(\mathbf{z}^*)$ as $\mu \to 0$ and $\nabla_{(\mathbf{z},\mathbf{s},\boldsymbol{\lambda})} L \to 0$. The problem in deemed infeasible if the maximum iterations is exceeded or when $L(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*) \to \infty$. These conditions can be used as the convergence and exit criteria for the algorithm. The problem is primal infeasible till final convergence. But the iterates are interior to the nonnegative orthant ($\mathbf{s}$, $\boldsymbol{\lambda}$). These two statements explain the name infeasible and interior-point in IIPM. Problem P2 in the given form is a minimization problem so the negative of objectives in (8), (9), (10) should be taken and the maximize definition is replaced by minimize.

Algorithm 1

Initialize: $t = 0, \mathbf{z}_t, \tau, (\boldsymbol{\lambda}_t, \mathbf{s}_t) > 0$;
repeat
  evaluate $L(\mathbf{z},\mathbf{s},\boldsymbol{\lambda})$, $\nabla_{(\mathbf{z},\mathbf{s},\boldsymbol{\lambda})} L$, μ_t, $\mathbf{F}_t$, $\mathbf{J}_t$;
  $t = t + 1$;
  solve (14) to obtain new search directions;
  find step size from (16);
  evaluate (15) to update variables;
until convergence
Output: $\mathbf{z}$.

B. Iterative Power Allocation

To solve P1$_a$ by Algorithm-1, the primal variable $\mathbf{z}$ is replaced by $\mathbf{p}$ and the constraint vector for UE$_j$ is

$$\mathbf{c}_j(\mathbf{p}) = \begin{bmatrix} P_{max} - p_j \\ \mathbf{1}_M^T \mathbf{r}_j - r_{th} \end{bmatrix}. \tag{17}$$

The first constraint arises due to the upper bound on variable p_j. As mentioned before variable $\mathbf{U}$ is implicit.

C. Iterative Base Station Association

To solve P1$_b$, Lagrangian Relaxations in [10] can be used. They are based on dualizing the constraints and the solution is obtained by subgradient or BB techniques. Under some strict conditions they may converge but rather slowly to an inferior bound or may require enumeration. Algorithm-1 can be applied to solve P1$_b$ after converting it to an NLP problem (P1$_{b1}$). To formulate P1$_{b1}$ new constraints are added for the integer variables. Each α_{ij} in (4) is replaced by two new constraints $\alpha_{ij}(\alpha_{ij} - 1) = 0$ and $\alpha_{ij} \in [0,1]$. α_{ij} is now a continuous variable in the unit interval. In this unit interval

$$\alpha_{ij}\left(\alpha_{ij} - 1\right) \leq 0, \quad \forall i, \forall j. \tag{18}$$

Primal variable $\mathbf{z}$ is replaced by $\boldsymbol{\alpha}$. The constraint vector for UE$_j$ is

$$\mathbf{c}_j(\boldsymbol{\alpha}) = \begin{bmatrix} -\alpha_{1j}(\alpha_{1j} - 1) \\ \vdots \\ -\alpha_{Mj}(\alpha_{Mj} - 1) \\ 1 - \mathbf{1}_M^T \boldsymbol{\alpha}_j \\ \mathbf{1}_M^T \mathbf{r}_j - r_{th} \end{bmatrix}. \tag{19}$$

$(\boldsymbol{\alpha}_j^T \mathbf{r}_j - r_{th})$ instead of $(\mathbf{1}_M^T \mathbf{r}_j - r_{th})$

The constraint $(1-\alpha_{ij})$ due to the upper bound on α_{ij} is not added to $\mathbf{c}_j(\boldsymbol{\alpha})$ since the inequality in (18) is true only in this unit interval.

D. Base Station Association by Convex Reformulation

By Schur Complement [11], the nonlinear constraint (18) can be written as an SDP constraint

$$\begin{bmatrix} \mathbf{D}_{\boldsymbol{\alpha}_j} & \boldsymbol{\alpha}_j \\ \boldsymbol{\alpha}_j^T & 1 \end{bmatrix} \geq 0, \quad \forall j, \tag{20}$$

where $\mathbf{D}_{\boldsymbol{\alpha}_j}$ is the diagonal matrix with $\boldsymbol{\alpha}_j$ in its diagonal and

$$trace(\mathbf{D}_{\boldsymbol{\alpha}_j}) = 1, \quad \forall j. \tag{21}$$

The convex reformulation P1_{b2} of P1_b is

$$\text{P1}_{b2}: \quad \underset{\boldsymbol{\alpha}}{\text{maximize}} \quad \sum_{i=1}^{M}\sum_{j=1}^{N} \alpha_{ij} r_{ij} \tag{22a}$$

$$\text{s.t.} \quad \boldsymbol{\alpha}_j^T \mathbf{r}_j \geq r_{th}, \quad \forall j, \tag{22b}$$

$$(20), (21). \tag{22c}$$

E. Two Stage Formulation

The iterative steps given in Algorithm-2 solve the two stage problem. BSA and PA are solved alternately.

Algorithm 2

Initialize: $\mathbf{p}$;
repeat
  solve for $\boldsymbol{\alpha}$ by P1_{b1} or P1_{b2};
  break on convergence;
  solve for $\mathbf{p}$ by P1_a;
  break on convergence;
until final convergence
Output: $\mathbf{p}, \boldsymbol{\alpha}$.

F. Single Stage Formulation

Algorithm-1 can be directly applied to solve P1 to simultaneously solve the BSA and PA subproblems. $\mathbf{p}$ and $\boldsymbol{\alpha}$ are the primal variables. Constraint vector $\mathbf{c}_j(\mathbf{p}, \boldsymbol{\alpha})$ is obtained by adding constraint $[P_{max} - p_j]$ to (19). Since the problem size is big when compared to the two stage formulation, both time and space complexity increase for each iteration.

IV. Numerical Results

Throughout the work $N_r = 2$, $\tau = 0.9$, $P_{max} = 0.1$, $\sigma_i^2 = 1$ and the initial values of $\mathbf{s}$, $\boldsymbol{\lambda}$ are such that $\mu = 0.25$. Perfect channel state information at the BSs and cooperation among the BSs is assumed. For comparison, P1_a is solved by Geometric Programming (GP) formulation [1] with single condensation method [12] for all the M^N combinations of BSA. Also for a given PA, P1_b is verified by IP solver CPLEX. The solution obtained for feasible cases is the same. Numerical results show that for a feasible instance if the conditions in Section III-A are satisfied then the IIPM will definitely converge and $\boldsymbol{\alpha}$ converges to a 0-1 solution. Initial values and required tolerance define the convergence.

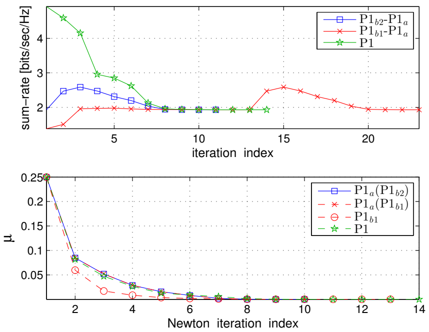

Fig. 1. Comparison of the three methods and convergence of μ

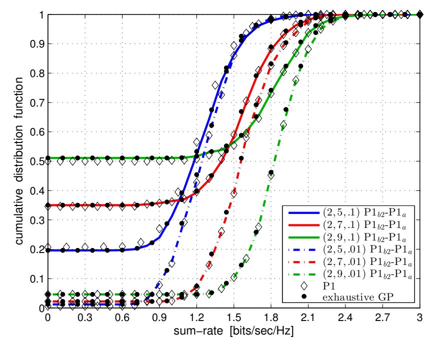

Fig. 2. cumulative distribution function of sum-rate

Fig.1 compares the discussed methods for a feasible instance of $M = 2, N = 8, r_{th} = 0.1$. Convergence of μ during Newton-iterations for P1, P1_a and P1_{b1} is also shown. Initialized values need not satisfy their bounds e.g., $p_j = 0.2, \alpha_{ij} = 1.5$ or 2 can be taken. Only the non-negativity of $\mathbf{s}$ and $\boldsymbol{\lambda}$ must be ensured throughout. P1_{b2}-P1_a converges before P1, but it cannot be generalized for every feasible case. The Lagrangian bound for P1 begins at a higher value since P1 has more variables to be initialized. Usually P1_{b1}-P1_a takes additional iterations to converge because μ is reinitialized to 0.25 each time the algorithm enters Newton-iteration to solve via IIPM. In Fig.1 there are two Newton-iterations for P1_{b1}-P1_a, one each for P1_{b1} and P1_a. After solving P1_{b1} by IIPM $\mu \to 0$. So μ is reinitialized to solve P1_a by IIPM (shown as $\text{P1}_a(\text{P1}_{b1})$) and again $\mu \to 0$. The upward rise in P1_{b1}-

$P1_a$ after almost converging is due to this reinitialization. Convergence of μ need not be monotonic.

Fig.2 shows the cumulative distribution function (cdf) of the sum-rate with different (M, N, r_{th}) for two cases of low QoS ($r_{th} = 0.01$) and high QoS ($r_{th} = 0.1$). Only $P1_{b2}$-$P1_a$ and P1 are considered since $P1_{b1}$-$P1_a$ requires more iterations. For this cdf plot, a right shift is an increase in objective value and an upward shift is an increase in the infeasibility. For a given (M, N, r_{th}), the plots for P1, $P1_a$-$P1_{b2}$ (except for P1 at high QoS) have the same optimal as the corresponding exhaustive GP. Increased infeasibility due to the chosen initial points is the reason for the deviation of P1 at high QoS. Different initial values e.g., $\mu = 1$ or 2, $\alpha_{ij} = 2, \forall i, j$ can rectify this. Increased interference causes a left and upward shift in the high QoS cdf plots compared to the corresponding low QoS plots. For a given r_{th}, with the increase in N the increased interference causes the plots shift right and upward. These explain the increased interference with QoS and N. Choosing an initial point for exhaustive GP was not easy, a feasible instance could easily be rendered unfeasible with an improper initial choice.

V. Conclusion

A primal-dual infeasible interior point method has been applied to solve the problem of sum-rate maximization with QoS for the multi-cell multi-user uplink. The MINLP problem is solved in a two stage formulation by separating the BSA and PA variables and also by a single stage formulation where BSA and PA are solved simultaneously. Simplified approaches to solve the NP-hard IP BSA problem has been shown where iterative IIPM and single step convex reformulation are used. Among the discussed methods $P1_a$-$P1_{b2}$ is efficient in terms of problem size and iterations. In any case the MINLP problem is converted to an NLP problem. As long as the problem is feasible, it is possible to obtain an optimal BSA and PA by the IIPM.

This IIPM has several advantages. The need to choose an initial primal feasible point is eliminated. Simultaneously solving for BSA and PA is possible. Additional recovery of the 0-1 BSA variables is not required. It can be applied to a broad class of utility functions though only sum-rate utility maximization has been considered. By replacing the objective (8a) in (8) with $-\mathbf{1}_N^T\mathbf{p}$, the problem of sum-power minimization can be solved without any modification to the algorithm. Though IIPM here has been applied to a conventional setup it can be applied to HetNets assuming open access to the Femtocells and also to a multi-carrier resource allocation problem, if different BSs are identified as different subcarriers.

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