Joint Base Station Association and Power Allocation for Uplink Sum-Power Minimization

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***Abstract*—In this paper, the problem of sum-power minimization with Quality of Service (QoS) for a multi-cell multi-user uplink is addressed. This Power Control (PC) problem is formulated as a Mixed Integer Non-linear Programming (MINLP) optimization problem and solved using a primal-dual infeasible-interior-point method (IIPM) after obtaining an equivalent non-linear programming (NLP) problem. Feasibility issues are handled by l_1-norm heuristics. Most of the iterative algorithms solve Power Allocation (PA) and Base Station Association (BSA) alternately with one of them fixed. With IIPM, PA and BSA are solved simultaneously thereby reducing it to a single stage problem. Simulation results are balanced as in the max-min case and converge to the optimum obtained by an existing exhaustive search. The advantage of l_1-norm infeasibility handing is also evident.**

***Index Terms*—Base station association, power allocation, mixed integer nonlinear program, primal-dual infeasible-interior-point method.**

I. Introduction

Spectrum scarcity, underutilization of network resources, increase in the demand for wireless services make the process of interference management and resource allocation crucial. With the user equipments (UEs) operating on the same radio resource, joint optimization of the network degrees-of-freedom e.g., PA, BSA and beamforming should be considered to realize the potential advantages. The broadcast nature of the shared wireless channel causes considerable interference among the network entities thereby deteriorating the performance. It couples the performance of each UE to every other UE performance making the multi-user network interference-limited. With coordination among the network entities, the technique of PC regulates the transmission power to provide an acceptable service and limits the interference perceived. This also achieves a balanced QoS among homogeneous UEs. Other advantages of PC include minimizing battery drain i.e., energy consumption of the UE, increase the number of active UEs and solving cross-layer design issues. PC over multiple cells is difficult since the BSA also determines the power level of each UE. Resource allocation and management is even more challenging in heterogeneous networks since the access points are plug-and-play type and the network topology is unplanned.

The considered QoS metric is UE rate. The rate QoS constraints can be reformulated as Signal to Interference Noise Ratio (SINR) constraints that are linear in the power variable for a given BSA. The PC problem is well understood in literature. The problem of sum-power minimization under SINR constraints is coupled to the problem of maximizing the minimum SINR i.e., the max-min problem where the objective is to achieve the same SINR level across all the UEs. The feasibility of the later leads to formulation of the former and finding a solution as in [1]. Solution to the max-min problem is equivalent to solving the PC problem. SINR balancing with unconstrained power was solved in [2] by a centralized approach and in [3] by an asynchronous approach. [3] shows some properties of the interference constraints, thus establishing the existence of iterative algorithms to solve the PC problem. The asynchronous approach is based on Fixed-Point Theory and the definition of interference as a standard function. The required monotonicity for standard interference function may not hold all the time limiting the use of this approach. Also the transmit power increases without bound as each user approaches the maximum attainable threshold. In [4] the asynchronous method is extended to the constrained power. Under sum-power constraint this problem was explained in [5], [1]. However maximum possible threshold is evaluated before applying the framework. To extend these to a multi-cell case, they must be solved for every BSA as in [6]. As the number of network entities increase this approach is practically not viable. In [7] joint PC, BSA and beamforming was considered. PA is implemented by distributed approach. However in this approach after every PA, each UE scans its BS set to change the association. In the worst case during each iteration there could be a different BSA for every UE. The problem along with macrodiversity is considered in [8]. To avoid exhaustive BSA-UE search, the BSA set for each UE is chosen based on the large scale fading factors thereby degrading the performance. The feasibility in all the cases is given by the spectral radius of an irreducible non-negative coupling matrix that includes network-wide parameters and the existence of a non-negative power vector is given by Perron-Frobenius Theory. Structure of the coupling matrix is different when the sum-power constraint is considered. Other extensions related to PC can be found in [9].

Mathematically the problem of BSA, PA and receive beamforming can be formulated as an MINLP optimization problem. For a fixed PA, BSA is an NP-hard Integer Programming (IP) problem. The non-convexity arises due to these discrete constraints. Further, the problem of PA is possible only if there is a given BSA. A primal-dual IIPM [10] is applied to simultaneously solve the BSA and PA subproblems. Section II gives the system model and problem formulation. NLP reformulation of the problem, IIPM framework and its application are explained in Section III-A. Section III-B gives the infeasibility handing by l_1-norm heuristic. Numerical results are explained in Section IV.

II. System and Problem Formulation

N single transmit antenna UEs (UE$_j$, $j = 1, 2, ..., N$) communicate with M BSs (BS$_i$, $i = 1, 2, ..., M$) each equipped with N_r receive antenna elements. With uncorrelated symbols and noise, the SINR γ_{ij} is given by

$$\gamma_{ij} = \frac{p_j \mathbf{u}_{ij}^H \mathbf{h}_{ij} \mathbf{h}_{ij}^H \mathbf{u}_{ij}}{\sum_{k \neq j} p_k \mathbf{u}_{ij}^H \mathbf{h}_{ik} \mathbf{h}_{ik}^H \mathbf{u}_{ij} + \sigma_i^2}. \tag{1}$$

p_j is the transmit power of UE$_j$. $\mathbf{h}_{ij}$ is the $N_r \times 1$ flat fading channel between the UE$_j$ and BS$_i$. Elements of $\mathbf{h}_{ij}$ are independent and identically distributed (i.i.d.) circular symmetric complex Gaussian random variables with zero mean and unit variance. Additive White Gaussian Noise at BS$_i$ has zero mean and variance σ_i^2. $\mathbf{u}_{ij}$ is the $N_r \times 1$ receive beamforming vector with the l_2-norm $\|\mathbf{u}_{ij}\|_2 = 1$. Minimum Mean Square Error (MMSE) beamforming vectors are considered.

$$\mathbf{u}_{ij} = \left(\sigma_i^2 \mathbf{I}_{N_r} + \sum_{k=1, k \neq j}^{N} p_k \mathbf{h}_{ik} \mathbf{h}_{ik}^H \right)^{-1} \mathbf{h}_{ij}, \tag{2}$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix, $(\cdot)^H$ is the Hermitian and $(\cdot)^{-1}$ is the inverse. $\mathbf{U} = [\mathbf{u}_{11}, \mathbf{u}_{21}, ..., \mathbf{u}_{MN}]$ is the overall MMSE matrix. Rate r_{ij} is given by

$$r_{ij} = \log_2(1 + \gamma_{ij}). \tag{3}$$

The association of UE$_j$ with BS$_i$ is given by an integer variable α_{ij}. If UE$_j$ is associated with BS$_i$ then $\alpha_{ij} = 1$ else $\alpha_{ij} = 0$ i.e.,

$$\alpha_{ij} \in \{0, 1\}, \quad \forall i, \forall j. \tag{4}$$

Though it is a multi-BS reception system, macrodiversity is not considered. Each UE is associated with only one BS. It is expressed as

$$\mathbf{1}_M^T \boldsymbol{\alpha}_j = 1, \quad \forall j. \tag{5}$$

$\mathbf{1}_M$ is an $M \times 1$ vector with all elements equal to unity. BSA vector of UE$_j$ is $\boldsymbol{\alpha}_j = [\alpha_{1j}, \alpha_{2j}, ..., \alpha_{Mj}]^T$. QoS constraint for UE$_j$ with threshold rate r_{th} and rate vector $\mathbf{r}_j = [r_{1j}, r_{2j}, ..., r_{Mj}]^T$ is

$$\boldsymbol{\alpha}_j^T \mathbf{r}_j \geq r_{th}, \quad \forall j. \tag{6}$$

With maximum transmit power P_{max}, UE$_j$ power is

$$p_j \in [0, P_{max}], \quad \forall j. \tag{7}$$

Overall power and BSA vectors are $\mathbf{p} = [p_1, p_2, ..., p_N]^T$ and $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^T, \boldsymbol{\alpha}_2^T, ..., \boldsymbol{\alpha}_N^T]^T$ respectively. $(\cdot)^T$ is the transpose. Further, primal variables $\boldsymbol{\alpha}$ and $\mathbf{p}$ are taken into an $N_1 \times 1$ vector $\mathbf{z} = [\mathbf{p}^T, \boldsymbol{\alpha}^T]^T$. Constraints w.r.t., $\mathbf{U}$ are not explicitly mentioned since $\mathbf{U}$ is updated by (2) whenever PA changes. The MINLP optimization problem is given by

$$\text{P1:} \quad \underset{\mathbf{z}}{\text{minimize}} \quad f_0(\mathbf{z}) = \mathbf{1}_N^T \mathbf{p} \tag{8a}$$

$$\text{s.t.} \quad (4), (5), (6), (7). \tag{8b}$$

III. Problem Solving

A. IIPM and application to Sum-Power minimization

IIPM framework [10] has been applied to solve uplink multi-cell sum-rate maximization with QoS in [11]. To solve P1 with IIPM, the MINLP problem should be converted to an NLP problem in continuous variables. The IIPM solves constrained NLP problem as a series of linear equations. For NLP equivalent, integer variables (4) are replaced by new constraints

$$\alpha_{ij}(\alpha_{ij} - 1) = 0 \quad \forall i, j, \tag{9a}$$

$$\alpha_{ij} \in [0, 1] \quad \forall i, j. \tag{9b}$$

α_{ij} is now a continuous variable in the unit interval. The reformulated NLP problem (P1$_{re}$) for P1 is given as

$$\text{P1}_{re}\text{:} \quad \underset{\mathbf{z}}{\text{minimize}} \quad f_0(\mathbf{z}) \tag{10a}$$

$$\text{s.t.} \quad (5), (6), (7), (9a), (9b). \tag{10b}$$

N_2 constraints of N UEs in (10b) are rearranged in a non-negative vector $\mathbf{c}(\mathbf{z}) = [\mathbf{c}_1^T(\mathbf{z}), \mathbf{c}_2^T(\mathbf{z}), ..., \mathbf{c}_N^T(\mathbf{z})]^T$ where constraint vector $\mathbf{c}_j(\mathbf{z})$ for UE$_j$ is

$$\mathbf{c}_j(\mathbf{z}) = \begin{bmatrix} -\boldsymbol{\alpha}_j \circ (\boldsymbol{\alpha}_j - 1) \\ 1 - \mathbf{1}_M^T \boldsymbol{\alpha}_j \\ \boldsymbol{\alpha}_j^T \mathbf{r}_j - r_{th} \\ P_{max} - p_j \end{bmatrix}. \tag{11}$$

($\circ$) is the Hadamard product. Lower bound on p_j and α_{ij} is not required in (11). In the unit interval (9b) alone (9a) is equivalent to $\alpha_{ij}\left(\alpha_{ij}-1\right) \leq 0, \quad \forall i, \forall j$. Hence the constraint $\left(1-\alpha_{ij}\right)$ due to the upper bound on α_{ij} is redundant in (11). The equality in (5) is replaced by the inequality $\leq$ since the constraints satisfying the equality will also satisfy the inequality. The last constraint arises due to the upper bound on variable p_j. So there is no increase in the number of variables and constraints. By adding non-negative slack vector $\mathbf{s}=[\mathbf{s}_1^T, \mathbf{s}_2^T, ..., \mathbf{s}_N^T]^T$, $\mathrm{P1}_{re}$ can be written as

$$
\begin{aligned}
\mathrm{P1}_s: \quad & \underset{\mathbf{z}}{\text{minimize}} \quad f_0(\mathbf{z}) && \text{(12a)}\\
& \text{s.t.} \quad \mathbf{c}(\mathbf{z})-\mathbf{s}=0, && \text{(12b)}
\end{aligned}
$$

A Lagrangian is formed for $\mathrm{P1}_s$ and is given by

$$
L(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda})=f_0(\mathbf{z})-\boldsymbol{\lambda}^T(\mathbf{c}(\mathbf{z})-\mathbf{s}). \tag{13}
$$

$\boldsymbol{\lambda}$ is the non-negative vector of Lagrangian variables corresponding to the dual feasibility. Along with the non-negativity of $\mathbf{s}$ and $\boldsymbol{\lambda}$ the Karash-Kuhn-Tucker (KKT) conditions in the vector form are

$$
\mathbf{F}=\left[\begin{array}{c}
\nabla_{(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda})} L(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda}) \\
\mathbf{c}(\mathbf{z})-\mathbf{s} \\
\mathbf{D}_{\boldsymbol{\lambda}} \mathbf{D}_{\mathbf{s}} \mathbf{1}_{N_2}
\end{array}\right]=0. \tag{14}
$$

The first equation set in $\mathbf{F}$ is the first order derivative condition w.r.t., $\mathbf{z}$, $\mathbf{s}$, $\boldsymbol{\lambda}$. $\nabla(\cdot)$ is the gradient. The second equation set is the primal feasibility condition and the third equation is the complementary slackness condition. $\mathbf{D}_{\mathbf{s}}$, $\mathbf{D}_{\boldsymbol{\lambda}}$ are the diagonal matrices with $\mathbf{s}$ and $\boldsymbol{\lambda}$ in their diagonal respectively.

With t as the iteration index and given initial values of $\mathbf{z}$, $\mathbf{s}$, $\boldsymbol{\lambda}$ the following steps constitute the algorithm. Search directions $[\Delta_{\mathbf{z}}^T, \Delta_{\mathbf{s}}^T, \Delta_{\boldsymbol{\lambda}}^T]^T$ called the Newton directions for the next iteration are evaluated as

$$
\mathbf{J}_t[\Delta_{\mathbf{z}_{t+1}}^T, \Delta_{\mathbf{s}_{t+1}}^T, \Delta_{\boldsymbol{\lambda}_{t+1}}^T]^T=\mathbf{F}_t+[\mathbf{0}_{N_1}^T,-\mu_t \mathbf{1}_{N_2}^T, \mathbf{0}_{N_2}^T]^T. \tag{15}
$$

$\mathbf{J}$ is the Jacobian of $\mathbf{F}$ w.r.t., $[\mathbf{z}^T, \mathbf{s}^T, \boldsymbol{\lambda}^T]^T$, $\mu_t=\frac{\boldsymbol{\lambda}_t^T \mathbf{s}_t}{N_2}$ is a complementarity measure. (15) forms a perturbed KKT equation system. The variables are updated as

$$
\begin{aligned}
[\mathbf{s}_{t+1}^T, \mathbf{z}_{t+1}^T]^T &= \left[\mathbf{s}_t^T, \mathbf{z}_t^T\right]^T+\delta_{\mathbf{s}_{t+1}}[\Delta_{\mathbf{s}_{t+1}}^T, \Delta_{\mathbf{z}_{t+1}}^T]^T, && \text{(16a)}\\
\boldsymbol{\lambda}_{t+1} &= \boldsymbol{\lambda}_t+\delta_{\boldsymbol{\lambda}_{t+1}} \Delta_{\boldsymbol{\lambda}_{t+1}}^T. && \text{(16b)}
\end{aligned}
$$

$\delta_{\mathbf{s}_{t+1}}$ and $\delta_{\boldsymbol{\lambda}_{t+1}}$ are step sizes and are a solution to

$$
\begin{aligned}
& \max\left\{\delta_{\mathbf{s}_{t+1}} \mid \delta_{\mathbf{s}_{t+1}} \in(0,1], \mathbf{s}_{t+1} \geq(1-\tau) \mathbf{s}_t\right\}, && \text{(17a)}\\
& \max\left\{\delta_{\boldsymbol{\lambda}_{t+1}} \mid \delta_{\boldsymbol{\lambda}_{t+1}} \in(0,1], \boldsymbol{\lambda}_{t+1} \geq(1-\tau) \boldsymbol{\lambda}_t\right\}, && \text{(17b)}
\end{aligned}
$$

and $\tau \in(0,1)$. These iterations called Newton iterations are repeated till the required convergence. The described sequence of steps are given in Algorithm-1.

Algorithm 1

Initialize: $t=0, \mathbf{z}_t, \tau,(\boldsymbol{\lambda}_t, \mathbf{s}_t)>0$;
repeat
 evaluate $L(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda})$, $\nabla_{(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda})} L$, μ_t, $\mathbf{F}_t$, $\mathbf{J}_t$;
 $t=t+1$;
 solve (15) to obtain new search directions;
 find step size from (17);
 evaluate (16) to update variables;
until convergence
Output: $\mathbf{z}$.

For the IIPM framework, some conditions must be satisfied. The problem is assumed feasible. The involved functions are continuous and continuously differentiable. At optimal values $(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ the conditions of Strict Complementarity and Linear Independence Constraint Qualification (LICQ) should hold. The Lagrangian bound $L(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*)$ is equal to the primal optimum $f_0(\mathbf{z}^*)$ as $\mu \rightarrow 0$ and $\nabla_{(\mathbf{z}, \mathbf{s}, \boldsymbol{\lambda})} L \rightarrow 0$. The problem in deemed infeasible if maximum iterations is exceeded or when $L(\mathbf{z}^*, \mathbf{s}^*, \boldsymbol{\lambda}^*) \rightarrow \infty$. These conditions are the convergence and exit criteria. Problem is primal infeasible till final convergence but the iterates are interior to the non-negative orthant ($\mathbf{s}$, $\boldsymbol{\lambda}$), hence the terms infeasible and interior in IIPM.

B. Feasibility

If the problem instance $\mathrm{P1}_s$ is infeasible then it is reasonable to serve as many QoS-satisfying UEs (UE^s) as possible while dropping the QoS-violating UEs (UE^v). To handle infeasibility of $\mathrm{P1}_s$, l_1-norm heuristic [12] is applied. A new variable $\mathbf{q}=[q_1, q_2, ..., q_N]^T$ is introduced and

$$
q_j \in[0,1] \quad \forall j. \tag{18}
$$

Further, QoS constraint (6) is modified as

$$
q_j+\boldsymbol{\alpha}_j^T \mathbf{r}_{\mathbf{j}} \geq r_{th}, \quad \forall j. \tag{19}
$$

The reformulated NLP problem ($\mathrm{P1}_f$) for P1 that handles infeasibility and can be solved via IIPM is

$$
\begin{aligned}
\mathrm{P1}_f: \quad & \underset{\mathbf{z}, \mathbf{q}}{\text{minimize}} \quad f_0(\mathbf{z})+\beta\|\mathbf{q}\|_1 && \text{(20a)}\\
& \text{s.t.} \quad \text{(5), (7), (9a), (9b), (18), (19)}\ . && \text{(20b)}
\end{aligned}
$$

β is a large positive constant. The constraint vector is

$$
\mathbf{c}_j(\mathbf{z}, \mathbf{q})=\left[\begin{array}{c}
-\boldsymbol{\alpha}_j \circ(\boldsymbol{\alpha}_j-1) \\
1-\mathbf{1}_M^T \boldsymbol{\alpha}_j \\
q_j+\boldsymbol{\alpha}_j^T \mathbf{r}_j-r_{th} \\
P_{max}-p_j \\
1-q_j \\
q_j
\end{array}\right]. \tag{21}
$$

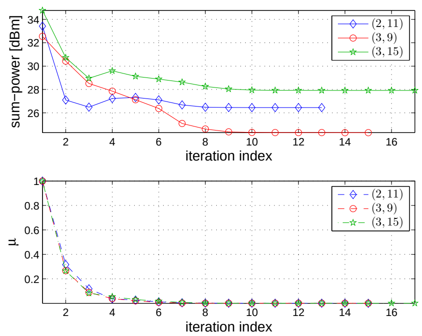

Fig. 1. convergence of sum-power objective and μ. $r_{th} = 0.1$.

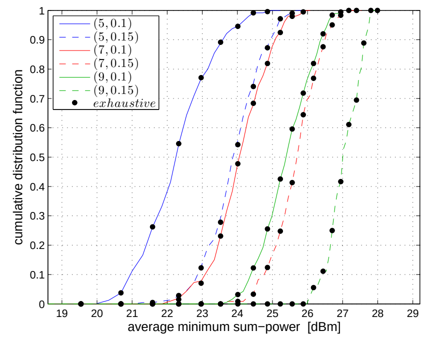

Fig. 2. cdf of minimum sum-power. $M = 2$.

Unlike the other variables, the lower bound of q_j has to be included in (21). For a feasible instance, P1$_f$ and P1$_s$ give the same result with $q_j = 0, \forall j$, while for an infeasible instance the number of QoS violations are minimized due to the sparsity requirement of l_1-norm. P1$_f$ is always feasible and the solution w.r.t., UE^s is balanced.

IV. Numerical Results

Throughout $N_r = 2$, $P_{max} = 0.1$, $\tau = 0.9$, $\sigma_i^2 = 1$, $\beta = 1e2$ and tolerance $= 1e-6$. Initial values of $\mathbf{s}$ and $\boldsymbol{\lambda}$ are such that $\mu = 1$. Perfect channel state information at the BSs and cooperation among the BSs is assumed. At times choosing an initial feasible solution is as hard as the original problem but with IIPM initialized values need not satisfy their corresponding bounds e.g., $p_j = 0.2$, $\alpha_{ij} = 1.5$ can be used. Only non-negativity of $\mathbf{s}$ and $\boldsymbol{\lambda}$ must be ensured throughout.

Fig.1 shows the convergence of sum-power objective and corresponding μ for various (M, N) for a feasible case. Convergence of μ appears to be monotonic, but in general this may not be the case. Convergence depends on the initial values and required tolerance. It can be seen that with the Newton iterations, sum-power objective converges as the corresponding $\mu \to 0$. At convergence each UE is associated with a BS i.e., α_{ij} converges to a 0-1 solution and has a minimum power level satisfying the QoS. The solution is balanced as the max-min case.

Fig.2 shows the cumulative distribution function (cdf) of the objective for various (N, r_{th}). For comparison, PA is solved as in [4] for all M^N BSA combinations. For a given PA at convergence, the BSA problem is also verified by IP solver CPLEX. Simulation results show that the IIPM converges to the same optimum as the exhaustive search. As expected, the cdf curve shifts to the right with N and r_{th} i.e., an increase in sum-power. Increase in interference being the reason.

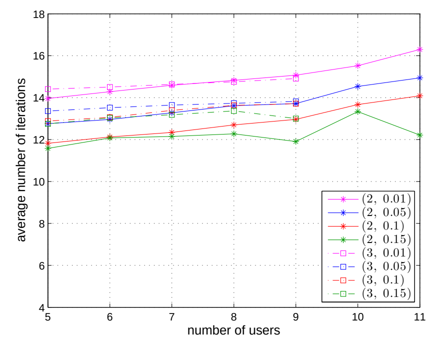

Fig. 3. average number of required iterations to converge.

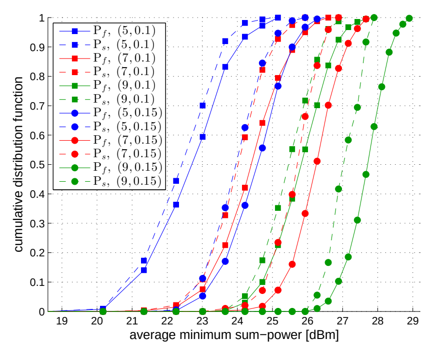

Fig. 4. cdf with (P1$_f$) and without (P1$_s$) feasibility handle. $M = 2$.

Fig.3 shows the average number of iterations required to converge for various feasible (M, r_{th}). Simulation results show that a feasible problem on an average requires 12 to 16 iterations to converge. Depending on the initialized values and tolerance, the number of iterations for an instance vary between 10 and 30.

Fig.4 shows the cdf of sum-power objective for P1$_s$ and P1$_f$ for various (N, r_{th}). Even for an infeasible instance, all UEs have an associated BS i.e., UE^v are not

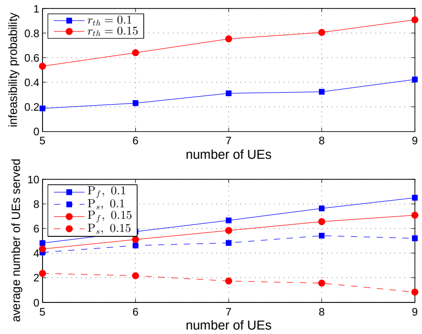

Fig. 5. probability of infeasibility (top) and number of UE served on an average (below). $M = 2$.

dropped. $q_j \neq 0$, $\forall$ UE$_j^v$ and $q_j = 0$, $\forall$ UE$_j^s$. $p_j = P_{max}$, $\forall$ UE$_j^v$, this increases the sum-power and explains the right shift in the cdf plot of P$_f$ w.r.t., P$_s$.

Fig.5 (top) shows that infeasibility increases with N and r_{th}. Fig.5 (below) compares P1$_s$ and P1$_f$ in terms of the average number of UEs served and explains the advantage of P1$_f$ over P1$_s$. For $r_{th} = 0.15$ number of feasible instances are very less hence P1$_s$ plot decreases with N. The corresponding P1$_f$ plot increases with N due to finite UE^s for some infeasible cases.

V. Conclusion

A simple primal-dual infeasible interior point method has been applied to solve the problem of sum-power minimization with QoS for the multi-cell multi-user uplink. The MINLP problem of PC is solved as a single stage optimization problem after obtaining an equivalent NLP problem. The optimum obtained is the same as the exhaustive search, therefore a significant computational advantage is gained. This approach also requires very few iterations though the computational effort for each iteration may vary with the problem size i.e., number of primal variables. The l_1-norm heuristic handles the infeasibility by serving as many UEs as possible making the framework more practical. IIPM has several advantages. The need to choose an initial primal feasible point is eliminated. Simultaneously solving for BSA and PA is possible. Additional recovery of the 0-1 BSA variables is not required. It can be applied to a broad class of utility functions though only sum-power minimization is shown. It can be extended to multi-carrier problem if different BSs are identified as subcarriers.

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