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# Optimal Modulation Index of the Mach-Zehnder Modulator in a Coherent Optical OFDM System Employing Digital Predistortion

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## Abstract

We study the impact of nonlinear distortions caused by the external optical modulator in coherent optical OFDM systems and determine by simulation the optimal modulation index that minimizes the OSNR penalty introduced by nonlinear distortion and modulation excess loss. To reduce this penalty a digital predistortion device implemented by a look-up table is proposed and the achievable gain in receiver sensitivity is quantified. These results are compared for the word lengths 6 bit and 8 bit of the digital-to-analog converter and look-up table, respectively.

## 1 Introduction

One widely discussed candidate for optical transmission systems that reach data rates beyond 100 Gbit/s is orthogonal frequency division multiplex (OFDM) combined with coherent detection (COOFDM). This is due to its resilience to chromatic and polarization mode dispersion [1], [2], its tight spectral shape that facilitates the generation of superchannels [3] and its potential to implement flexible bandwidth allocation. Its sensitivity to nonlinearities, however, is one of the major drawbacks of COOFDM [4]. An important origin of nonlinearities is the external optical modulator realized by two Mach-Zehnder modulators (MZMs) that exhibit a sine-shaped electro-optical characteristic. The MZM nonlinear characteristic in COOFDM systems has first been studied by Tang et al. for binary phase shift keying (BPSK) [4] and in [5] for 4- to 128-QAM. Digital predistortion to overcome the MZM nonlinearity has been investigated in [6]–[8] but the achievable receiver sensitivity gain has not been quantified yet. In addition, quantization impairments have to be taken into account when implementing digital predistortion for real-time COOFDM systems at the envisaged data rates. In [6], the limited word length of the digital-to-analog converter (DAC) has been modeled by quantizing the OFDM transmit signal, but quantization errors in the transmitter digital signal processing (DSP) units, especially in the digital predistortion device, have not been studied yet.

The main contribution of this paper is the definition and determination of the optimal modulation index for a COOFDM system with 4-, 16- and 64-QAM and the investigation of a look-up table (LUT) based predistortion device that allows detailed performance evaluation of digital predistortion in the presence of transmitter quantization errors. Furthermore, the receiver sensitivity gain through digital predistortion is identified and compared for the aforementioned modulation orders.

This paper is organized as follows. The system model establishing the basis of our studies is introduced in Section 2. In Section 3, we define modulation index (MI) and modulation excess loss (EL) and derive an analytical expression for the latter. The optimal MI in terms of minimum combined receiver sensitivity penalty and EL is determined in Section 4 through numerical simulations and LUT based digital predistortion is studied in Section 5. In Section 6 we summarize our findings.

## 2 System model

The system model used in this work is based on [9] and has been extended by digital predistortion. Its block diagram is depicted in **Fig. 1**. The OFDM transmit-

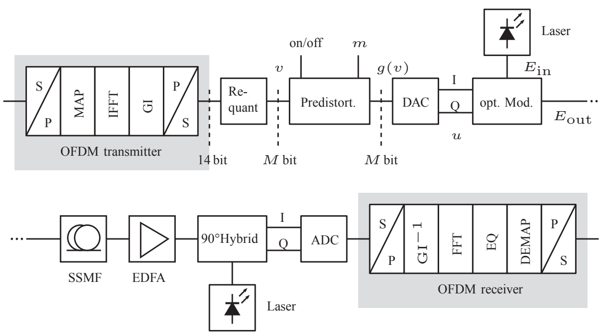


**Figure 1** Block diagram of COOFDM system.

ter consists of mapper (MAP), inverse fast Fourier transform (IFFT) and guard interval (GI) insertion. The IFFT dimension is 256 resulting in the same number of subcarriers of which 165 are modulated. The IFFT block is modeled by a custom Very High

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Speed Hardware Description Language (VHDL) design [10] with an output word length of 14 bit. The DAC, working at a sampling rate of 32 GSa/s, usually has a lower word length $M$ of 6 bit to 8 bit. Consequently, the word length of the OFDM transmitter output has to be reduced which is realized by the block *Requant*. It employs the shift and clip (SAC) method we have introduced in [9] and is followed by a digital predistortion unit that will be described in more detail in Section 5. After digital-to-analog conversion the inphase (I) and quadrature (Q) signals drive two nested MZMs in quadrature configuration, thus modulating amplitude and phase of the laser-generated continuous wave. The electric transfer function of the MZMs is assumed to be frequency-flat, since Barros et al. have shown in [7] that the effect of the electric transfer function is negligible. The modulated signal is transmitted over 80 km of standard single mode fiber (SSMF) and therefore affected by chromatic dispersion (CD) whose compensation is enabled by insertion of an GI 8 samples long (3.125% of the basic OFDM symbol length). Nonlinear fiber effects are neglected under the assumption that the signal power fed into the fiber is below 1 mW. The received signal is amplified by an erbium doped fiber amplifier (EDFA) and fed into a 90° hybrid for coherent detection before it is sampled by an analog-to-digital converter (ADC) that likewise operates at 32 GSa/s. As this work is focused on transmitter quantization effects the ADC word length is assumed to be unlimited. Similarly, quantization effects are neglected in the OFDM receiver that consists of GI removal ($\mathrm{GI}^{-1}$), fast Fourier transform (FFT), equalizer (EQ) and demapper (DEMAP). In [9] we have introduced a technique to effectively remove the impact of transmitter quantization errors on channel estimation. Here, perfect channel knowledge is assumed for equalization to make the results independent of the influence that quantization errors have on channel estimation. Using the parameters mentioned above and assuming polarization multiplex, data rates of 80, 160 and 240 Gbit/s are achieved for 4-, 16- and 64-QAM, respectively.

# 3 Modulation index and modulation excess loss

An MZM biased at minimum transmission point exhibits the well-known electro-optical characteristic

$$\frac{E_{\mathrm{out}}(t)}{E_{\mathrm{in}}(t)} = \frac{1}{\sqrt{a_{\mathrm{I}}}} \sin\left(\frac{u(t)}{U_\pi}\frac{\pi}{2}\right), \qquad (1)$$

where $E_{\mathrm{out}}(t)$ and $E_{\mathrm{in}}(t)$ are the input and output electric fields, $a_{\mathrm{I}} \geq 1$ is the insertion loss, $u(t)$ is the electric driving voltage and $U_\pi$ is the half-wave switching voltage. The time $t$ will be omitted in the remainder of this paper for convenience. It becomes apparent from (1) that the MZM is approximately linear for small $u$ but shows a strong nonlinear characteristic for values of $u$ approaching $\pm U_\pi$. For quantifying the degree of modulation the modulation index (MI) $m$ is introduced:

$$m = \frac{u_{\mathrm{rms}}}{U_\pi}\pi, \qquad (2)$$

where $u_{\mathrm{rms}}$ is the root mean square value of $u$. With $u_{\mathrm{rms}}$ a statistical quantity is chosen for the definition of MI, because OFDM signals have a noise-like, approximately Gaussian distribution. Specifically, it has been shown in [11] that the probability density function $p(u)$ of $u$ can be modeled with good approximation by

$$p(u) = \frac{1}{u_{\mathrm{rms}}\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{u}{u_{\mathrm{rms}}}\right)^2}. \qquad (3)$$

It is therefore of little value to use, for instance, the maximum of $u$ for the definition of MI, since this value occurs rarely. To illustrate the meaning of $m$ we define the driving voltage $u_{90}$ for which $\Pr[-u_{90} < u < u_{90}] = 0.9$, where $\Pr[\cdot]$ stands for probability, and provide some values of $u_{90}$ in Table 1. From (1) and Table 1 it can also be seen that $E_{\mathrm{out}}$ is small with respect to $E_{\mathrm{in}}$ for small $m$. In this case the mean optical output power $P_{\mathrm{out}}$ of the MZM will be reduced significantly compared to the mean optical input power $P_{\mathrm{in}}$. The optical power loss caused by modulation is called modulation excess loss and defined by

| $m$ | 0.1 | 0.5 | 1.0 | 1.5 | 1.8 | 2.0 |
|---|---|---|---|---|---|---|
| $u_{90}/U_\pi$ | 0.05 | 0.26 | 0.52 | 0.79 | 0.94 | 1.05 |

**Table 1** 90% range $u_{90}$ in dependence of $m$

$$a_{\mathrm{E}} = \frac{P_{\mathrm{in}}/a_{\mathrm{I}}}{P_{\mathrm{out}}}. \qquad (4)$$

Furthermore,

$$\begin{aligned} P_{\mathrm{out}} &= K\mathrm{E}\left[E_{\mathrm{out}}^2\right] \\ &= \frac{K}{a_{\mathrm{I}}}\mathrm{E}\left[\sin^2\left(\frac{u}{U_\pi}\frac{\pi}{2}\right)E_{\mathrm{in}}^2\right], \end{aligned} \qquad (5)$$

where $\mathrm{E}[\cdot]$ is the expectation operator and $K$ is a proportionality constant. The coherent wave $E_{\mathrm{in}}$ is statistically independent from the term $\sin^2(\ldots)$. In addition, $P_{\mathrm{in}} = K\mathrm{E}\left[E_{\mathrm{in}}^2\right]$. Hence,

$$P_{\mathrm{out}} = \frac{P_{\mathrm{in}}}{a_{\mathrm{I}}} \int_{-\infty}^{\infty} \sin^2\left(\frac{u}{U_\pi}\frac{\pi}{2}\right) p(u)\mathrm{d}u. \qquad (6)$$

From (4) using eqs. (2), (3) and (6) then follows

$$a_{\mathrm{E}} = 2\left(1 - \mathrm{e}^{-\frac{m^2}{2}}\right)^{-1}. \qquad (7)$$

As can be seen from eq. (7) the EL $a_{\mathrm{E}}$ neither depends on the number of subcarriers nor the subcarrier modulation scheme but on the MI alone. It should further be noted that the EL caused by modulation of the optical carrier with an OFDM signal cannot be less than 3 dB, because $\lim\limits_{m\to\infty} a_{\mathrm{E}} = 2$. To verify (7) the expression is compared with numerical results that have

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been obtained by simulation of the system described in Section 2 excluding quantization effects. Both results are shown in **Fig. 2** and are in good agreement.

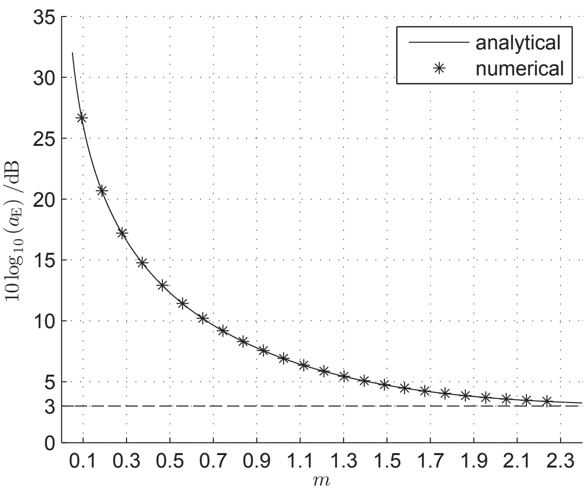


**Figure 2** Comparison of analytical and numerical computation of EL $a_\mathrm{E}$; limit of $a_\mathrm{E}$ (dashed line).

# 4 Optimal modulation index

To study the effect of MZM nonlinearity on system performance the optical signal-to-noise ratio (OSNR) required to reach a bit error rate (BER) of $10^{-3}$, $\gamma_\mathrm{r}$ (in dB), is determined for different values of $m$ by simulation. Quantization effects are neglected in this section in order to isolate the effect of MZM nonlinearity. Moreover, digital predistortion is not applied here. Let $\gamma_0$ be the required OSNR in dB for BER = $10^{-3}$ of an idealized system with linear external modulator. Then the OSNR penalty is defined by

$$\gamma_\mathrm{p} = \gamma_\mathrm{r} - \gamma_0. \tag{8}$$

and is shown in **Fig. 3** along with the EL. For small

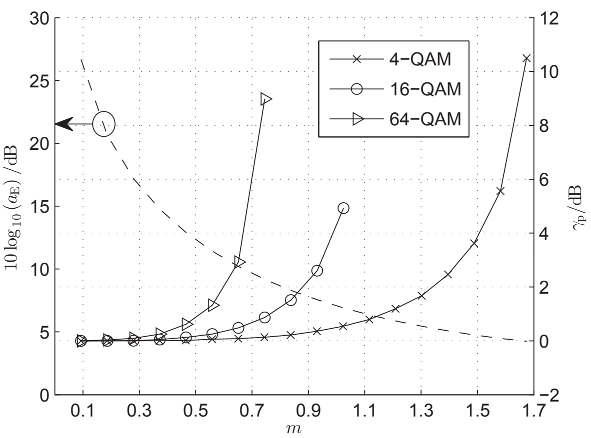


**Figure 3** OSNR penalty $\gamma_\mathrm{p}$ due to MZM nonlinearity for different modulation orders (solid lines) and EL $a_\mathrm{E}$ (dashed line).

$m$ the OSNR penalty is negligible as the driving signal is in the quasi-linear region of the MZM characteristic but shows a steep rise when $m$ is further increased. Compared to 4-QAM higher modulation orders are obviously more sensitive to nonlinearities: for an allowable OSNR penalty of 2 dB $m$ must be below 0.6 (0.9) for 64-QAM (16-QAM) whereas 4-QAM allows an MI as high as 1.3. Keeping the MI low reduces the OSNR penalty significantly. However, this comes at the cost of a high EL which will effectively reduce the available OSNR at the receiver side, because the transmit laser power $P_\mathrm{in}$ is limited. Therefore, a trade-off between OSNR penalty through MZM nonlinearity and OSNR penalty through modulation has to be found. For this purpose we define the combined OSNR penalty

$$\gamma_\mathrm{c} = \gamma_\mathrm{p} + 10\log_{10}(a_\mathrm{E}) \tag{9}$$

that should be minimized with respect to $m$. From **Fig. 4** the optimal modulation index can be obtained, namely the value of $m$ for which $\gamma_\mathrm{c}$ is minimum. The

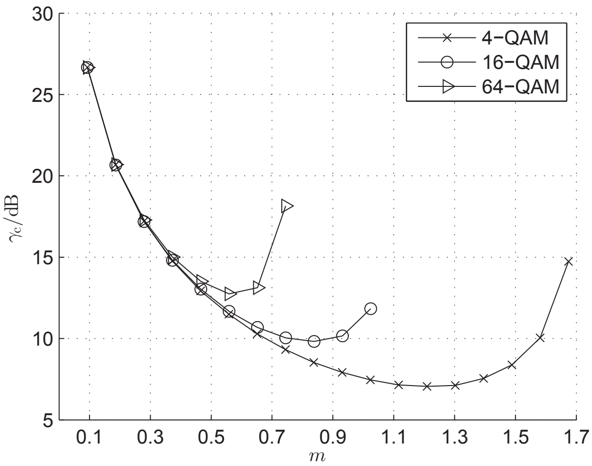


**Figure 4** Combined OSNR penalty $\gamma_\mathrm{c}$ as a function of $m$ for modulation orders 4-, 16- and 64-QAM.

higher the modulation order the lower is the optimal MI as the sensitivity to nonlinearity increases with increasing modulation order, whereas the EL is independent thereof. For the same reasons the minimum of $\gamma_\mathrm{c}$ increases with increasing modulation order. It should be noted that depending on the optimal fiber launch power and the available laser input power $P_\mathrm{in}$ some EL might even be desired to avoid fiber nonlinearities. Optimization of MI in the presence of fiber nonlinear effects is still to be investigated and outside the scope of this work.

# 5 Digital predistortion

As demonstrated in the previous section MZM nonlinearity provokes significant reduction of system performance, especially for modulation orders 16- and 64-QAM. To mitigate this effect a predistortion device

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can be used that exhibits the inverse characteristic of the MZM thus yielding an overall linear characteristic. However, perfect compensation of the MZM characteristic by its inverse function is impossible since (1) is $2\pi$-periodic and thus the iverse function exhibits ambiguities. Values $|u| > U_\pi$ cannot be mapped to a predistorted value that will result in the desired MZM output value and are therefore clipped. This leads to the predistortion function

$$g(v) = \begin{cases} A & \text{if } v > 1 \\ \frac{2}{\pi} A \arcsin(v) & \text{if } |v| \leq 1 \\ -A & \text{if } v < -1 \end{cases}, \tag{10}$$

where $v$ is the input of the predistortion device and $A$ is its maximum output level. An analog implementation of (10) can only approximate the required arcsine characteristic [12], whereas in the digital domain an LUT is able to implement arbitrary functions and can be made reconfigurable. Hence, the latter has been chosen in this work to implement the required predistortion function. Yet a digital realization adds quantization noise that has to be taken into account. In the following we will derive the predistortion function of the LUT and study its performance.

The proposed LUT has an input and output word length of $M$ bit, respectively. It stores an output vector for each possible input vector and is assumed to be reconfigurable, so that the implemented function $g(v)$ can be adapted to the dynamic range of $u$. The discrete valued LUT input $v$ as well as its output $g(v)$ can be interpreted as signed integer numbers in $[-2^{M-1}, 2^{M-1}-1]$. We assume a fast LUT device so that static and dynamic predistortion characteristics ar identical. Consequently, we can drop time $t$ in the following and the DAC simply converts $g(v)$ to a voltage

$$u = \frac{U_0}{2^{M-1}} g(v). \tag{11}$$

The MZM characteristic that is supposed to be linearized by $g(v)$ is given in (1):

$$\begin{aligned} \frac{E_\text{out}}{E_\text{in}} \sqrt{a_\text{I}} &= \sin\left(\frac{u}{U_\pi}\frac{\pi}{2}\right) \\ &\overset{(11)}{=} \sin\left(g(v)\frac{U_0}{U_\pi 2^{M-1}}\frac{\pi}{2}\right) \overset{!}{=} Rv, \end{aligned} \tag{12}$$

where $R$ is a positive proportionality constant. From (12) we derive the predistortion function:

$$g(v) = \frac{U_\pi}{U_0} 2^{M-1} \frac{2}{\pi} \arcsin(Rv) \quad \text{for } |Rv| \leq 1. \tag{13}$$

In order to reduce quantization noise, the range of $g(v)$ has to be fully exploited which leads to the condition

$$\begin{aligned} g\left(-2^{M-1}\right) &\overset{!}{=} -2^{M-1} \\ \Leftrightarrow \quad R &\overset{(13)}{=} \frac{1}{2^{M-1}} \sin\left(\frac{U_0}{U_\pi}\frac{\pi}{2}\right). \end{aligned} \tag{14}$$

Equations (13) and (14) imply that $|U_0| \leq U_\pi$ which limits the range of $m$. Further increase of $m$ can be achieved by (a) finding a predistortion function for $|U_0| > U_\pi$ or by (b) scaling and clipping of $v$. Implementing option (a) would require to reduce the number of output words that represent the predistortable range of $u$, thus leading to an increase in quantization noise. For this reason option (b) has been chosen and implemented by scaling $v$ with $R > 1/2^{M-1}$ if the desired MI cannot be achieved otherwise. In this case $U_0 = U_\pi$ and

$$g(v) = \begin{cases} 2^{M-1} - 1 & \text{if } Rv > \frac{2^{M-1}-1}{2^{M-1}} \\ -2^{M-1} & \text{if } Rv < -1 \\ \frac{U_\pi}{U_0} 2^{M-1} \frac{2}{\pi} \arcsin(Rv) & \text{otherwise} \end{cases}. \tag{15}$$

The LUT function is finally obtained by quantizing $g(v)$ given by (15) for all possible input values $v$. Some examples of LUT characteristics for $M = 8$ bit are shown in **Fig. 5**. Note that the predistortion device in Fig. 1 consists of two identical LUTs for inphase and quadrature signal, respectively.

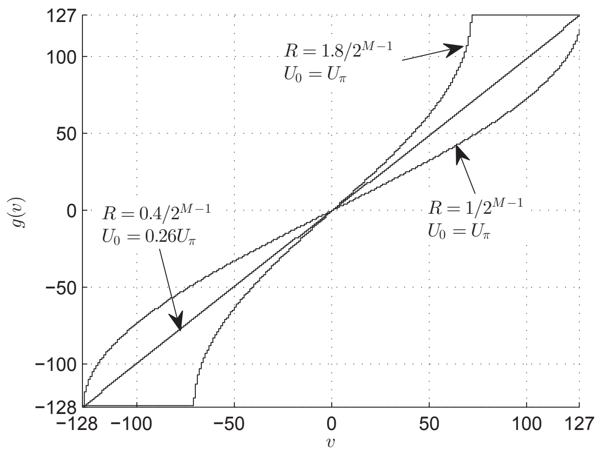


**Figure 5** Predistortion characteristic with LUT for $M = 8$ bit and different values of $R$.

To study the effectiveness of the proposed predistortion device, the OSNR required for a BER of $10^{-3}$ has been simulated using the system model introduced in Section 2. Applying the SAC method for requantization a number of most significant bits is cut so that quantization and clipping noise are minimized before predistortion [9]. The predistorted system is compared to a system that does not apply predistortion and the resulting OSNR gain as a function of $m$ is shown in **Fig. 6** for $M = 6$ bit and 8 bit. The improvement by predistortion is negligible for small $m$ since the MZM characteristic is approximately linear in this region. With increasing $m$ the OSNR gain increases significantly where the sensitivity to nonlinearity of higher modulation orders is indicated by the rise of the OSNR gain already at small $m$ compared to 4-QAM. Accordingly, the maximum OSNR gain is observed for

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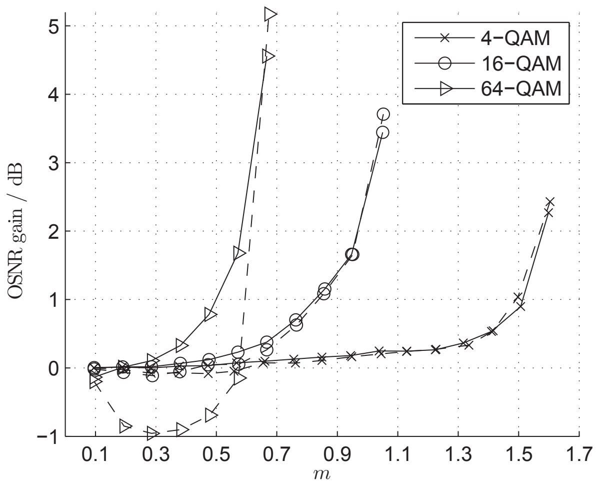


**Figure 6** OSNR gain through digital predistortion for $M$ = 6 bit (dashed line) and 8 bit (solid line).

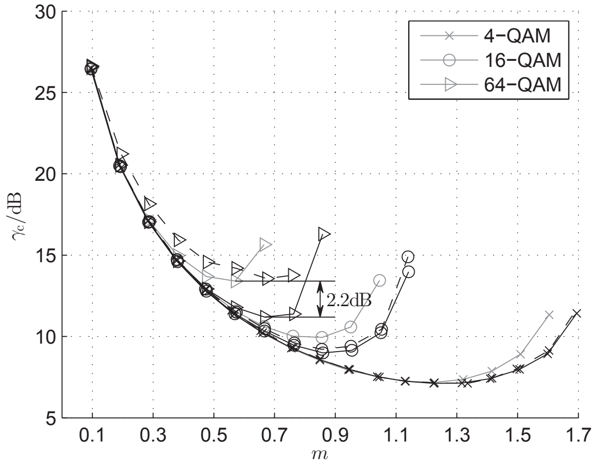


**Figure 7** Combined OSNR penalty using LUT predistortion for $M$ = 6 bit (dashed line) and 8 bit (solid line) compared to non-predistorted system at $M$ = 8 bit (gray line).

64-QAM, followed by 16-QAM. Fig. 6 also shows that quantization errors introduced by the LUT predistortion can reduce the achievable OSNR gain. This effect is especially observed for 6 bit and 64-QAM where quantization noise dominates for small $m$ and thus causes the predistorted system to perform even worse than the non-predistorted system. With increasing $m$ distortion dominates again and the predistorted system exhibits a significant performance gain of over 5 dB. While some important amount of OSNR gain is clearly achieved by digital predistortion, the OSNR penalty observed in Fig. 3 cannot completely be eliminated because clipping occurs for large $m$ that leads to additional distortion. Furthermore, the residual OSNR penalty through MZM nonlinearity as well as the EL have to be taken into account when evaluating the gain through digital predistortion. Thus, the combined OSNR penalty $\gamma_c$ has been determined and is compared to the non-predistorted system in **Fig. 7**. These results show that for 4-QAM the combined OSNR penalty $\gamma_c$ can be reduced for large $m$ although the reduction of the minimum $\gamma_c$ is negligible. At 16-QAM and $M = 8\,\mathrm{bit}$ the minimum $\gamma_c$ can be reduced by 1.0 dB and the greatest improvement in receiver sensitivity is achieved for 64-QAM, where $\gamma_c$ is reduced by 2.2 dB. For $M$ = 6 bit the performance of the predistorted system is reduced, especially for 64-QAM. In contrast, 4-QAM is robust against quantization effects.

# 6 Conclusion

We have shown that digital predistortion of the nonlinear MZM characteristic using an LUT can improve receiver sensitivity in COOFDM systems by 2.2 dB (1.0 dB) for 64-QAM (16-QAM) by introducing a combined OSNR penalty that takes into account nonlinear distortion effects, transmitter quantization errors and modulation excess loss. When 4-QAM is used predistorted and non-predistorted systems exhibit similar performance. Our results show, that an LUT input/output word length of 8 bit (6 bit) is sufficient for 64-QAM (16-QAM). Furthermore, we have determined the optimal modulation index in terms of minimal combined OSNR penalty for predistorted and non-predistorted systems and have derived an analytical expression for the modulation excess loss in COOFDM systems.

# 7 Acknowledgment

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