Filter Optimization for Carrier-Frequency- and Timing-Offset in Universal Filtered Multi-Carrier Systems

Xiaojie Wang *, Thorsten Wild †, and Frank Schaich †

*Institute of Telecommunications, University of Stuttgart, E–mail: gdi73302@gmail.com
†Alcatel-Lucent Bell Labs, Lorenzstr. 10, 70435, Stuttgart, Germany
E–mail: {thorsten.wild, frank.schaich}@alcatel-lucent.com

***Abstract*— Universal Filtered Multi-Carrier (UFMC) is a novel multi-carrier modulation technique which can be seen as a generalization of filtered OFDM and filter bank based multi-carrier (FBMC-FMT). Being a candidate waveform technology for 5G wireless systems, it combines the simplicity of OFDM with the advantages of FBMC. The FIR-filter, used in UFMC to filter a group of subcarriers, is a key design parameter to gain more robustness in relaxed synchronization conditions, i.e. time-frequency misalignment. It was shown in previous work that very significant SIR improvement can be achieved for UFMC by optimizing the FIR-filter, taking carrier frequency offset into account. In this paper, we optimize the FIR-filter design in UFMC by taking both carrier frequency and timing offset into account in an uplink multi-user FDMA scenario. From the simulation results, up to 3.6 dB SIR improvement can be achieved with the optimized FIR filter compared to UFMC with non-optimized Dolph-Chebyshev filter and 15.1 dB SIR gain against classical CP-OFDM system respectively, provided that the normalized carrier frequency and timing offset are uniformly distributed in the interval $\pm 5\%$.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is today's most prominent multi-carrier modulation technique. It is applied in standards such as Long Term Evolution (LTE) and WiFi, because of its simple and efficient modulation and demodulation stages using Fast Fourier Transformation (FFT) and inverse FFT (IFFT). Furthermore, it allows scalar equalization per-subcarrier at the receiver. However, it is well known that the rectangular symbol shape of OFDM is neither well localized in time nor in frequency, due to the comparatively high spectral side-lobe level. Strict synchronicity is enforced in LTE to maximize system performance [1]. While being reasonable in today's networks typically delivering high-rate traffic to high-end devices like smart phones and tablets, such an approach is unfeasible for new types of wireless services such as Internet of Things (IoT) and the tactile Internet. So, future 5G wireless communication systems have to be able to deal with a very diverse variety of traffic types ranging from regular high-rate traffic, sporadic small packet and urgent low latency transmission. In order to reduce the signaling overhead and the battery consumption for low-end devices (e.g. simple sensor elements) in 5G, they should be allowed to transmit with relaxed synchronization conditions regarding time-frequency misalignments. When doing so, the inter-carrier interference introduced to other well localized users, is to be investigated.

A future 5G candidate technology for replacing OFDM is filter-bank based multi-carrier (FBMC) [2]. Each subcarrier is individually filtered, strongly enhancing robustness against inter-carrier interference (ICI) effects. However, typical FBMC systems utilize filters, whose length is multiple times of samples per multi-carrier symbol. Hence, its drawback is its long filter length, making it disadvantageous for communication in short uplink bursts, as required in potential application scenarios of 5G systems [3], like low latency communication or energy-efficient Machine-Type Communication (MTC). Universal Filtered Multi-Carrier (UFMC) is a novel multi-carrier modulation technique [4], which is a generalization of filtered OFDM and FBMC (in its filtered multi-tone (FMT) variant). While the former filters the entire band and the latter filters each subcarrier individually, UFMC filters subband-blocks, i.e. groups of subcarriers. This allows reducing the filter length considerably, compared to FBMC. Furthermore, quadrature amplitude modulation (QAM) is applicable in UFMC (in contrast to the FBMC case [2]), making UFMC compatible to all kinds of Multiple Input Multiple Output (MIMO). UFMC can also rely on FFT-based receive processing with per-subcarrier equalization. As UFMC is very close in nature to OFDM, it is also known as Universal Filtered OFDM (UF-OFDM) [5].

The focus of this paper is to optimize the finite impulse response (FIR) filter for UFMC to achieve more robustness against carrier frequency and timing offset. This robustness is here mainly meant w.r.t. an uplink multi-user Frequency Division Multiple Access (FDMA) scenario. While individual per-user timing- and frequency offset can be estimated and corrected, in a multi-user scenario, residual inter-carrier-interference remains, when users are misaligned to each other [7]. According to our previous work [6], a significant improvement in terms of Signal to Interference Ratio (SIR) can be achieved by optimizing the filter, if only carrier frequency offset (CFO) is present. Since no Cyclic Prefix (CP) is used in UFMC systems, even a small timing offset introduces ICI and Inter Symbol Interference (ISI). In contrast to UFMC, CP-OFDM systems are very robust against positive timing offset as long as it is within the CP length. Thus, the FIR-filter in UFMC is optimized in this paper to overcome the impact of CFO and timing misalignment jointly. Furthermore, the system performance is quantified in terms of SIR and compared with CP-OFDM systems. Related work [7] introduces an open-loop based synchronization procedure for systems applying UFMC, called Autonomous Timing Advance (ATA). In [7], UFMC is evaluated regarding the amount of timing and frequency offsets the system is able to tolerate, using non-optimized Dolph-Chebyshev filters. The filter optimization presented in this paper thus provides a further performance improvement for [7]. In an envisaged 5G system this filter optimization is not meant to be executed "on the fly". Instead, a set of multi-carrier parameters (like a filter "codebook") is provided by the system for supporting a certain expected range of time-frequency misalignments, depending on device capabilities and system design parameters. For a transmission the filter parameters are chosen out of this set, e.g. controlled by the basestation.

The remaining part of this paper is structured as follows. Section II describes the system model of UFMC and analyzes the impact of CFO and timing offset. Section III discusses the optimization criteria for FIR-filter design in UFMC. Section IV shows the simulation results and quantifies the system performance in terms of SIR. Section V summarizes this paper.

II. System Model of UFMC

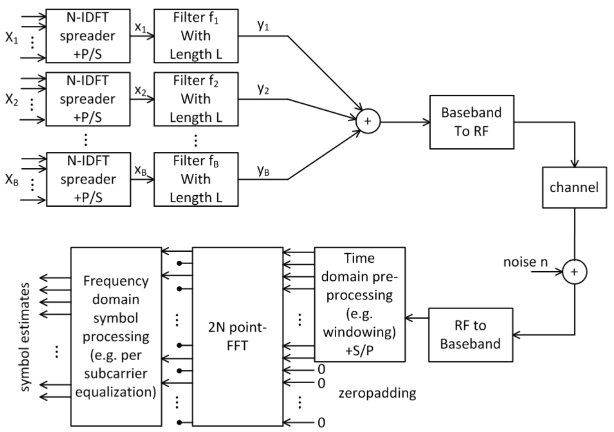

Fig. 1: System model of UFMC

The system model of UFMC is shown in Fig. 1 ([8], [9]). The overall bandwidth is divided into B sub-bands. Each sub-band can be allocated with n_i consecutive subcarriers and the sub-band may correspond to Physical Resource Block (PRB) in LTE. The total number of subcarriers is N. With the N-point IDFT operation, the frequency domain signal X_i is transformed into time domain x_i. Then this output signal x_i is filtered by a FIR-filter f_i with the length of L. That results in a symbol length of $N+L-1$ because of the linear convolution between x_i and f_i. Then, all the filtered sub-band signals y_i are added together and transmitted. At the receiver, FFT-based detection is used to transform the received time domain signal into frequency domain. Since the UFMC symbol has the length of $N+L-1$, zeros are padded to perform the $2N$-point FFT. If the channel impulse response is ideally Delta-Dirac function $\delta(t)$, time domain preprocessing is not used and CFO and timing offset are not taken into account, the estimated m-th symbol $Y_m(k)$ can be written as

$$Y_m(k) = \sum_{i=1}^{B} \tilde{X}_{i,m}(k) F_{i,m}(k) \quad k = 0, \cdots, 2N-1, \tag{1}$$

where $\tilde{X}_i$ and F_i are the 2N-point FFT of the time domain signal x_i and f_i. It can be easily observed from $\tilde{X}_i$ that all even subcarriers contain data, while all odd subcarriers contain interference from all the data symbols thus they are dropped.

Extensive works regarding CFO and timing offset have been done for CP-OFDM systems in [10] and [11]. But the impact of CFO and timing offset in UFMC systems have not been studied. Thus, we study the effect of carrier frequency and timing offset in a UFMC system in the following w.r.t. an uplink multi-user FDMA scenario.

A. Timing Offset

In OFDM systems, positive timing offset (signal arrives later than the receiver expected), has no effect on the SIR as long as it is within the CP duration. Once the timing offset exceeds the CP-length or it is negative (signal arrives earlier than the receiver expected), it starts to degrade the system performance. This non-symmetric effect of timing errors is shown in [12]. In contrast to OFDM, any timing offset in UFMC systems affect the system performance, since no CP is inserted. Even with a small timing offset, the orthogonality between subcarriers is destroyed, which causes ICI. Additionally, the previous (positive delay) or subsequent (negative delay) symbol introduces ISI. But the filter ramp-up and ramp-down indicate a soft protection against timing offset, since relatively small energy is contained. So, carefully multi-carrier parameter design can contribute in mitigation of ISI.

If an integer number of delay samples n_{to} (positive or negative) is considered, the received symbol within a receiver window consists of $N+L-1-|n_{\text{to}}|$ samples from the symbol m and $|n_{\text{to}}|$ samples from the symbol $m+1$ or $m-1$. In order to handle the negative and positive delay consistently, we define a symbol index m' for the previous or subsequent symbol. That is $m' = m+1$, if timing offset is positive i.e. $n_{\text{to}} > 0$. And $m' = m-1$, if timing offset is negative i.e. $n_{\text{to}} < 0$. By applying rectangular windowing and circular shift approach for the symbol m and m' similar as in [12], we get the symbol estimates in case of timing offset n_{to}

$$\hat{Y}_m(k) = W_1(k) * \left(Y_m(k) e^{\phi_1(k)}\right) + W_2(k) * \left(Y_{m'}(k) e^{\phi_2(k)}\right) \tag{2}$$

where $\phi_1(k) = e^{-j\pi k n_{\text{to}}/N}$ and $\phi_2(k) = e^{\text{sign}(n_{\text{to}}) j\pi k(N+L-1-|n_{\text{to}}|)/N}$ represent linear phase shift due to circular shift in time domain. $*$ denotes the convolution operator. Furthermore, W_1 and W_2 are the 2N-point FFT of the rectangular window w_1 and w_2 respectively. With respect to the timing offset n_{to}, these two rectangular windows can be determined as e.g. for positive timing offset $w_1 = [\mathbf{0}_{[1\times n_{\mathrm{to}}]}, \mathbf{1}_{[1\times(N+L-1-n_{\mathrm{to}})]}, \mathbf{0}_{[1\times(N-L+1)]}]^T$ and $w_2 = [\mathbf{1}_{[1\times n_{\mathrm{to}}]}, \mathbf{0}_{[1\times(2N-n_{\mathrm{to}})]}]^T$, where $\mathbf{0}$ and $\mathbf{1}$ are the vectors, whose elements are all zeros and all ones. Analogously, if the timing offset n_{to} is negative, we get $w_1 = [\mathbf{1}_{[1\times(N+L-1+n_{\mathrm{to}})]}, \mathbf{0}_{[1\times(N-L+1-n_{\mathrm{to}})]}]^T$ and $w_2 = [\mathbf{0}_{[1\times(N+L-1+n_{\mathrm{to}})]}, \mathbf{1}_{[1\times(-n_{\mathrm{to}})]}, \mathbf{0}_{[1\times(N-L+1)]}]^T$. It can be easily proven with (2) and the corresponding above mentioned windows that UFMC systems have a symmetric effect of timing offset rather than the non-symmetric behavior of OFDM system. The reason is that the SIR for every subcarrier remains the same under positive and negative timing offset $\mathrm{SIR}(k, n_{\mathrm{to}}) = \mathrm{SIR}(k, -n_{\mathrm{to}})$. This claim is verified with the results in [7]. Furthermore, UFMC is expected to be more robust against small negative timing offset than OFDM, since the energy within the filter ramp-up and ramp-down is comparatively small.

B. Joint CFO and Timing Offset

Besides timing offset, CFO also occurs due to Doppler effect and local oscillator frequency mismatch between transmitter and receiver. The effect of CFO alone was studied in our previous work in [6]. Assume that the CFO remains unchanged for the two considered symbols, with (2) the symbol estimates under timing and carrier frequency offset can be written as

$$\hat{Y}_m(l) = \sum_{k=0}^{2N-1} \Gamma_1(l-k) Y_m(k) e^{\phi_1(k)} + \sum_{k=0}^{2N-1} \Gamma_2(l-k) Y_{m'}(k) e^{\phi_2(k)} \tag{3}$$

where $\Gamma_1 = C_i * W_1$ and $\Gamma_2 = C_i * W_2$. And C_i is frequency domain description of the CFO, which can be written as

$$C_i(k) = \frac{\sin\left(\frac{\pi}{2N}(2\epsilon_i - k)(N+L-1)\right)}{2N\sin\left(\frac{\pi}{2N}(2\epsilon_i - k)\right)} \cdot e^{j\frac{\pi}{2N}(2\epsilon_i - k)(N+L-2)}$$
$$k = 0, ..., 2N-1;$$

where ϵ_i denotes the normalized CFO (rCFO), which is normalized to the subcarrier spacing [6]. From (1) and (3), it is clear that all the odd subcarriers of Y_m, which contain interference due to 2N-point FFT, have to be considered in case of time-frequency misalignment. Thus, we separate the signal part $Y_{m,\mathrm{S_k}}$ from interference part $Y_{m,\mathrm{ICI_k}}$ for a considered subcarrier k. Without loss of generality, the considered subcarrier k can be assumed to be allocated to the sub-band or PRB i. It can be written as

$$Y_m(l) = Y_{m,\mathrm{S_k}}(l) + Y_{m,\mathrm{ICI_k}}(l) \tag{4}$$

where $Y_{m,\mathrm{S_k}}(l)$ denotes the spread signal in subcarrier l from the subcarrier k and $Y_{m,\mathrm{ICI_k}}(l)$ is the corresponding ICI from all other subcarriers. They are straightforward

$$Y_{m,\mathrm{S_k}}(l) = \begin{cases} X_{i,m}\left(\frac{k}{2}\right) F_{i,m}\left(\frac{k}{2}\right) & \text{if } l = k \\ X_{i,m}\left(\frac{k}{2}\right) F_{i,m}\left(\frac{l}{2}\right) \frac{\sin\left(\frac{\pi}{2}(k-l)\right)}{N\sin\left(\frac{\pi}{2N}(k-l)\right)} e^{j\frac{\pi}{2}(k-l)\left(1-\frac{1}{N}\right)} & \\ \quad \text{if } l \text{ odd} & \\ 0 & \text{otherwise} \end{cases}$$

and

$$Y_{m,\mathrm{ICI_k}}(l) = Y_m(l) - Y_{m,\mathrm{S_k}}(l)$$

where $X_{i,m}$ is the N-point FFT of $x_{i,m}$. Using (4), (3) can be reformulated as

$$\hat{Y}_m(k) = S_m(k) + S_{m'}(k) + I_{m,\mathrm{ICI}}(k) + I_{m',\mathrm{ICI}}(k) \tag{5}$$

with

$$S_m(k) = \sum_{l=0}^{2N-1} \Gamma_1(k-l) Y_{m,\mathrm{S_k}}(l) e^{j\phi_1(k)} \tag{6}$$

$$S_{m'}(k) = \sum_{l=0}^{2N-1} \Gamma_2(k-l) Y_{m',\mathrm{S_k}}(l) e^{j\phi_2(k)} \tag{7}$$

$$I_{m,\mathrm{ICI}}(k) = \sum_{l=0}^{2N-1} \Gamma_1(k-l) Y_{m,\mathrm{ICI_k}}(l) e^{j\phi_1(k)} \tag{8}$$

$$I_{m',\mathrm{ICI}}(k) = \sum_{l=0}^{2N-1} \Gamma_2(k-l) Y_{m',\mathrm{ICI_k}}(l) e^{j\phi_2(k)} \tag{9}$$

where $S_m(k)$ is the signal part of m-th symbol, while $I_{m,\mathrm{ICI}}(k)$ and $I_{m',\mathrm{ICI}}(k) + S_{m'}$ is the corresponding ICI and ISI term.

III. Leakage-based Filter Optimization

A FIR filter f_i with the length of L is introduced for the subband i in UFMC system. This filter can be designed to gain more robustness against time-frequency misalignment in the real wireless communication system. In an uplink multi-user FDMA scenario, users are allocated with different subbands. Since the number of subcarriers and the filters for different subbands can be designed differently in the considered multi-user scenario, we focus on the optimization of the filter f_i for an arbitrary considered subband i without any knowledge of other subbands. Thus, we assume that only one subband of interest i is active.

From (5), the symbol estimate in each subcarrier contains signal contribution from both symbol m and m'. Additionally, ISI and ICI are present due to timing and frequency offset. Using the equations (6) to (9), we are able to calculate the expected energy of $S_m(k)$, $S_{m'}(k)$, $I_{m,\mathrm{ICI}}(k)$ and $I_{m',\mathrm{ICI}}(k)$ in matrix form respectively. Since only the symbol estimates in even subcarriers are of interest, we drop odd subcarriers and the subband index i for simplicity reason. $k' = 2k$ is defined for $k = 0, \cdots, N-1$.

$$E\left[\|S_m(k')\|^2\right] = \mathbf{f}^H \mathbf{S}_m^{k'} \mathbf{f} \tag{10}$$

$$E\left[\|S_{m'}(k')\|^2\right] = \mathbf{f}^H \mathbf{S}_{m'}^{k'} \mathbf{f} \tag{11}$$

$$E\left[\|I_{m,\mathrm{ICI}}(k')\|^2\right] = \mathbf{f}^H \mathbf{I}_{m,\mathrm{ICI}}^{k'} \mathbf{f} \tag{12}$$

$$E\left[\|I_{m',\mathrm{ICI}}(k')\|^2\right] = \mathbf{f}^H \mathbf{I}_{m',\mathrm{ICI}}^{k'} \mathbf{f} \tag{13}$$

where $\mathbf{f}$ is a vector, which contains all the filter coefficients. Obviously, (10) and (11) have only non-zero values for $k'/2 \in S_i$, where S_i is a set that contains all the subcarrier indexes within the considered subband i. (12) and (13) correspond to out-of-band-leakage (OBL) due to timing and frequency offset for $k'/2 \notin S_i$ and in-band-distortion (IBD) for $k'/2 \in S_i$. The

OBL is a interference to other subbands, while IBD is self interference.

In contrast to OFDM systems, the SIR differs in different subcarrier indexes due to the filtering approach in UFMC systems. Hence, a direct maximization of SIR results in a challenging optimization task. Thus, we use the optimization criteria maximizing Signal to out-of-band Leakage Ratio (SLR) and maximizing Signal to in-band Distortion plus out-of-band Leakage Ratio (SDLR) in case of timing and frequency offset. These optimization criteria lead to a generalized eigenvalue problem, which has a closed-form solution [13].

A. maximizing SLR

First of all, the so called SLR under a normalized timing offset $\Delta n = n_{\mathrm{to}}/(N+L-1)$ and rCFO ϵ is defined as

$$\mathrm{SLR}\ (\Delta n, \epsilon) = \frac{\sum\limits_{k'/2 \in S_i} \mathbf{f}^H \mathbf{S}_m^{k'}(\Delta n, \epsilon)\mathbf{f}}{\sum\limits_{k'/2 \notin S_i} \mathbf{f}^H \left(\mathbf{I}_{m,\mathrm{ICI}}^{k'}(\Delta n, \epsilon) + \mathbf{I}_{m',\mathrm{ICI}}^{k'}(\Delta n, \epsilon)\right)\mathbf{f}}. \tag{14}$$

It is sufficient to consider a normalized timing offset within the interval of half symbol duration, i.e. $-0.5 \leq \Delta n \leq 0.5$. The numerator represents the sum of the total signal energy of the m-th symbol within the considered subband i, while the denominator is the total OBL energy which causes interference to neighboring subbands. If the distributions of Δn and ϵ, i.e. $p_{\Delta n}(\Delta n)$ and $p_\epsilon(\epsilon)$, are known and they are independent of each other, we define the SLR as

$$\begin{aligned}\mathrm{SLR} &= \\ &\frac{\mathbf{f}^H \left(\sum\limits_{k'/2 \in S_i} \sum\limits_{\Delta n} \sum\limits_{\epsilon} \mathbf{S}_m^{k'}(\Delta n, \epsilon) p_{\Delta n}(\Delta n) p_\epsilon(\epsilon)\right)\mathbf{f}}{\mathbf{f}^H \left(\sum\limits_{k'/2 \notin S_i} \sum\limits_{\Delta n} \sum\limits_{\epsilon} \mathbf{I}_{\mathrm{ICI}}^{k'}(\Delta n, \epsilon) p_{\Delta n}(\Delta n) p_\epsilon(\epsilon)\right)\mathbf{f}} \\ &= \frac{\mathbf{f}^H \mathbf{S} \mathbf{f}}{\mathbf{f}^H \mathbf{Z}_{\mathrm{OBL}} \mathbf{f}}\end{aligned} \tag{15}$$

where $\mathbf{I}_{\mathrm{ICI}}^{k'}(\Delta n, \epsilon) = \mathbf{I}_{m,\mathrm{ICI}}^{k'}(\Delta n, \epsilon) + \mathbf{I}_{m',\mathrm{ICI}}^{k'}(\Delta n, \epsilon)$. The first optimization task, maximizing SLR, is

$$\begin{aligned} \mathbf{f}_{o,\mathrm{SLR}} &= \arg\max_{\mathbf{f}} \mathrm{SLR} \\ &\text{subject to } \|\mathbf{f}\|^2 = 1. \end{aligned} \tag{16}$$

It aims at minimizing the inference to other well-synchronized subbands, if the considered subband is time and frequency misaligned. In this approach, interferences within the subband are not taken into account. The reason is that they have no effect on other well-synchronized subbands. By maximizing the SLR, the total energy of OBL can be minimized while the total signal energy is maximized. The solution of (16) is given by

$$\mathbf{f}_{o,\mathrm{SLR}} \propto \text{ max. eigenvector} \left(\mathbf{Z}_{\mathrm{OBL}}^{-1}\mathbf{S}\right) \tag{17}$$

the eigen-vector corresponding to the largest eigen-value of the matrix $\mathbf{Z}_{\mathrm{OBL}}^{-1}\mathbf{S}$ [13], where $(\cdot)^{-1}$ denotes the matrix inverse operator.

In this approach, we dropped the self interference (IBD) term in the optimization and try to minimize the OBL. If all the subbands use the optimized filter using this approach, the interference between subbands is reduced and consequently the average SIR can be improved.

B. maximizing SDLR

In the second approach, the IBD is also taken into account. The SDLR is similarly defined as

$$\begin{aligned}\mathrm{SDLR} &= \\ &\frac{\mathbf{f}^H \left(\sum\limits_{k'/2 \in S_i} \sum\limits_{\Delta n} \sum\limits_{\epsilon} \mathbf{S}_m^{k'}(\Delta n, \epsilon) p_{\Delta n}(\Delta n) p_\epsilon(\epsilon)\right)\mathbf{f}}{\mathbf{f}^H \left(\sum\limits_{k'=0}^{2N-2} \sum\limits_{\Delta n} \sum\limits_{\epsilon} \mathbf{I}^{k'}(\Delta n, \epsilon) p_{\Delta n}(\Delta n) p_\epsilon(\epsilon)\right)\mathbf{f}} \\ &= \frac{\mathbf{f}^H \mathbf{S} \mathbf{f}}{\mathbf{f}^H \mathbf{Z}_{\mathrm{I}} \mathbf{f}}\end{aligned} \tag{18}$$

where $\mathbf{I}^{k'}(\Delta n, \epsilon) = \mathbf{I}_{\mathrm{ICI}}^{k'}(\Delta n, \epsilon) + \mathbf{S}_{m'}^{k'}(\Delta n, \epsilon)$. With the definition of SDLR above, the total energy of ICI and ISI in all the even subcarriers k' is to be minimized. Analogously, the second optimization task can be formulated as

$$\begin{aligned} \mathbf{f}_{o,\mathrm{SDLR}} &= \arg\max_{\mathbf{f}} \mathrm{SDLR} \\ &\text{subject to } \|\mathbf{f}\|^2 = 1. \end{aligned} \tag{19}$$

The solution of (19) is thus

$$\mathbf{f}_{o,\mathrm{SDLR}} \propto \text{ max. eigenvector} \left(\mathbf{Z}_{\mathrm{I}}^{-1}\mathbf{S}\right). \tag{20}$$

In this approach, all the interferences including IBD and OBL (from both considered symbols m and m') are to be minimized. Both of the approaches, maximizing SLR and SDLR, cannot guarantee a SIR improvement for every individual subcarrier. However, the average SIR within one subband may be improved. Furthermore, these optimizations are carried out per subband requiring only some basic multi carrier parameters (FFT and subband size) and expected timing and frequency offset distribution. Thus, they can be computed offline for various range of timing and frequency offset to build a filter “codebook”. Before transmission, the filter parameters are chosen out of this set, e.g. controlled by the basestation for supporting a certain expected range of time-frequency misalignments.

IV. RESULTS

To show the potential of the filter optimization, simulations are carried out for UFMC with the optimized filter using maximizing SDLR and SLR. The system parameters are as follows: FFT size $N = 128$, filter length $L = 16$, subband size $L_B = 12$ and total number of subbands $B = 10$. QPSK is used for modulation. No guard subcarriers are used between subbands and all subbands use the same type of filter. The filters are frequency shifted to the center frequency of each subband. For comparison purpose, the simulation is also carried out for conventional CP-OFDM system and UFMC with Dolph-Chebyshev filters with the side lobe attenuation of 40 dB. The cyclic prefix length of OFDM is chosen to be $N_g = 15$ such that the resulting symbol length remains the same for UFMC and OFDM, that is $N + L - 1 = N + N_g = 143$. The scenario considered in the simulation is as follows: one subband (User of Interest, UoI) is at the receiver perfectly synchronized without any timing or frequency offset. All the other 9 subbands are impaired with the timing offset samples n_{to} and rCFO ϵ. Furthermore, timing offset and frequency offset are assumed to be uniformly distributed.

In the following table I, we show the average SIR (in dB) for different timing and frequency offset ranges. The

TABLE I: Average SIR comparison for different system settings in various timing and frequency offset ranges

		Relative Timing offset Δn Range			
ϵ		$\pm 5\%$	$\pm 10\%$	$\pm 20\%$	$\pm 50\%$
$\pm 5\%$	CP-OFDM	17.56	16.38	14.11	12.15
	UFMC-CW	29.10	20.10	16.04	13.68
	UFMC-Opt	32.71	20.11	16.06	13.70
$\pm 10\%$	CP-OFDM	17.24	16.15	14.01	12.12
	UFMC-CW	27.26	19.81	15.95	13.65
	UFMC-Opt	27.86	19.80	15.97	13.67
$\pm 20\%$	CP-OFDM	16.22	15.40	13.66	12.00
	UFMC-CW	23.61	18.86	15.61	13.53
	UFMC-Opt	23.52	18.80	15.64	13.56
$\pm 50\%$	CP-OFDM	13.12	12.86	12.20	11.38
	UFMC-CW	17.29	15.60	14.05	12.85
	UFMC-Opt	18.47	15.74	14.17	12.92

Note: The average SIR value for 'UFMC-Opt' is selected from the biggest value of the maximizing SLR and SDLR approach.

average SIR values are calculated by assuming that the timing and frequency offsets are uniformly distributed in the corresponding interval, e.g. a timing and frequency offset range of $\pm 5\%$ means the timing and frequency offsets are uniformly distributed in the interval $\Delta n \in [-0.05, 0.05]$ and $\epsilon \in [-0.05, 0.05]$ respectively. And 'UFMC-CW' denotes the system setting of UFMC with Dolph-Chebyshev filter with a side-lobe attenuation of 40 dB. Furthermore, the biggest average SIR value by using max. SLR and SDLR filter in UFMC can be found under 'UFMC-Opt'. All values under timing offset range of $\pm 5\%$ are obtained by using max. SLR approach, while others are obtained by using max. SDLR approach. With the optimized filter, a average SIR gain of 3.6 dB and 15.1 dB can be achieved against UFMC with Chebyshev-filter (40 dB side-lobe attenuation) and CP-OFDM respectively in the timing and frequency offset range of $\pm 5\%$. Even if the frequency offset is uniformly distributed in its maximum interval $\pm 50\%$ and the timing offset $\pm 5\%$, 1.2 dB and 5.4 dB gain is achievable.

In the Fig. 2, the average SIR are computed for rCFO $\epsilon = 0.05$ and timing offset in the range of ± 0.05. The used filters in UFMC are optimized in the time and frequency offset range of $\pm 5\%$ using max. SLR and max. SDLR, which are shown in Fig. 3. Both the optimized filters are Dirac-like filters, since the frequency offset range is small and the timing offset range is within the ramp-up and ramp-down of the UFMC symbols. This means UFMC converges to zero-prefix OFDM in this case. Both of the method max. SLR and max. SDLR work well in the timing offset range of $\pm 5\%$ and max. SLR approach slightly outperforms max. SDLR in that timing offset range. OFDM is robust against positive timing offset, as long as the delay sample doesn't exceed the CP length. In contrast to OFDM, UFMC is able to protect against both positive and negative timing offset. The average SIR for OFDM is 17.36 dB, while UFMC with Chebyshev filter has the value of 27.97 dB. With the optimized filter using max. SDLR and max. SLR, an additional SIR gain of 0.9 and 1.2 dB can be achieved against UFMC with the Chebyshev filter respectively.

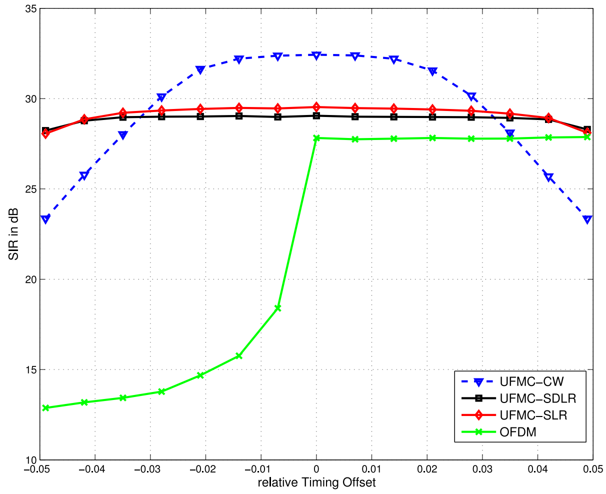

Fig. 2: SIR Comparison for different system settings under rCFO $\epsilon = 0.05$

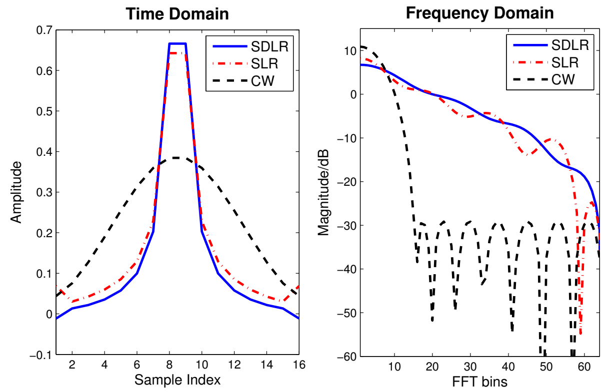

Fig. 3: Filter, optimized for timing and frequency offset in the range of $\pm 5\%$ in time and frequency domain

In the Fig. 4, SIR is compared between the aforementioned four different systems with rCFO $\epsilon = 0.5$ and timing offset again in the range ± 0.05. The used filters are optimized in the timing offset range of $\pm 5\%$ and rCFO range of $\pm 50\%$, shown in Fig 5. For this large rCFO, UFMC is very robust compared to OFDM. The average SIR for OFDM amount to 10.41 dB. UFMC with the Chebyshev filter has the SIR of 13.26 dB. Using the filter optimized by max. SDLR, only 0.2 dB can be additionally gained over UFMC with the Chebyshev filter.

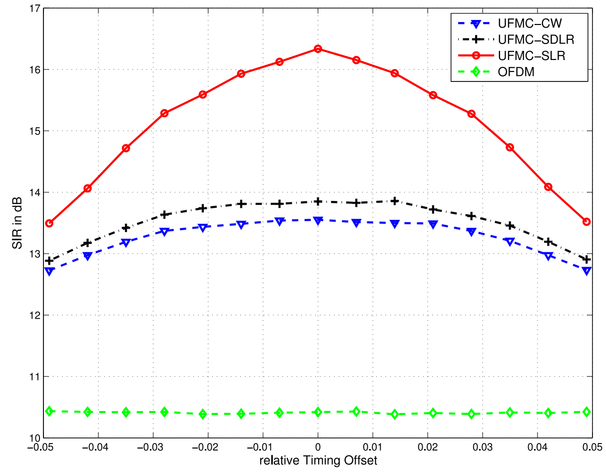

Fig. 4: SIR Comparison for different system settings under rCFO $\epsilon = 0.5$

In this case, the filter optimized by maximizing SLR has similar characteristics of Chebyshev filter with a side lobe attenuation of 11.6 dB. Both the optimized filters have a smaller main lobe width than Chebyshev filter with side lobe attenuation of 40 dB. In the meanwhile, the side lobe level of them are larger than that of Chebyshev filter with side lobe attenuation of 40 dB.

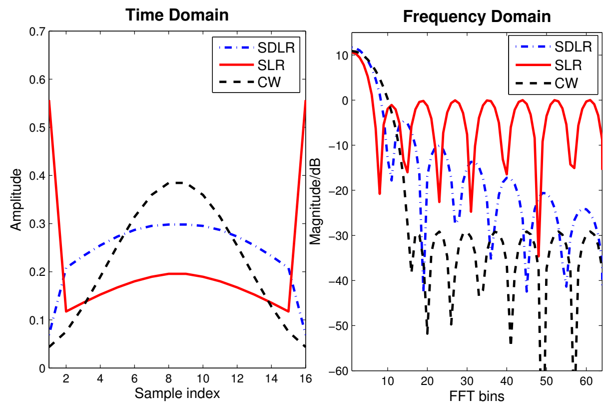

Fig. 5: Filter, optimized for timing offset range of $\pm 5\%$ and rCFO range of $\pm 50\%$ in time and frequency domain

However, 1.7 dB of SIR gain is achievable (again compared with Chebyshev filter), if the filter is optimized by using the criterion maximizing SLR.

V. Summary

In this paper we analyzed a UFMC system in the presence of joint carrier frequency and timing offset. We extended our work of filter optimization for UFMC system from carrier frequency offset to joint timing and frequency offset. The optimization methods we used, maximizing SLR and SDLR, are generalized eigen-value problem and have closed-form solution. Both of the two methods do not require any other knowledge of other subbands, thus each user can optimize its own FIR filter with some knowledge of expected timing and frequency offset distribution before transmission. By optimizing FIR filter, interferences between different sub-bands can be reduced. The main limitation of these optimization is that both of them work well only in a relatively small range of timing offset. However, up to 3.6 dB SIR gain can be achieved in timing and frequency offset range of $\pm 5\%$ against non-optimized Dolph-Chebyshev filter (with side-lobe attenuation of 40 dB) and 15.1 dB against classical CP-OFDM. The results also show that UFMC is more robust than CP-OFDM system in a relaxed synchronization condition, since UFMC is not so sensitive to frequency offset and negative timing offset compared to CP-OFDM.

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