Symbol Error Probability of M-QAM in Multihop Communication Systems with Regenerative Relays

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***Abstract*—We analyze the performance of multihop communication systems with regenerative relays in terms of the average symbol error probability (SEP) for M-ary quadrature amplitude modulation (M-QAM). We derive an exact analytical closed-form expression for the average SEP in time-invariant additive white Gaussian noise channels as well as a generic expression for slow, frequency-flat fading channels, which might be easily evaluated numerically for a wide variety of different fading distributions. For the important case of independent but not necessarily identical Nakagami-m fading on all hops, the average SEP is furthermore given analytically in closed-form. Our results are valid for general rectangular signal point constellations as well as an arbitrary number of intermediate relay stations.**

I. Introduction

Multihop transmission is likely to play an important role in future wireless communication systems since it represents an effective low-cost solution for coverage extension and capacity enhancement of cellular networks as well as a fundamental enabling-technology for wireless ad-hoc and sensor networks [1], [2]. The basic idea of this approach is to split up the link between a source and a destination node into multiple shorter hops, so that the source communicates with the destination only indirectly via a set of intermediate relay stations. These relay stations generally do not require any backhaul connection to a fixed wireline network and might be either part of a (deployed) network infrastructure or they may simply correspond to other users nearby, who in return might revert to other users again when transmitting their own data, thus establishing a collaborative system of mutual support. Since the individual hops are generally much shorter than the distance between source and destination, the detrimental effects of path loss can be mitigated and therefore the total transmit power might be reduced compared to systems without relays [1]. If less transmit power is used, this also leads to a reduction of both the inter- and intra-cell interference level and might facilitate a cell capacity gain since multiple nodes within one cell might transmit data at the same time if they are far enough apart [2].

In the past few years, such multihop communication systems have received a lot of research attention, what is to some extent due to the increasingly popular concept of cooperative communications that has emerged over the past decade [3]. In [4]–[7], Hasna and Alouini as well as Karagiannidis *et al.* thoroughly studied the performance of relayed transmission with different types of relays over Rayleigh and Nakagami-m fading channels, mainly focusing on the average (end-to-end) bit error rate and information outage probability. In [8], the authors have analyzed the average symbol error probability (SEP) of a cooperative amplify-and-forward (AF) system employing M-ary phase shift keying (M-PSK) in Rayleigh fading scenarios and they have presented an exact SEP expression containing only a single integral with finite integration limits, which contains the non-cooperative single-relay case as a special case. Besides, high SNR asymptotics of the average SEPs for general cooperative AF systems recently have been presented in [9] for a wide variety different modulation formats and the average SEP of decode-and-forward (DF) systems with conditional relaying has been investigated in [10]. An exact SEP analysis for DF systems with unconditional relaying has, however, to the best of our knowledge not been presented in literature before. Even for non-cooperative multihop systems, where each node processes only the signals received from its immediate predecessor node—what we will consider herein—the exact average SEP of common modulation schemes is not known yet. In order to close this gap, this correspondence presents an exact average SEP analysis of M-ary quadrature amplitude modulation (M-QAM) in non-cooperative multihop communication systems with an arbitrary number of regenerative decode-and-forward relays, which all perform hard decisions on the signals received from their predecessor node before forwarding them to the next station. In this regard, we consider both, simple additive white Gaussian noise (AWGN) channels as well as slow, frequency-flat fading channels with a special emphasis on the important Nakagami-m fading case.

The remainder of this paper is organized as follows: In Section II, we analyze the average SEP in AWGN whereas the corresponding performance in flat fading channels is investigated in Section III. Some numerical results are presented in Section IV and the conclusions are finally given in Section V.

II. SEP Analysis for AWGN Channels

We consider general symmetrical $I \times J$ rectangular signal point constellations, where $I = 2^\nu$, $\nu \in \mathbb{N}$ and $J = 2^\kappa$, $\kappa \in \mathbb{N}$. The distance between two adjacent signal points will be denoted by d in the following and we assume that the decision thresholds at the receiver are placed exactly in the middle between always two neighboring signal points. Furthermore, we label the individual signal points by $s_{i,j}$ ($i = 1, \ldots, I$, $j = 1, \ldots, J$), where i denotes the row and j the column of the signal point in the corresponding constellation diagram. An example for such a signal point constellation is depicted in Fig. 1 for the important case of 16-QAM modulation.

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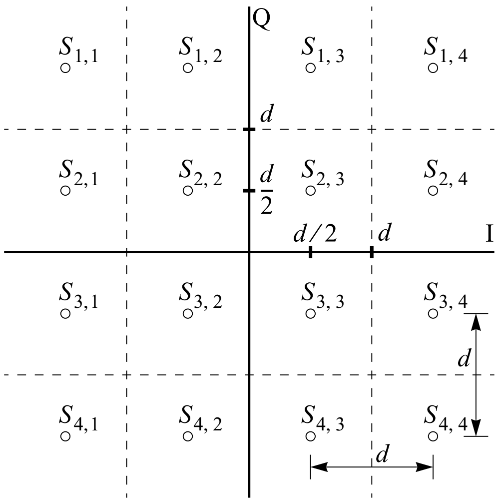

Fig. 1. 4×4 signal point constellation (16-QAM) with symbol labelings and decision thresholds (dashed lines).

A. Dual-Hop Transmission

In dual-hop systems, the data transmission between the source and the destination node generally is subdivided into two different phases: In the first phase, the source transmits a certain data symbol s to the relay, which receives in case of time-invariant AWGN channels $y_{\text{relay}} = s + n_{\text{relay}}$ with $n_{\text{relay}} \sim \mathcal{CN}(0, \sigma^2_{\text{relay}})$, where $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex normal distribution with mean μ and variance σ^2. The relay then performs a hard decision on this noisy symbol and forwards its estimate $\hat{s}_{\text{relay}}$ in the second phase to the actual destination node, which thereupon receives $y_{\text{dest}} = \hat{s}_{\text{relay}} + n_{\text{dest}}$, where $n_{\text{dest}} \sim \mathcal{CN}(0, \sigma^2_{\text{dest}})$. Performing another hard decision on this symbol then eventually yields the final estimate $\hat{s}_{\text{dest}}$ of the originally transmitted symbol s. Clearly, the end-to-end transmission can be error-free even if the estimate of the intermediate relay station was wrong, i.e., if $\hat{s}_{\text{relay}} \neq s$. This might happen if the erroneous decision at the relay is reversed by another erroneous decision at the destination.

For calculating the average probability P_c of *correct* end-to-end transmission, we consequently have to consider all possible combinations of transmitted symbols s, corresponding estimates at the relay $\hat{s}_{\text{relay}}$, and decisions at the destination for which $\hat{s}_{\text{dest}} = s$ again. Hence, it can easily be seen that P_c generally can be expressed as

$$P_c = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{I}\sum_{n=1}^{J} P[\hat{s}_{\text{relay}} = s_{m,n} | s = s_{i,j}] \times P[\hat{s}_{\text{dest}} = s_{i,j} | \hat{s}_{\text{relay}} = s_{m,n}]\, P[s = s_{i,j}], \quad (1)$$

where $P[s = s_{i,j}]$ denotes the probability that the source transmits $s_{i,j}$, $P[\hat{s}_{\text{relay}} = s_{m,n} | s = s_{i,j}]$ the probability that the relay decides in favor of $s_{m,n}$ if the source has transmitted $s_{i,j}$, and $P[\hat{s}_{\text{dest}} = s_{i,j} | \hat{s}_{\text{relay}} = s_{m,n}]$ the probability that the destination decides in favor of $s_{i,j}$ if the relay has forwarded $s_{m,n}$. The corresponding average SEP is then simply given by

$$P_e = 1 - P_c. \quad (2)$$

For simplicity, we always assume in the following that all constellation symbols are sent with equal probability, i.e., that $P[s = s_{i,j}] = \frac{1}{M}\ \forall\ i,j$. An extension of our results to the more general case with different a priori probabilities is straightforward, but would significantly complicate notation. If we additionally exploit the symmetry of the considered signal point constellations, (2) combined with (1) simplifies to

$$P_e = 1 - \frac{4}{M} \sum_{i=1}^{I/2}\sum_{j=1}^{J/2}\sum_{m=1}^{I}\sum_{n=1}^{J} P[\hat{s}_{\text{relay}} = s_{m,n} | s = s_{i,j}] \times P[\hat{s}_{\text{dest}} = s_{i,j} | \hat{s}_{\text{relay}} = s_{m,n}]. \quad (3)$$

Hence, the calculation of the average SEP actually can be traced back to the calculation of the probability that we decide in favor of a symbol $s_{m,n}$ if the symbol $s_{i,j}$ has been transmitted. Please note that this represents a major difference to error calculations of conventional single-hop systems, where it is generally only necessary to determine the probability that the estimated symbol is unequal to the transmitted symbol, without having to care about which particular constellation symbol actually has been estimated if an error occurs.

For notational brevity, we introduce the short-hand notation

$$P_{i,j}^{m,n} := P[\hat{s} = s_{m,n} | s = s_{i,j}] = P\left[s_{i,j} + n \in \mathcal{D}_{m,n}\right] \quad (4)$$

in the following, where $n = n_R + j\, n_I$ ($n_R, n_I \in \mathbb{R}$) denotes a complex Gaussian random variable with zero mean and variance $\sigma^2/2$ per dimension and $\mathcal{D}_{m,n}$ is the decision region associated with $s_{m,n}$. In general, the probability according to (4) clearly depends on the distance between the signal points $s_{i,j}$ and $s_{m,n}$, but also on the type of the signal point $s_{m,n}$, which might be either a corner point, a horizontal or vertical edge point, or an interior point of the considered signal point constellation. In order to distinguish these different cases, we will denote the corresponding probabilities $P_{i,j}^{m,n}$ by $P_{c\,i,j}^{\ m,n}$, $P_{h\,i,j}^{\ m,n}$, $P_{v\,i,j}^{\ m,n}$, and $P_{0\,i,j}^{\ m,n}$, respectively, i.e., we have

$$P_{i,j}^{m,n} = \begin{cases} P_{c\,i,j}^{\ m,n} & \text{if } m \in \{1, I\} \wedge n \in \{1, J\} \\ P_{h\,i,j}^{\ m,n} & \text{if } m \in \{1, I\} \wedge n \in \{2, \ldots, J-1\} \\ P_{v\,i,j}^{\ m,n} & \text{if } m \in \{2, \ldots, I-1\} \wedge n \in \{1, J\} \\ P_{0\,i,j}^{\ m,n} & \text{if } m \in \{2, \ldots, I-1\} \\ & \quad \wedge n \in \{2, \ldots, J-1\} \end{cases} \quad (5)$$

If $s_{m,n}$ is a corner point, the probability that we decide in favor of $s_{m,n}$ in case that $s_{i,j}$ has been transmitted can easily be shown to be given by

$$P_{c\,i,j}^{\ m,n} = P\left[\left(|n-j| - \tfrac{1}{2}\right) d < n_R\right] P\left[\left(|m-i| - \tfrac{1}{2}\right) d < n_I\right] \quad (6)$$

which can be calculated by means of the Gaussian Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-u^2/2\right) du$ as

$$P_{c\,i,j}^{\ m,n} = Q\left(\frac{\left(|n-j| - \frac{1}{2}\right) d}{\sigma/\sqrt{2}}\right) Q\left(\frac{\left(|m-i| - \frac{1}{2}\right) d}{\sigma/\sqrt{2}}\right). \quad (7)$$

Noting that the mean energy per transmit symbol $\bar{E}_s$ of the considered M-QAM constellations is generally given by $\bar{E}_s =$

$(I^2+J^2-2)d^2/12$, we can relate the average SNR $\gamma = \bar{E}_s/\sigma^2$ to the inter-symbol distance d as $\frac{d}{\sigma/\sqrt{2}} = \sqrt{\gamma\,\chi}$, where we have introduced for brevity the short-hand notation

$$\chi = \frac{24}{I^2+J^2-2}. \tag{8}$$

Hence, (7) can be expressed in terms of the SNR γ as

$$P_{c\,i,j}^{\;m,n}(\gamma) = Q\left(\sqrt{\gamma\chi}\left(|n-j|-\tfrac{1}{2}\right)\right) Q\left(\sqrt{\gamma\chi}\left(|m-i|-\tfrac{1}{2}\right)\right). \tag{9}$$

Likewise, we get for the other signal point types

$$P_{h\,i,j}^{\;m,n}(\gamma) = \left[Q\left(\sqrt{\gamma\chi}\left(|n-j|-\tfrac{1}{2}\right)\right) - Q\left(\sqrt{\gamma\chi}\left(|n-j|+\tfrac{1}{2}\right)\right)\right] Q\left(\sqrt{\gamma\chi}\left(|m-i|-\tfrac{1}{2}\right)\right) \tag{10}$$

$$P_{v\,i,j}^{\;m,n}(\gamma) = \left[Q\left(\sqrt{\gamma\chi}\left(|m-i|-\tfrac{1}{2}\right)\right) - Q\left(\sqrt{\gamma\chi}\left(|m-i|+\tfrac{1}{2}\right)\right)\right] Q\left(\sqrt{\gamma\chi}\left(|n-j|-\tfrac{1}{2}\right)\right) \tag{11}$$

$$P_{0\,i,j}^{\;m,n}(\gamma) = \left[Q\left(\sqrt{\gamma\chi}\left(|m-i|-\tfrac{1}{2}\right)\right) - Q\left(\sqrt{\gamma\chi}\left(|m-i|+\tfrac{1}{2}\right)\right)\right] \left[Q\left(\sqrt{\gamma\chi}\left(|n-j|-\tfrac{1}{2}\right)\right) - Q\left(\sqrt{\gamma\chi}\left(|n-j|+\tfrac{1}{2}\right)\right)\right]. \tag{12}$$

Denoting the SNR on the source-to-relay link by γ_R and the SNR on the relay-to-destination link by γ_D, the exact average SEP hence can be expressed based on (3) as

$$P_e(\gamma_R, \gamma_D) = 1 - \frac{4}{M} \sum_{i=1}^{I/2} \sum_{j=1}^{J/2} \sum_{m=1}^{I} \sum_{n=1}^{J} P_{i,j}^{m,n}(\gamma_R)\, P_{m,n}^{i,j}(\gamma_D), \tag{13}$$

with $P_{m,n}^{i,j}(\gamma)$ according to (5) with (9), (10), (11), and (12).

B. Systems with Multiple Relays

In the general case of a multihop system with N consecutive relays, the basic idea for determining the average SEP is the same as before, i.e., we have to consider all possible combinations of correct and erroneous decisions at the intermediate relay stations and the actual destination node leading to error-free end-to-end transmission. If we denote the SNR on the i-th hop ($i = 0, \ldots, N$) by γ_i, we consequently obtain

$$P_e = 1 - \frac{1}{M} \sum_{\{m_i,n_i\}_{i=0}^{N}} P_{m_0,n_0}^{m_1,n_1}(\gamma_0) P_{m_1,n_1}^{m_2,n_2}(\gamma_1) \times P_{m_2,n_2}^{m_3,n_3}(\gamma_2) \cdots P_{m_{N-1},n_{N-1}}^{m_N,n_N}(\gamma_{N-1}) P_{m_N,n_N}^{m_0,n_0}(\gamma_N) \tag{14}$$

which can be reformulated in a more compact form as

$$P_e = 1 - \frac{1}{M} \sum_{\{m_i,n_i\}_{i=0}^{N}} P_{m_N,n_N}^{m_0,n_0}(\gamma_N) \prod_{i=0}^{N-1} P_{m_i,n_i}^{m_{i+1},n_{i+1}}(\gamma_i), \tag{15}$$

where the summation has to be taken over all possible index tuples $\{m_0, n_0, m_1, n_1, \ldots, m_N, n_N\}$ with $m_i = 1, \ldots, I$ and $n_i = 1, \ldots, J$ and with $P_{i,j}^{m,n}(\gamma)$ according to (5).

III. Average SEP in Fading Channels

In slow, frequency-flat fading channels, the average SEP generally can be calculated by averaging our result for AWGN channels according to (15) over the joint distribution of the SNRs of the individual hops. In this regard, we assume that all hops are independent of each other, what should be approximately fulfilled in almost all cases of practical interest. If we denote the probability density function of the SNR on the i-th hop by $p_{\gamma_i}(\gamma)$, the average SEP is generally given by

$$\bar{P}_e = 1 - \frac{1}{M} \underbrace{\int_0^\infty \cdots \int_0^\infty}_{(N+1)-\text{fold}} \sum_{\{m_i,n_i\}_{i=0}^{N}} P_{m_N,n_N}^{m_0,n_0}(\gamma_N) \times p_{\gamma_N}(\gamma_N) d\gamma_N \prod_{i=0}^{N-1} P_{m_i,n_i}^{m_{i+1},n_{i+1}}(\gamma_i) p_{\gamma_i}(\gamma_i) d\gamma_i, \tag{16}$$

with $P_{i,j}^{m,n}(\gamma)$ according to (5). After changing the order of integration and summation and rearranging the terms in an appropriate way, this can be rewritten as

$$\bar{P}_e = 1 - \tfrac{1}{M} \sum_{\{m_i,n_i\}_{i=0}^{N}} \bar{P}_{m_N,n_N}^{m_0,n_0}(N) \prod_{i=0}^{N-1} \bar{P}_{m_i,n_i}^{m_{i+1},n_{i+1}}(i), \tag{17}$$

where we have introduced for brevity the short-hand notation

$$\bar{P}_{i,j}^{m,n}(k) = \int_0^\infty P_{i,j}^{m,n}(\gamma)\, p_{\gamma_k}(\gamma) d\gamma, \tag{18}$$

which simply corresponds to the average probability that the $(k+1)$-th node detects the symbol $s_{m,n}$ if node k has forwarded the symbol $s_{i,j}$. This probability generally can be calculated based on (5) and as before, we have to distinguish four different cases again, depending on the actual values of m and n. Introducing the short-hand notation

$$\Upsilon_k(c_1, c_2) = \int_0^\infty Q\left(\sqrt{\gamma\chi}\left(c_1 - \tfrac{1}{2}\right)\right) \times Q\left(\sqrt{\gamma\chi}\left(c_2 - \tfrac{1}{2}\right)\right) p_{\gamma_k}(\gamma) d\gamma, \tag{19}$$

where $c_1, c_2 \in \mathbb{R}$, it can easily be seen based on (9) – (12) that for $m \in \{1, I\} \wedge n \in \{1, J\}$, we have

$$\bar{P}_{i,j}^{m,n}(k) = \Upsilon_k\left(|n-j|, |m-i|\right), \tag{20}$$

for $m \in \{1, I\} \wedge n \in \{2, \ldots, J-1\}$

$$\bar{P}_{i,j}^{m,n}(k) = \Upsilon_k\left(|n-j|, |m-i|\right) - \Upsilon_k\left(|n-j|+1, |m-i|\right) \tag{21}$$

for $m \in \{2, \ldots, I-1\} \wedge n \in \{1, J\}$

$$\bar{P}_{i,j}^{m,n}(k) = \Upsilon_k\left(|n-j|, |m-i|\right) - \Upsilon_k\left(|n-j|, |m-i|+1\right) \tag{22}$$

and for $m \in \{2, \ldots, I-1\} \wedge n \in \{2, \ldots, J-1\}$

$$\bar{P}_{i,j}^{m,n}(k) = \Upsilon_k\left(|n-j|, |m-i|\right) - \Upsilon_k\left(|n-j|, |m-i|+1\right) - \Upsilon_k\left(|n-j|+1, |m-i|\right) + \Upsilon_k\left(|n-j|+1, |m-i|+1\right). \tag{23}$$

If $c_1 \geq 1/2$ and $c_2 \geq 1/2$, (19) can be rewritten by capitalizing on [11, eq. (4.8)] as

$$\zeta_k(c_1, c_2) = \Upsilon_k(c_1, c_2)\big|_{c_1, c_2 \geq \frac{1}{2}} \tag{24}$$

$$= \frac{1}{2\pi}\left[\int_0^{\frac{\pi}{2}-\xi} M_{\gamma_k}\left(\frac{-\chi\left(c_1 - \frac{1}{2}\right)^2}{2\sin^2\theta}\right) d\theta + \int_0^{\xi} M_{\gamma_k}\left(\frac{-\chi\left(c_2 - \frac{1}{2}\right)^2}{2\sin^2\theta}\right) d\theta\right], \tag{25}$$

where $M_{\gamma_k}(s)$ denotes the moment-generating function (mgf) of γ_k and with the short-hand notation $\xi = \arctan \frac{2c_2-1}{2c_1-1}$. Please note that (25) might be easily evaluated numerically for a wide variety of different fading distributions since it involves only single integrals with finite integration limits. However, it can easily be seen from (20) – (23) that for calculating the average SEP we generally also have to evaluate (19) for $c_1, c_2 < \frac{1}{2}$, but fortunately these cases can be traced back to the previously considered case by exploiting that $Q(-x) = 1 - Q(x)$. By doing so, we obtain for arbitrary c_1 and c_2

$$\Upsilon_k(c_1, c_2) = \begin{cases} \zeta_k(c_1, c_2) & \text{if } c_1, c_2 \geq \frac{1}{2} \\ 2\,\zeta_k\left(c_1, \frac{1}{2}\right) - \zeta_k\left(c_1, 1-c_2\right) & \text{if } c_1 \geq \frac{1}{2}, c_2 < \frac{1}{2} \\ 2\,\zeta_k\left(\frac{1}{2}, c_2\right) - \zeta_k\left(1-c_1, c_2\right) & \text{if } c_1 < \frac{1}{2}, c_2 \geq \frac{1}{2} \\ 1 - 2\zeta_k\left(\frac{1}{2}, 1-c_2\right) - 2\,\zeta_k\left(1-c_1, \frac{1}{2}\right) & \\ \quad +\zeta_k\left(1-c_1, 1-c_2\right) & \text{else} \end{cases} . \quad (26)$$

For many scenarios of practical interest, (19) and hence also the average SEP actually might be even given in closed-form. For the important case of Nakagami-m fading with integer fading parameter m_k and average SNR $\bar{\gamma}_k$ on the k-th hop, for example, we have $M_{\gamma_k}(s) = (1 - s\bar{\gamma}_k/m_k)^{-m_k}$ and by making use of [12, eq. (6)], we obtain

$$\begin{aligned} \zeta_k(c_1, c_2) &= \frac{1}{4} - \frac{1}{2\pi} \sum_{\nu=0}^{m_k-1} \binom{2\nu}{\nu} \\ &\times \left[\frac{\lambda_k(c_1, c_2)}{\left[4\left(1 + \frac{(c_1-\frac{1}{2})^2}{\alpha_k}\right)\right]^\nu} + \frac{\lambda_k(c_2, c_1)}{\left[4\left(1 + \frac{(c_2-\frac{1}{2})^2}{\alpha_k}\right)\right]^\nu} \right] \\ &+ \frac{1}{2\pi} \sum_{\nu=1}^{m_k-1} \sum_{\eta=1}^{\nu} T_{\eta\nu} \left(c_1 - \frac{1}{2}\right)\left(c_2 - \frac{1}{2}\right) \\ &\times \frac{\left(\alpha_k + (c_1 - \frac{1}{2})^2\right)^{-\eta} + \left(\alpha_k + (c_2 - \frac{1}{2})^2\right)^{-\eta}}{\alpha_k^{-\nu}\left(\alpha_k + (c_1 - \frac{1}{2})^2 + (c_2 - \frac{1}{2})^2\right)^{\nu-\eta+1}} \end{aligned} \quad (27)$$

where $\alpha_k = \frac{2m_k}{\bar{\gamma}_k \chi}$, $T_{\eta\nu} = \frac{\binom{2\nu}{\nu}}{\binom{2(\nu-\eta)}{\nu-\eta} 4^\eta (2(\nu-\eta)+1)}$, and

$$\lambda_k(a, b) = \sqrt{\frac{(a-\frac{1}{2})^2}{\alpha_k + (a-\frac{1}{2})^2}} \arctan\left(\sqrt{\frac{\alpha_k + (a-\frac{1}{2})^2}{(b-\frac{1}{2})^2}}\right). \quad (28)$$

Inserting (27) in (26) then yields together with (20) – (23) as well as (17) the desired exact analytical closed-form expression for the average SEP in Nakagami-m fading.

IV. Numerical Results

Figure 2 depicts the average SEP of a dual-hop system in AWGN versus the average SNR $\bar{\gamma}_R$ of the source-to-relay link for 16-QAM as well as 8-QAM and several different average SNRs $\bar{\gamma}_D$ of the relay-to-destination link. Obviously, for small values of $\bar{\gamma}_R$, the average SEP is basically independent of $\bar{\gamma}_D$, because in this case the overall SEP is dominated by the source-to-relay link. However, for large $\bar{\gamma}_R$, it is just the other way around and therefore increasing $\bar{\gamma}_R$ does not lead to a further reduction of the error probability. In this regard, the observed error floors depend only on the statistical properties of the relay-to-destination link since for $\bar{\gamma}_R \to \infty$ the source-to-relay link is error-free and hence the overall SEP simply corresponds to the average SEP on the second hop. Finally, it can be seen that there is a perfect match between our analytical results and results obtained from Monte-Carlo simulations, what verifies the accuracy of our theoretical analysis.

An increase in the number of hops generally results in a loss of spectral efficiency since always only one node may transmit at a time. This loss might be compensated by increasing the modulation order, for example, but actually it is not obvious at a first glance whether it is better to employ a higher-order modulation scheme with multiple hops or to transmit directly from the source to the destination with a more robust modulation scheme. For that reason, Fig. 3 shows a comparison between the average SEP for direct transmission and QPSK modulation as well as for a dual-hop system with 16-QAM for different Nakagami-m fading parameters. In this regard, we assume that the relay is placed on the direct connection line between source and destination at the relative distance x from the source, i.e., for $x = 0.5$, for example, the relay would be exactly in the middle of them. Furthermore, we assume that the received power decays with distance as $P_{\text{rx}} \sim x^{-\nu}$, where ν is the path loss exponent, which is always set to $\nu = 4$ in the following. Please note that this represents a simple yet useful model for the path loss in urban environments, for instance. Finally, we assume that the relay transmits at the same power as the source and that the fading parameters of both hops of the dual-hop system are equal and correspond to the fading parameter of the direct link of the single-hop reference system. Given the average SNR $\bar{\gamma}_0$ of the direct link, the average SNRs of the dual-hop system then simply are given by $\bar{\gamma}_0\, x^{-\nu}$ and $\bar{\gamma}_0\,(1-x)^{-\nu}$, respectively.

As can be seen from Fig. 3, the position of the relay is very crucial for the performance of the dual-hop system. If the relay is placed exactly in the middle between source and destination ($x = 0.5$), the average SEP might be significantly reduced compared to direct transmission and the performance gain obviously is an increasing function of the fading parameter m. However, if the relay is located closer to the source ($x = 0.25$), the performance of the dual-hop system is always much worse than the one for the single-hop system.

The impact of the relay position on the average SEP is illustrated in more detail in Fig. 4. Apparently, in Rayleigh fading ($m = 1$) the average SEP can only be marginally reduced compared to direct transmission and only if the relay is located in the middle between source and destination whereas otherwise the performance is getting worse. In less severe fading ($m > 1$), the range of possible values of x for which we obtain a reduction of the error probability in case of dual-hop transmission is increased and also the performance gain becomes more significant. This is because in less severe fading the relative difference between the average SEPs on the first and the second hop becomes more pronounced and the overall SEP is more and more governed by the average SEP of the weaker hop, which might be smaller than the average SEP of the direct transmission even though a higher-order modulation scheme is used due to the reduced path loss.

Finally, it should be noted that 16-QAM generally entails a higher complexity and requires amplitude information of the received signal/channel whereas in case of QPSK phase information is sufficient. On the other hand, relayed transmission might also be useful for combating shadowing effects due to shielding obstacles, for example, which we did not take into account in our considerations, but which increases the number of scenarios where multihop transmission might be advantageous compared to direct transmission.

V. Conclusion

We have analyzed the average SEP of multihop transmission with rectangular M-QAM for an arbitrary number of regenerative relays. We have derived an exact analytical closed-form expression for time-invariant AWGN channels as well as a generic expression based on the mgfs of the SNRs of the individual hops for flat fading channels, which was given in closed-form for the important case of Nakagami-m fading on all hops. Based on these results, we have investigated whether relayed transmission actually outperforms direct transmission for a given spectral efficiency and it turned out that this strongly depends on the relay position and the fading severity.

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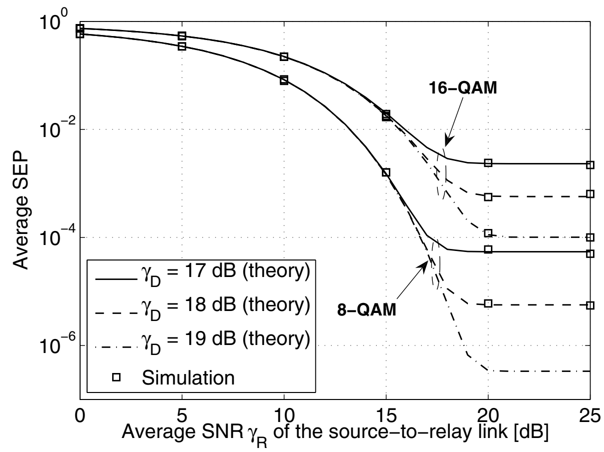

Fig. 2. Average SEP for dual-hop transmission in AWGN for 8-QAM and 16-QAM and several different SNRs γ_D on the relay-to-destination link.

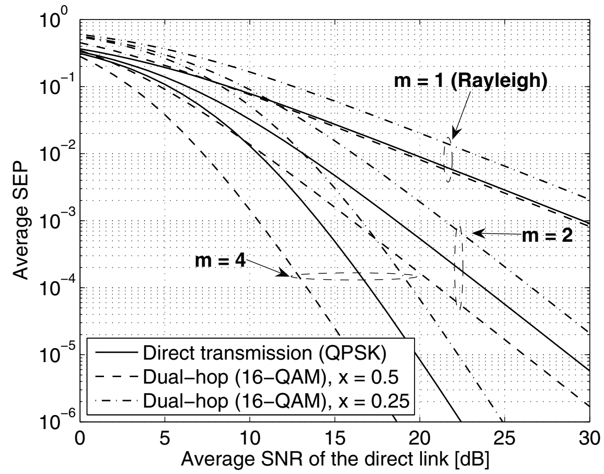

Fig. 3. Comparison of the average SEP for direct transmission with QPSK and dual-hop transmission with 16-QAM for two different relative locations x of the relay station and a path loss exponent of $\nu = 4$.

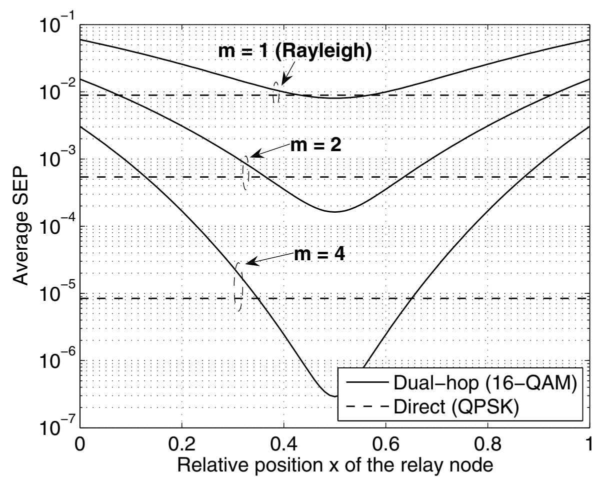

Fig. 4. Impact of the relative relay position on the average SEP for dual-hop transmission with 16-QAM, a path loss exponent of $\nu = 4$, and $\bar{\gamma}_0 = 20\,\text{dB}$.