Fiber Capacity Limits With Optimized Ring Constellations

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***Abstract*—In this letter, we calculate theoretical limits to fiber-optic transport capacity under the constraint of requiring the use of concentric multiring modulation alphabets. Subject to these conditions, we vary ring amplitude ratios and occupation probabilities to determine the capacity-optimized ring constellations. The technique used for fiber transmission calculations accurately captures all fiber nonlinear effects, with the accuracy limited by the noise and signal statistical properties only.**

***Index Terms*—Amplitude and phase modulation, channel capacity, Kerr fiber nonlinearity, optical fiber communication.**

I. Introduction

The concept of channel capacity as the maximum information rate that can be carried by a communication channel within a given bandwidth was first introduced by Shannon in 1948 [1], who also established a closed-form capacity formula for a memoryless additive white Gaussian noise channel. An extension of Shannon's work to optical fiber communications, including Kerr fiber nonlinearity has continuously attracted attention in various circles [2]–[5], and has most recently been approached using numerical solutions of the underlying nonlinear Schrödinger equation for bandlimited [6] and minimum bandwidth [7] modulated Nyquist pulse streams. These studies are not restrictive in that they include all fiber nonlinear effects. Furthermore, they provide an integrated connection among nonlinearity compensation, signaling constellation, and Nyquist filtering options. It was shown in [7] that a capacity of 5.6 bit/s·Hz^{-1} can be achieved for a 2000-km single polarization wavelength-division multiplexing (WDM) transmission using multiring phase-modulated formats with uniform ring spacings and equal frequencies of occupation.

In this letter, we extend the investigations of [6] and [7] by optimizing the concentric ring constellations in terms of ring amplitude ratios and frequencies of occupation. We show that the seeming anomaly of a drop of capacity for ring constellations with more than one ring below the one-ring case in the high SNR regime [6], [7] vanishes. Moreover, the maximum achievable capacity can be improved by up to 0.5 bit/s·Hz^{-1}, depending on the number of rings. This result was obtained using a common type of dispersion map with small but nonzero residual dispersion per span.

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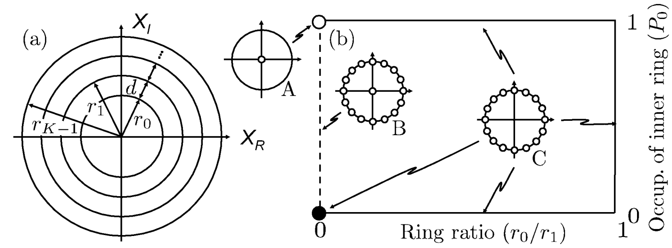

Fig. 1. (a) Constellation with K rings (K-ASK/PSK). (b) Parameter space for the two-ring constellation (i.e., $K = 2$), with pictures of the constellation at the extremes. Constellation A is for the empty circle, B for the dashed line, and C for the full line and filled circle.

II. Capacity Calculation and Constellation

The channel capacity for a specified random channel input X, giving rise to the random channel output Y, is given by the relation $C/B = \max_{p(x)}\{H(Y) - H(Y \mid X)\}$ [1], where B is the bandwidth assigned to each WDM channel, and $H(Y)$ and $H(Y \mid X)$ are referred to as the entropy of Y and the entropy of Y conditioned on X, respectively, and $\max_{p(x)}$ is the maximization over all input distributions $p(x)$. Mutual information assumes optimum coding. We numerically evaluate fiber capacity using a discrete memoryless channel [1] model that lower bounds capacity. Such a model is in part motivated by the complete removal of the channel memory associated with signal–signal intrachannel nonlinearities [8] through reverse propagation [9] at the receiver.

We estimate the capacity for a constellation of K concentric rings with discrete amplitudes [amplitude-shift keying (K-ASK)] and essentially continuous random phase (phase-shift keying (PSK) with 2^{20} points on a uniform angular grid), as depicted in Fig. 1(a). For numerical simulation, up to 2048 symbols are randomly selected from all possible symbols of the K-ASK/PSK constellation. To verify convergence and stability, we performed capacity calculation with different random sequences of lengths 128, 512, 2048, and, in some instances, 8192 symbols. The variance in capacity decreases with an increasing number of symbols. We estimated the variance to be a few tenths of bits/second/hertz for 2048 symbols.

A single state of polarization is considered in this letter. For optical dispersion compensation, we use a singly periodic dispersion map with -1050 ps/nm precompensation and a residual dispersion per span of 20 ps/nm applied after each of the 20 100-km spans. The dispersion is brought to zero at the receiver. Such a map is efficient in reducing distortions from

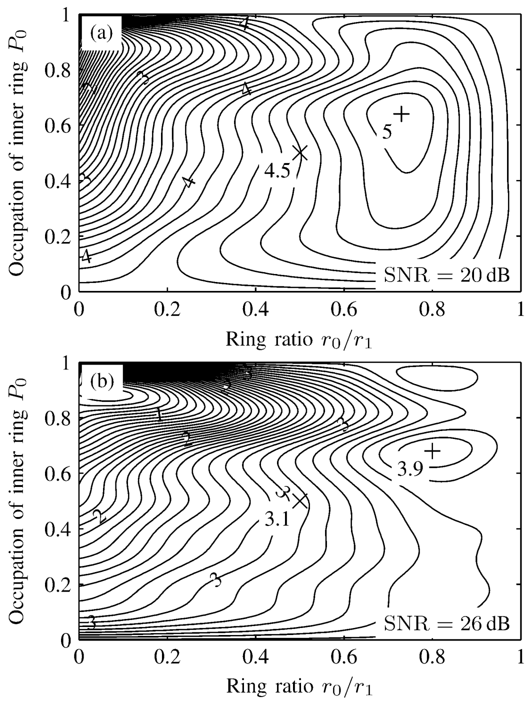

Fig. 2. Capacity per unit bandwidth as a function of the ring ratio r_0/r_1 and the occupancy of the inner ring P_0 for two-ring constellations at an SNR of: (a) 20 and (b) 26 dB.

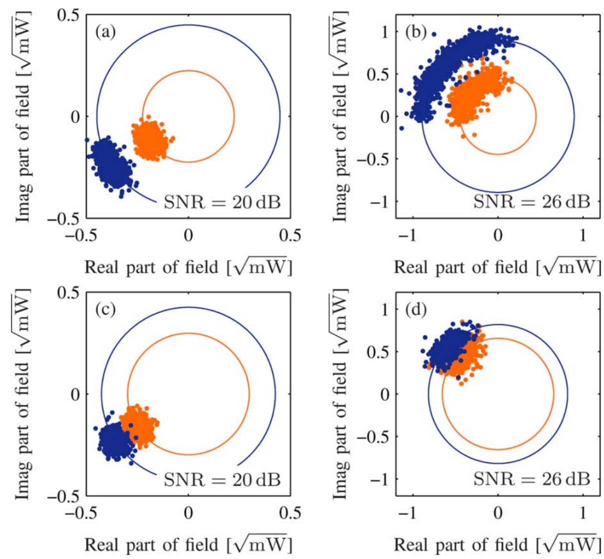

Fig. 3. Signal constellation after transmission, back rotated by the original modulation angle: (a) and (b) without, and (c) and (d) with optimization of the ring ratio r_0/r_1 and the frequency of occupation of the inner ring P_0.

signal–signal intrachannel nonlinearities [10] in the absence of intrachannel nonlinearity compensation [6]. The noise from distributed Raman amplification with gain exactly compensating loss (e.g., see [7, Refs. 26 and 27]) is continuously added during propagation, thereby accurately capturing all signal noise nonlinear interactions.

III. Optimization of the Two-Ring Constellation

We first examine the case of $K = 2$ concentric rings (i.e., 2-ASK/PSK), and evaluate the capacity for all possible ratios of ring radii (r_0/r_1) and frequencies of occupation P_0 and $P_1 = 1 - P_0$. As shown in Fig. 1(b), we varied both P_0 and r_0/r_1 between 0 and 1. Fig. 1(b) (insets) illustrates the extreme cases of the two-ring constellations.

For each fiber launch power, we exhaustively searched the parameter space of Fig. 1(b) for the optimum parameter set $\{r_0/r_1, P_0\}$ that maximizes capacity. Examples are presented in Fig. 2(a) for an SNR of 20 dB near the capacity peak, and Fig. 2(b) for an SNR of 26 dB deep into the nonlinear regime of transmission. The contour lines are spaced at 0.1 bit/s$\cdot$Hz^{-1}. The points marked with "$\times$" refer to the initial two-ring constellations of [6] and [7], where the outer ring had twice the radius of the inner ring and both rings had the same frequency of occupation. The points marked with "+" denote the capacity-optimized two-ring constellations. For both SNR values, we observe that the optimum values of r_0/r_1 and P_0 are larger than for the initial constellation, i.e., the two rings get closer together and points on the inner ring are more frequently transmitted. For an SNR of 20(26) dB, the optimum ring ratio is about 0.7(0.8) and the frequency of occupation of the inner ring is about 0.6(0.7). The gain in capacity by optimizing the signal constellation compared to the initial constellation is 0.5(0.8) bit/s$\cdot$Hz^{-1} for an SNR of 20(26) dB. Most of the capacity increase is due to optimizing r_0/r_1; optimization of P_0 improves the capacity by less than 0.1(0.3) bit/s$\cdot$Hz^{-1} for an SNR of 20(26) dB and is therefore less consequential.

The constellation evolution is shown in Fig. 3, again for the two SNR values of 20 and 26 dB. In Fig. 3, each received signal point is back rotated by its original phase angle. The clouds for the initial constellation in Fig. 3(a) and (b) show a significantly larger angular spread than the clouds for the optimized constellations in Fig. 3(c) and (d). Thus, more symbols can be placed on one ring without too much overlap of the corresponding clouds, thus leading to higher capacities. The reduced angular spread outweighs the larger radial overlap in the optimized constellations.

IV. Optimization of the Multiring Constellation

To optimize general multiring constellations for $K > 2$, an exhaustive search would involve $2K - 2$ dimensions [K ring radii $r_k (k = 0, 1, \ldots, K-1)$ and K frequencies of occupation $P_k (k = 0, 1, \ldots, K-1)$], which quickly becomes computationally untractable. For instance, using ten points for each dimension results in $10^{2\times 4-2} = 1$ million simulation cases already for $K = 4$. Therefore, we consider the following restrictions inspired from the two-ring constellation optimization. First, we define a constant distance d between each pair of rings, as depicted in Fig. 1(a), i.e., $r_k = r_0 + kd$. With this restriction and the constraint of a fixed total average power per WDM channel, the K radii are determined by a single parameter, which we choose to be the ratio of the innermost ring radius to the outermost ring radius r_0/r_{K-1}. By increasing or decreasing r_0/r_{K-1}, the rings simultaneously move closer together or further apart (like accordion pleats). The constellation investigated in [6] and [7], where $r_k = (k+1)d$, is given by $r_0/r_{K-1} = 1/K$.

Second, we assume equal frequencies of occupation $P_k = 1/K$ for all rings, as in [6] and [7]. This assumption is motivated by the fact that we observed only minor improvements in capacity when optimizing P_0 in the two-ring case. Thus, for any

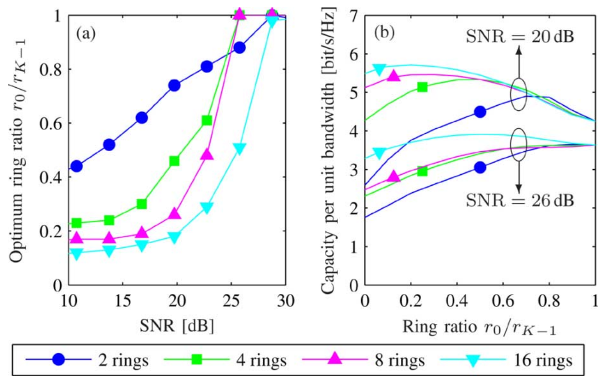

Fig. 4. (a) Optimum ring ratio r_0/r_{K-1} as a function of the SNR. (b) Capacity per unit bandwidth as a function of the ring ratio r_0/r_{K-1} for equal frequency of occupation for all rings.

number of rings K, we are left with a single optimization parameter for our multiring constellations, the ring ratio $r_0/r_{K-1} \in [0, 1]$. The two extremes $r_0/r_{K-1} = 0$ and $r_0/r_{K-1} = 1$ refer to the case where the inner ring collapses to the point (0,0) and the case where all rings have the same radius, which degenerates to a single ring.

Fig. 4(a) shows the optimum ring ratio r_0/r_{K-1}, which maximizes capacity, as a function of the SNR for different numbers of rings. For $K = 2$, the figure shows the optimum ring ratio under the constraint $P_0 = P_1 = 1/2$. We always observe an increasing optimum ring ratio for increasing SNR, which we discussed for the two-ring case along with Fig. 3. For an SNR of about 25–30 dB, the optimum r_0/r_{K-1} approaches 1 for any K, and the optimum constellation degenerates to one ring.

The variation of capacity as a function of r_0/r_{K-1} is shown in Fig. 4(b) for two different SNR values of 20 and 26 dB. Again, the graphs for $K = 2$ apply to $P_0 = P_1 = 1/2$. Each line of Fig. 4(b) is drawn from 11 points obtained by scanning r_0/r_{K-1} from 0 to 1 by a step of 0.1. The positions of the markers refer to the constellations of [6] and [7], i.e., $r_0/r_{K-1} = 1/K$. The gain in capacity decreases for an increasing number of rings for an SNR of 20 dB, and is almost independent of K for an SNR of 26 dB.

Finally, Fig. 5 shows the capacity as a function of SNR. Fig. 5(a) presents the results for the constellations of [6] and [7], where $r_0/r_{K-1} = 1/K$ and $P_k = 1/K$. (Slight differences compared to the curves in [7] are due to statistical variations and the use of back propagation at the receiver here, instead of the transmitter.) Note that the unexpected drop in the capacity below the one-ring case for multiring constellations at SNRs greater than 25 dB is eliminated by constellation optimization in Fig. 5(b). The multiring capacities now converge to the one-ring capacity at high SNR, which is expected from the fact that r_0/r_{K-1} approaches 1 at high SNR, resulting in a degenerated one-ring constellation. The maximum capacity, occurring for an SNR of 20 dB independent of the number of rings, is improved by 0.5 bit/s·Hz^{-1} for $K = 2$ rings. The improvement is negligible for more than two rings.

V. Conclusion

We presented a ring constellation optimization of capacity limit estimates for transport in optically routed WDM networks of 2000 km in size. It was found that the capacity increases by about 0.5 bit/s·Hz^{-1} from optimization of the two-ring constellation, while the increase was negligible for a larger number of rings.

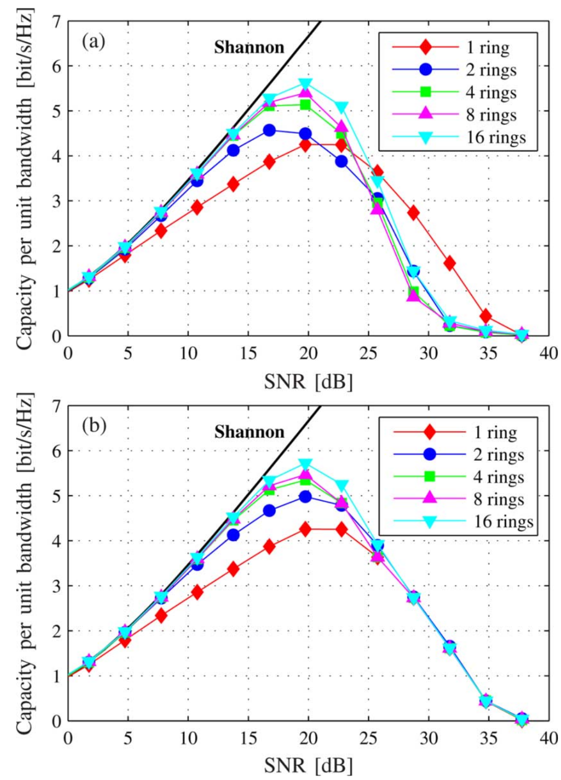

Fig. 5. Capacity per unit bandwidth as a function of the SNR: (a) without and (b) with optimization of the ring ratio r_0/r_{K-1}.

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