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Switch-and-Stay Transmit Diversity for Cooperative Decode-and-Forward Systems

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***Abstract*—We consider the application of switch-and-stay transmit diversity (SSTD) to a three-node cooperative diversity system with decode-and-forward relaying. The basic idea of SSTD generally is to use a certain transmit antenna as long as the induced signal quality satisfies a given requirement while the transmitter switches to another antenna element if this is no longer the case. Applying this principle to the source node of a cooperative diversity system is significantly more complex than for conventional single-hop systems since the selected antenna element generally affects both the source-to-relay and the source-to-destination link. In this regard, we consider three different approaches, namely relay-driven, destination-driven, as well as jointly-driven optimal switching, always targeting at minimizing the outage probability between source and destination. For all cases, we determine exact analytical closed-form expressions for the resulting outage probability assuming Rayleigh-fading on all hops as well as asymptotic expressions, based on which it is possible to directly quantify the achievable diversity order.**

I. Introduction

Cooperative diversity systems are well-known to offer many advantages over conventional point-to-point transmission and therefore they have received a considerable amount of research attention in recent years [1], [2]. Among others, significant performance gains might be achieved by establishing virtual multiple antenna systems and exploiting spatial diversity in a distributed fashion, even if all involved nodes are equipped with a single antenna element only. Clearly, further performance improvements are possible if all nodes themselves have multiple co-located antenna elements, thus being able to combine the benefits of conventional multiple-input multiple-output (MIMO) transmission with those of cooperative diversity systems [3], [4]. However, the proper application of traditional MIMO schemes to such cooperative diversity systems is generally not straightforward since multiple channels as well as the signal processing at the relay stations have to be taken into account. Therefore, the refinement and redesign of existing MIMO schemes such that they also account for intermediate relay stations and their mode of operation represents an important and interesting field of research.

In this paper, we address the problem of performing switch-and-stay transmit diversity (SSTD) at the source node of a three-node cooperative diversity system with regenerative relaying. In general, SSTD represents a low-complexity antenna selection technique at the transmitter-side, similar to switch-and-stay combining at the receiver-side [5]–[9]. For conventional point-to-point transmission systems, this scheme has been extensively analyzed in literature, see for example [10]. The basic idea of this approach is to use a certain transmit antenna as long as the induced signal quality is satisfactory whereas otherwise the transmitter switches to another antenna element. As always only one antenna element is active, only one radio frequency chain is required, thus allowing for a rather cheap and low-complexity implementation in practical systems. Besides, compared to an alternative approach where before each transmission always the best antenna is selected—which has recently been investigated for cooperative diversity systems in [11]—SSTD may significantly reduce the number of switching instants and hence further reduce the complexity and power consumption. The main difficulty of applying SSTD to cooperative diversity systems is that there is generally no node which has perfect channel state information (csi) of all involved hops and hence could determine the switching instants in an optimal way. Depending on how and where the decision whether to switch or not is made, in fact different approaches are possible, which might lead to different performance results. In this regard, we propose and analyze a couple of different strategies, where our goal generally is to minimize the end-to-end outage probability in order to maximize the transmission reliability for a given information rate.

The remainder of this paper is organized as follows: In Section II, we outline our system and channel model whereas the different switching strategies are presented and analyzed in Section III. An asymptotic high SNR analysis is performed in Section IV, followed by selected numerical results in Section V and finally some concluding remarks in Section VI.

II. System Model

We consider a standard three-node cooperative decode-and-forward system, where a source node S wants to transmit data to a destination node D with the assistance of an intermediate regenerative relay station R. In this regard, every end-to-end transmission between S and D is subdivided into two different time slots of equal length. During the first phase, the source encodes a certain message and broadcasts it with transmit power P_T over the channel. The relay node then tries to decode the received message and upon success reencodes it again using the same code as the source node before forwarding it during the second phase to the destination with the same transmit power P_T[1]. If the decoding at the relay station

[1] A generalization of our results to the case with different transmit powers at the source node and the relay station is basically straightforward, but would significantly complicate notation in the following.

fails, it simply remains silent during the second phase. After both phases, the destination combines the message originally received from the source node with the message forwarded by the relay station—if applicable—using conventional maximum ratio combining before performing the actual decoding.

In the following, the source node is always assumed to be equipped with $N \geq 2$ different antenna elements, out of which always only one antenna is selected for the actual data transmission using a switch-and-stay approach, i.e., a certain antenna element is used as long as the induced signal quality is satisfactory whereas otherwise the source should switch to an (arbitrary) other antenna element. In this regard, the switching might be driven by either the relay station, the destination or by both of them together, where the relevant nodes always feed back their request to the source node via a zero-delay error-free feedback channel. In general, the optimal switching instants are determined such that the end-to-end outage probability conditioned on the available csi is minimized. For notational convenience, both R and D are assumed to have one antenna element only, but our results are basically directly applicable to the case with multiple antenna elements at each of these nodes as well if appropriate diversity combiners are used. Besides, all channels are assumed to be statistically independent of each other and to undergo frequency-flat block-fading, where the SNR on the R-D link will be denoted by γ_D, and the ones on the S-R and S-D link after possible antenna switching by γ_R and γ_S, respectively. Furthermore, $F_{\gamma_D}(\gamma)$ always specifies the cumulative distribution function (cdf) of γ_D and $p_{\gamma_D}(\gamma)$ the corresponding probability density function (pdf) while the cdfs and pdfs of the SNRs $\gamma_{R,0}$ and $\gamma_{S,0}$ on the S-R and S-D links for the case that always the same antenna element is used at the source node, i.e., without any antenna switching, will be denoted by $F_{\gamma_{R,0}}(\gamma)$, $F_{\gamma_{S,0}}(\gamma)$, $p_{\gamma_{R,0}}(\gamma)$, and $p_{\gamma_{S,0}}(\gamma)$. Finally, the relay is assumed to have always perfect csi of the S-R link while the destination has perfect csi of both the R-D and the S-D links.

Aside from a generic analysis based on the pdfs and cdfs of the various hops, we always consider in the following the important case with Rayleigh fading on all hops as a concrete example, assuming average power gains Ω_R, Ω_S, and Ω_D on the S-R, S-D, and R-D links, respectively, thus yielding to

$$F_{\gamma_{S,0}}(\gamma) = 1 - e^{-\frac{\gamma}{\Omega_S \bar{\gamma}}} \quad (1)$$

$$F_{\gamma_{R,0}}(\gamma) = 1 - e^{-\frac{\gamma}{\Omega_R \bar{\gamma}}} \quad (2)$$

$$F_{\gamma_D}(\gamma) = 1 - e^{-\frac{\gamma}{\Omega_D \bar{\gamma}}}, \quad (3)$$

as well as $p_{\gamma_k}(\gamma) = \frac{\partial}{\partial \gamma} F_{\gamma_k}(\gamma)$ for $\gamma_k \in \{\gamma_{S,0}, \gamma_{R,0}, \gamma_D\}$, where $\bar{\gamma} = P_T/\sigma^2$ denotes the average SNR in terms of the transmit power over the noise variance σ^2, which is without loss of generality assumed to be the same for all nodes[2].

III. Antenna Switching Strategies

In the following, we propose three different switching strategies, where the switching is initiated either by the relay station, the destination, or by both of them together. In this regard, we always aim at minimizing the end-to-end outage probability conditioned on the available csi. Besides, we compare the performance of the various approaches to the conventional case without any antenna switching at the source node, i.e., where the source is equipped with a single antenna element only.

In general, an outage between source and destination occurs if the corresponding mutual information for a given set of SNR values γ_R, γ_S, and γ_D on the various hops falls below a given target information rate R. Clearly, if the relay is not able to correctly decode the message transmitted by the source, this happens iff $\frac{1}{2}\log_2(1+\gamma_S) < R$ whereas otherwise this is the case iff $\frac{1}{2}\log_2(1+\gamma_D+\gamma_S) < R$, where the factor $\frac{1}{2}$ is due to the fact that every end-to-end transmission requires two different time intervals. The relay itself is able to correctly decode and hence forward the message if the S-R link is not in outage, i.e., if $\frac{1}{2}\log_2(1+\gamma_R) \geq R$. Hence, the end-to-end outage probability can be readily expressed as [12]–[14]

$$P_{\text{out}} = \text{Prob}\left[(\gamma_R < \gamma_T) \wedge (\gamma_S < \gamma_T)\right] + \text{Prob}\left[(\gamma_R \geq \gamma_T) \wedge (\gamma_D + \gamma_S \leq \gamma_T)\right], \quad (4)$$

where we have introduced for brevity the short-hand notation $\gamma_T = 2^{2R} - 1$. Exploiting the assumed independence between the various SNRs, (4) generally might be calculated based on the cdfs and pdfs of γ_R, γ_S, and γ_D as

$$P_{\text{out}} = F_{\gamma_R}(\gamma_T)\, F_{\gamma_S}(\gamma_T) + (1 - F_{\gamma_R}(\gamma_T)) \int_0^{\gamma_T} F_{\gamma_D}(\gamma_T - \gamma_S)\, p_{\gamma_S}(\gamma_S)\, d\gamma_S, \quad (5)$$

where the actual distributions of γ_R and γ_S depend on the concrete switching strategy that is considered.

A. No Switching

As a reference, we always consider the case that no antenna switching is performed, which has already been thoroughly analyzed in terms of the outage probability in [12], [13], for example. Clearly, in this case we have $F_{\gamma_R}(\gamma_R) = F_{\gamma_{R,0}}(\gamma_R)$ as well as $F_{\gamma_S}(\gamma_S) = F_{\gamma_{S,0}}(\gamma_S)$ and consequently we obtain a generic expression for the corresponding outage probability by simply plugging these relationships in (5). Considering the important case of Rayleigh-fading on all hops as a concrete example and making use of (1) – (3), we furthermore obtain after some basic manipulations the closed-form result

$$P_{\text{out,A}} = 1 - e^{-\frac{\gamma_T}{\Omega_S \bar{\gamma}}} + \frac{\Omega_D}{\Omega_D - \Omega_S}\, e^{-\frac{\gamma_T}{\Omega_R \bar{\gamma}}} \left(e^{-\frac{\gamma_T}{\bar{\gamma}\,\Omega_S}} - e^{-\frac{\gamma_T}{\bar{\gamma}\,\Omega_D}} \right), \quad (6)$$

which is perfectly in line with the expression provided in [12].

B. Relay-Driven Switching

The relay station is assumed to have always perfect csi of the S-R link, but no csi of the S-D and R-D link, respectively. Hence, the best thing the relay can do is to control the antenna switching at the source node such that the probability that the relay is able to correctly decode the message received from the source and hence to be able to forward the message to the destination is maximized. Clearly, this can be achieved

[2]The case with different noise variances might be traced back to the case considered here by adjusting the average power gains appropriately.

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by always comparing the current SNR on the source-to-relay link to the outage threshold γ_T and if it falls below γ_T, the source should switch to the other antenna element since otherwise an outage would occur on the S-R link. If the current SNR exceeds γ_T, in contrast, no switching should be performed since then the S-R link is definitely not in outage with the currently active antenna element. In fact, this strategy is identical to conventional outage-optimal SSTD for single-hop systems and hence we readily obtain similar to [7], [10]

$$F_{\gamma_R}(\gamma) = \begin{cases} F_{\gamma_{R,0}}(\gamma)\, F_{\gamma_{R,0}}(\gamma_T) & \gamma \le \gamma_T \\ F_{\gamma_{R,0}}(\gamma) - F_{\gamma_{R,0}}(\gamma_T) + F_{\gamma_{R,0}}(\gamma)\, F_{\gamma_{R,0}}(\gamma_T) & \gamma > \gamma_T \end{cases} \quad (7)$$

while at the same time $F_{\gamma_S}(\gamma_S) = F_{\gamma_{S,0}}(\gamma_S)$ as the S-D link is not taken into account when deciding whether to switch or not. Plugging (7) in (5) yields as generic result for the corresponding outage probability

$$P_{\text{out,B}} = \left(F_{\gamma_{R,0}}(\gamma_T)\right)^2 F_{\gamma_{S,0}}(\gamma_T) + \left[1 - \left(F_{\gamma_{R,0}}(\gamma_T)\right)^2\right] \times \int_0^{\gamma_T} F_{\gamma_D}(\gamma_T - \gamma_{S,0})\, p_{\gamma_{S,0}}(\gamma_{S,0})\, d\gamma_{S,0}, \quad (8)$$

which reduces for the important case of Rayleigh fading on all hops again by making use of (1) – (3) to

$$P_{\text{out,B}} = e^{-\frac{\gamma_T}{\Omega_R \bar{\gamma}}} \left(2 - e^{-\frac{\gamma_T}{\Omega_R \bar{\gamma}}}\right) \times \left[1 - e^{-\frac{\gamma_T}{\Omega_S \bar{\gamma}}} + \frac{\Omega_D}{\Omega_D - \Omega_S} \left(e^{-\frac{\gamma_T}{\bar{\gamma}\Omega_S}} - e^{-\frac{\gamma_T}{\Omega_D \bar{\gamma}}}\right)\right] + \left(1 - e^{-\frac{\gamma_T}{\Omega_R \bar{\gamma}}}\right)^2 \left(1 - e^{-\frac{\gamma_T}{\Omega_S \bar{\gamma}}}\right). \quad (9)$$

Please note that with this approach only one feedback bit is required, with which the relay indicates whether the source should switch to another antenna element or not.

C. Destination-Driven Switching

The destination is always assumed to have perfect csi of both the S-D and the R-D link, but it is generally not aware in advance whether the relay will forward the message received from the source or not. If the message is forwarded, the destination can coherently combine the signals received from both source and relay, thus leading to an effective SNR which is simply the sum of the SNRs of both hops. Otherwise, the destination only has the signal directly received from the source and hence the effective SNR corresponds to the SNR on the S-D link in that case. Clearly, if the SNR γ'_S on the S-D link for the currently active transmit antenna at the source exceeds γ_T, we generally should not switch since even without the support of the relay station no outage would occur. If, in contrast, $\gamma'_S + \gamma_D < \gamma_T$, we should switch in any case as even if the relay can decode and forward the message received from the source—what is not known in advance—an outage cannot be avoided. The corresponding (conditional) outage probability for this case can easily be shown based on (5) to be given by

$$P_{\text{out}}^{\text{switch}} = F_{\gamma_{R,0}}(\gamma_T)\, F_{\gamma_{S,0}}(\gamma_T) + \left(1 - F_{\gamma_{R,0}}(\gamma_T)\right) F_{\gamma_{S,0}}(\gamma_T - \gamma_D). \quad (10)$$

In all other cases, i.e., if $\gamma_T - \gamma_D \le \gamma'_S < \gamma_T$, the optimal action (given the available csi) depends on the probability that the relay is able to successfully decode and hence forward the message. If we switch, the outage probability would also be given by (10) in that case whereas otherwise we clearly would obtain

$$P_{\text{out}}^{\text{stay}} = F_{\gamma_{R,0}}(\gamma_T), \quad (11)$$

i.e., an outage would occur iff an outage occurs on the S-R link. Hence, if $P_{\text{out}}^{\text{switch}} < P_{\text{out}}^{\text{stay}}$, the destination should request to switch while in all other cases it should request that the source keeps the currently active antenna element. In this regard, we can show that there is always a unique γ_D^* such that $P_{\text{out}}^{\text{switch}} < P_{\text{out}}^{\text{stay}}$ for all $\gamma_D \ge \gamma_D^*$ and likewise $P_{\text{out}}^{\text{switch}} \ge P_{\text{out}}^{\text{stay}}$ for all $\gamma_D \le \gamma_D^*$. This is rather obvious since $P_{\text{out}}^{\text{switch}}$ is a non-increasing function of γ_D while $P_{\text{out}}^{\text{stay}}$ is independent of it and at the same time $P_{\text{out}}^{\text{switch}}\big|_{\gamma_D \ge \gamma_T} \le P_{\text{out}}^{\text{stay}}$. Besides, $P_{\text{out}}^{\text{switch}}$ is always smaller than $P_{\text{out}}^{\text{stay}}$ if $P_{\text{out}}^{\text{switch}}\big|_{\gamma_D = 0} = F_{\gamma_{S,0}}(\gamma_T) < P_{\text{out}}^{\text{stay}} = F_{\gamma_{R,0}}(\gamma_T)$. Hence, the optimal value for γ_D^* generally can be readily expressed as

$$\gamma_D^* = \begin{cases} 0 & \text{if } F_{\gamma_{S,0}}(\gamma_T) \le F_{\gamma_{R,0}}(\gamma_T) \\ \text{solution of } P_{\text{out}}^{\text{switch}}(\gamma_D) = P_{\text{out}}^{\text{stay}} & \text{otherwise} \end{cases}, \quad (12)$$

where the range of possible values for γ_D^* can be restricted to $0 \le \gamma_D^* \le \gamma_T$. Based on (12), (10) and (11), it is consequently for virtually arbitrary fading distributions possible to determine γ_D^* always at least efficiently numerically. For the important case of Rayleigh fading on all hops, we even can derive an analytical closed-form solution by combining (1) and (2) with (10) – (12), yielding after some basic manipulations to

$$\gamma_D^* = \gamma_T + \Omega_S \bar{\gamma} \ln\left(1 + e^{-\frac{\gamma_T}{\Omega_S \bar{\gamma}}} - e^{-\frac{\gamma_T}{\bar{\gamma}} \max\left\{0, \frac{1}{\Omega_S} - \frac{1}{\Omega_R}\right\}}\right). \quad (13)$$

Putting everything together, we can say that for minimizing the end-to-end outage probability, no switching should be performed if either $\gamma'_S \ge \gamma_T$ or $\gamma'_S + \gamma_D \ge \gamma_T$ and $\gamma_D \le \gamma_D^*$ whereas in all other cases the source should switch to the other antenna element. Furthermore, the corresponding average end-to-end outage probability is then generally given by

$$P_{\text{out,C}} = \int_0^{\gamma_T} \int_0^{\infty} P_{\text{out}}^{\text{switch}}\, p_{\gamma_D}(\gamma_D)\, p_{\gamma_{S,0}}(\gamma_S)\, d\gamma_D\, d\gamma_S - \int_{\gamma_T - \gamma_D^*}^{\gamma_T} \int_{\gamma_T - \gamma_S}^{\gamma_D^*} P_{\text{out}}^{\text{switch}}\, p_{\gamma_D}(\gamma_D)\, p_{\gamma_{S,0}}(\gamma_S)\, d\gamma_D\, d\gamma_S + \int_{\gamma_T - \gamma_D^*}^{\gamma_T} \int_{\gamma_T - \gamma_S}^{\gamma_D^*} P_{\text{out}}^{\text{stay}}\, p_{\gamma_D}(\gamma_D)\, p_{\gamma_{S,0}}(\gamma_S)\, d\gamma_D\, d\gamma_S, \quad (14)$$

with $P_{\text{out}}^{\text{switch}}$ and $P_{\text{out}}^{\text{stay}}$ according to (10) and (11), respectively. For the concrete example of Rayleigh fading on all hops, we get after inserting (1) – (3) in (14) and solving the corresponding integrals (a detailed derivation is omitted here due to space constraints) the expression according to (15), which is given on top of the next page. Finally, please note that for determining the optimal value of γ_D^* according to (12), the

$$
\begin{aligned}
P_{\text{out,C}} &= \left(1-e^{-\frac{\gamma_T}{\Omega_S\,\bar\gamma}}\right)^2 \left(1-e^{-\frac{\gamma_T}{\Omega_R\,\bar\gamma}}\right) + \left(1-e^{-\frac{\gamma_T}{\Omega_S\,\bar\gamma}}\right) e^{-\frac{\gamma_T}{\Omega_R\bar\gamma}} \left[1-e^{-\frac{\gamma_T}{\Omega_D\bar\gamma}} + \frac{\Omega_S}{\Omega_S-\Omega_D}\left(e^{-\frac{\gamma_T}{\Omega_D\bar\gamma}} - e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}}\right)\right] \\
&\quad + e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}} \left(1-e^{-\frac{\gamma_T}{\Omega_R\bar\gamma}}\right) \left[\left(-e^{-\frac{\gamma_D^*}{\Omega_D\bar\gamma}}\right)\left[e^{-\frac{\gamma_T-\gamma_D^*}{\Omega_S\bar\gamma}} - e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}}\right] - \frac{\Omega_D}{\Omega_D-\Omega_S}\left[e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}} - e^{-\frac{1}{\Omega_S\bar\gamma}(\gamma_T-\gamma_D^*)-\frac{\gamma_D^*}{\Omega_D\bar\gamma}}\right]\right] \\
&\quad - e^{-\frac{\gamma_T}{\Omega_R\bar\gamma}} \left[\frac{\Omega_D}{\Omega_S-\Omega_D}\left(e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}} - e^{-\frac{1}{\Omega_S\bar\gamma}(\gamma_T-\gamma_D^*)-\frac{\gamma_D^*}{\Omega_D\bar\gamma}}\right) - e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}-\frac{\gamma_D^*}{\Omega_D\bar\gamma}}\left(e^{\frac{\gamma_D^*}{\Omega_S\bar\gamma}}-1\right) - \frac{1}{\Omega_D-\Omega_S}\,e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}}\right. \\
&\quad \times \left.\left[e^{-\frac{\gamma_D^*}{\bar\gamma}\left(\frac{1}{\Omega_D}-\frac{1}{\Omega_S}\right)}\,\Omega_S\left(e^{-\frac{\gamma_T-\gamma_D^*}{\Omega_S\bar\gamma}} - e^{-\frac{\gamma_T}{\Omega_S\bar\gamma}}\right) - e^{-\frac{\gamma_T}{\bar\gamma}\left(\frac{1}{\Omega_D}-\frac{1}{\Omega_S}\right)}\frac{1}{\frac{1}{\Omega_D}-\frac{2}{\Omega_S}}\left(e^{\frac{\gamma_T}{\bar\gamma}\left(\frac{1}{\Omega_D}-\frac{2}{\Omega_S}\right)} - e^{\frac{\gamma_T-\gamma_D^*}{\bar\gamma}\left(\frac{1}{\Omega_D}-\frac{2}{\Omega_S}\right)}\right)\right]\right] \quad (15)
\end{aligned}
$$

$$
\begin{aligned}
P_{\text{out,D}} &= \left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_S}}\right)\left[1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_S}} + e^{-\frac{\gamma_T}{\bar\gamma}\left(\frac{1}{\Omega_R}+\frac{1}{\Omega_S}\right)}\right] - \frac{\Omega_S}{\Omega_S-\Omega_D}e^{-\frac{\gamma_T}{\bar\gamma}\left(\frac{1}{\Omega_S}+\frac{1}{\Omega_R}\right)}\left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_S}}\right) \\
&\quad + \frac{\Omega_D}{\Omega_D-\Omega_S}\left(e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_S}} - e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_D}}\right)e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_R}}\left[1+\left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_R}}\right)\left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_S}}\right)\right] \\
&\quad + \frac{\Omega_D}{\Omega_S-\Omega_D}e^{-\frac{\gamma_T}{\bar\gamma}\left(\frac{1}{\Omega_R}+\frac{1}{\Omega_S}\right)}\left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_R}}\right)\left(1-e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_D}}\right) + e^{-\frac{2\,\gamma_T}{\bar\gamma\,\Omega_R}}\frac{\Omega_S\,\Omega_D}{(\Omega_S-\Omega_D)\,(\Omega_S-2\,\Omega_D)}\left(e^{-\frac{2\gamma_T}{\bar\gamma\,\Omega_S}} - e^{-\frac{\gamma_T}{\bar\gamma\,\Omega_D}}\right) \quad (16)
\end{aligned}
$$

destination generally has to know the outage probability of the S-R link. However, since this probability depends only on the corresponding channel distribution and is therefore expected to change only at a rather slow pace, it has to be signaled only very infrequently using a low rate control channel, thus making this approach very suitable for practical implementations.

D. Jointly-Driven Optimal Switching

As a last approach, we consider the case where the antenna switching at the source node is driven jointly by the relay station and the destination. This way, it is possible to perform optimal switching in the sense that the source always switches the antenna if and only if with the currently active antenna element an outage would occur. For that purpose, the destination always feeds back one out of three possible states to the source node, requesting either to switch, not to switch, or to do whatever the relay has requested. In this regard, it requests—as for destination-driven switching—not to switch if the current SNR γ'_S on the S-D link exceeds γ_T since in this case even without the support of the relay station an outage can be avoided. Besides, it requests to switch in any case if $\gamma'_S + \gamma_D < \gamma_T$ since otherwise definitely an outage would occur, even if the relay forwards the message received from the source. In all other cases, i.e., if $\gamma'_S < \gamma_T$ while at the same time $\gamma'_S + \gamma_D > \gamma_T$, it depends on the S-R link whether or not an outage occurs. If the relay can correctly decode and forward the message, no outage occurs, otherwise yes. Therefore, the destination tells the source node to follow the request of the relay in that case, which always indicates whether the S-R link is with the currently active antenna element in outage or not.

Hence, the source node should always do what the destination has requested and antenna switching is always only performed iff $\gamma'_S + \gamma_D < \gamma_T$ or $\gamma'_S < \gamma_T$ and $\gamma'_R < \gamma_T$, where γ'_R denotes the current SNR on the S-R link. In this case, the outage probability is given by $P_{\text{out}}^{\text{switch}}$ according to (10) whereas in all other cases no outage occurs. Consequently, the average end-to-end outage probability can readily be shown to be generally given for arbitrary fading distributions by

$$
\begin{aligned}
&P_{\text{out,D}} = \\
&\int_0^{\gamma_T}\int_0^{\gamma_T-\gamma_S}\left[F_{\gamma_{R,0}}(\gamma_T)F_{\gamma_{S,0}}(\gamma_T) + F_{\gamma_{S,0}}(\gamma_T-\gamma_D)\right. \\
&\times\left.\left(1-F_{\gamma_{R,0}}(\gamma_T)\right)\right]p_{\gamma_D}(\gamma_D)\,p_{\gamma_{S,0}}(\gamma_S)\,d\gamma_D\,d\gamma_S \\
&+\int_0^{\gamma_T}\int_{\gamma_T-\gamma_S}^{\infty}\left[F_{\gamma_{R,0}}(\gamma_T)F_{\gamma_{S,0}}(\gamma_T) + F_{\gamma_{S,0}}(\gamma_T-\gamma_D)\right. \\
&\times\left.\left(1-F_{\gamma_{R,0}}(\gamma_T)\right)\right]F_{\gamma_{R,0}}(\gamma_T)p_{\gamma_D}(\gamma_D)p_{\gamma_{S,0}}(\gamma_S)d\gamma_D d\gamma_S. \quad (17)
\end{aligned}
$$

For the concrete example with Rayleigh fading on all hops, this outage probability can be given analytically in closed-form again by inserting (1) – (3) in (17) and solving the corresponding integrals, thus yielding to the final result according to (16), which is given at the top of this page.

Please note that with this jointly-driven switching strategy, more feedback information is required than with the other approaches outlined before. In fact, the destination always has to feed back two bits to the source (for coding three different states) while the relay has to feed back one bit. However, in return this way the optimum performance of SSTD can be achieved and no control information has to be exchanged between relay and destination, as required for destination-driven switching for determining the optimal value of γ_D^*.

IV. Asymptotic Analysis and Diversity Order

In the following, we perform an asymptotic analysis of the various switching strategies for high average SNRs $\bar\gamma \to \infty$ and the important case with Rayleigh fading on all hops. The asymptotic expressions are on the one hand much simpler and hence more intuitive than the exact results derived before and on the other hand they can readily be used for determining the end-to-end diversity order that can be achieved with the different schemes. In this regard, the basic idea generally is to perform first-order series expansions of the pdfs and cdfs of the various SNRs and then to use these approximations in the corresponding generic expressions according to (5), (8), (14), and (17). In fact, for a general Rayleigh fading link with average power gain Ω_k, the pdf and cdf of the corresponding SNR γ_k can be readily expanded into a power series as

$$p_{\gamma_k}(\gamma) = \frac{1}{\Omega_k \bar{\gamma}} + o\left(\frac{1}{\bar{\gamma}}\right) \quad (18)$$

$$F_{\gamma_k}(\gamma) = \frac{\gamma}{\Omega_k \bar{\gamma}} + o\left(\frac{1}{\bar{\gamma}^2}\right), \quad (19)$$

where $o\left(\frac{1}{\bar{\gamma}}\right)$ denotes higher-order terms of $\frac{1}{\bar{\gamma}}$, which can be reasonably neglected in the high SNR regime. Making use of these approximations in (5), we obtain for the case without any antenna switching at the source node

$$P_{\text{asymp,A}} = \frac{\gamma_T^2}{\bar{\gamma}^2\,\Omega_S}\left(\frac{1}{\Omega_R} + \frac{1}{2\,\Omega_D}\right), \quad (20)$$

from which we can directly conclude that the diversity order always equals two in that case since $P_{\text{asymp,A}} \sim \frac{1}{\bar{\gamma}^2}$. For relay-driven switching, in contrast, we likewise obtain by plugging (18) and (19) in (8) after some basic manipulations

$$P_{\text{asymp,B}} = \frac{1}{\Omega_S \Omega_D\,\bar{\gamma}^2}\,\frac{1}{2}\gamma_T^2 \quad (21)$$

and consequently also in this case always a diversity order of two is achieved. For the case of destination-driven switching, we first note that for the asymptotic case we obtain by solving (12) based on the approximations according to (18) and (19)

$$\gamma_D^*|_{\bar{\gamma}\to\infty} = \max\left\{0, \gamma_T\left(1 - \frac{\Omega_S}{\Omega_R}\right)\right\}. \quad (22)$$

Plugging then (18) and (19) in (14), solving the integrals and neglecting all higher-order terms of $\frac{1}{\bar{\gamma}}$ again, we get

$$P_{\text{asymp,C}} = \frac{1}{\bar{\gamma}^3}\left[\frac{\gamma_T^3}{\Omega_S^2}\left(\frac{1}{\Omega_R} + \frac{1}{2\,\Omega_D}\right) + \frac{{\gamma_D^*}^2\,\gamma_T}{2\,\Omega_S\Omega_D\Omega_R} - \frac{1}{\Omega_S^2\Omega_D}\left(\frac{1}{2}\gamma_T{\gamma_D^*}^2 - \frac{1}{3}{\gamma_D^*}^3\right)\right], \quad (23)$$

with γ_D^* according to (22). From this expression, it is quite obvious that with destination-driven switching always a diversity order of three can be achieved, independent of N.

Finally, for the case of optimal jointly-driven switching, we obtain similar to the cases considered before by making use of (18) and (19) in (17) and after some basic calculations

$$P_{\text{asymp,D}} = \frac{\gamma_T^3}{3\,\Omega_S^2\,\Omega_D\,\bar{\gamma}^3}. \quad (24)$$

It can be seen that also in this case the diversity order always equals three, just as for destination-driven considered before.

V. Numerical Results

In the following, we always assume that all hops undergo independent but not necessarily identically distributed Rayleigh fading and we normalize the average power gain Ω_S of the direct link between source and destination to one. Assuming for traceability as in [11] an exponential path loss model with a path loss coefficient ν and that the relay is placed at the relative position x on the connecting line between source and destination while neglecting any shadow fading effects, the average power gains of the S-R and R-D links can be readily calculated as $\Omega_R = x^{-\nu}$ and $\Omega_D = (1-x)^{-\nu}$, respectively.

Fig. 1 shows the average end-to-end outage probability as a function of the average SNR $\bar{\gamma}$ for the various different schemes, assuming that the relay is placed exactly in the middle between source and destination as well as $\nu = 4$. It can be seen that there is a perfect match between our calculated values and results obtained from Monte Carlo simulations, what verifies the validity of our theoretical analysis. Furthermore, the high SNR asymptotes are obviously really tight for sufficiently high average SNRs. Besides, it can be seen that with all schemes the performance can be considerably improved compared to the reference case without any switching[3], particularly with destination-driven or optimal switching since with these two approaches a higher diversity order is achieved. In the low SNR regime, relay-driven switching obviously outperforms destination-driven switching whereas it is exactly the other way around for high average SNRs. This is because in the low SNR regime basically always the relay is needed for avoiding an outage and the probability that the relay is able to decode and forward the message received from the source is increasing with relay-driven switching. In the high SNR regime, in contrast, the higher diversity order of destination-driven switching comes into its own, therefore clearly outperforming the other approach in that region.

Fig. 2 shows similar results as before, but with the relay placed at the relative position $x = 0.65$. In this case, the performance differences between the various approaches are generally more pronounced and it can be seen that with optimal switching in the high SNR regime an effective SNR gain of about 5 dB can be achieved compared to destination-driven switching. Likewise, with relay-driven switching an

[3]This comparison is fair from a hardware complexity perspective since in all cases the required number of radio frequency chains would be the same.

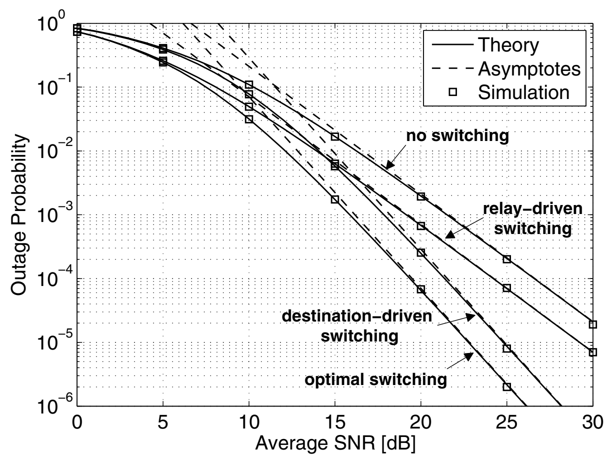

Fig. 1. Information outage probability as a function of the average SNR $\bar{\gamma}$ for R = 2 bit/s/Hz, a path loss coefficient of $\nu = 4$, and with the relay station located exactly in the middle between source and destination ($x = 0.5$).

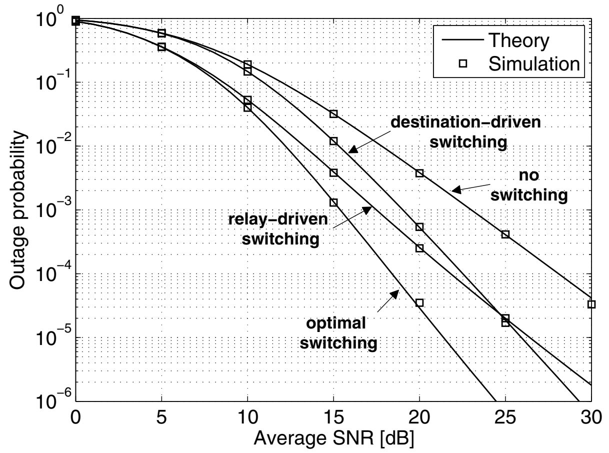

Fig. 2. Information outage probability as a function of the average SNR $\bar{\gamma}$ for R = 2 bit/s/Hz, a path loss coefficient of ν = 4, and with the relay station located slightly closer to the destination than to the source (x = 0.65).

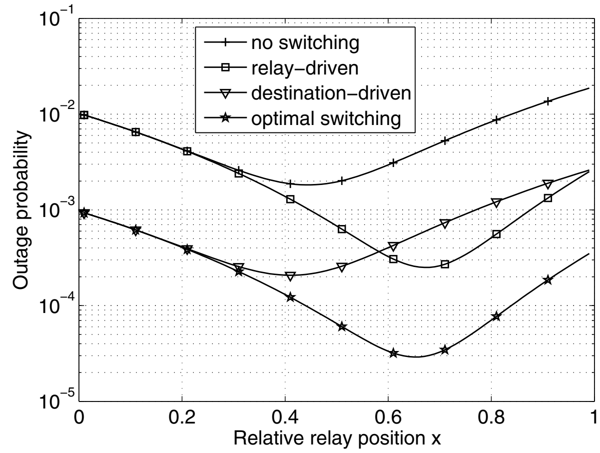

Fig. 3. Outage probability as a function of the relative relay position x for $\bar{\gamma}$ = 20 dB, R = 2 bit/s/Hz, and a path loss exponent of ν = 4.

effective SNR gain of even more than 6 dB can be achieved compared to the case without any switching at all.

Finally, Fig. 3 illustrates the impact of the relay position on the outage performance at an average SNR of $\bar{\gamma}$ = 20 dB. Clearly, if the relay is placed next to the source node (x = 0), relay-driven switching does not yield any gain over the case without any switching and optimal switching no gain over destination-driven switching. This is because in this case the S-R link is for all schemes basically always perfect such that no outage occurs on this link. If the relay is placed next to the destination, in contrast (x = 1), relay-driven and destination-driven switching lead to exactly the same performance, because in this case the R-D link is basically perfect while the performance of the S-R and S-D links are identical. Hence, it does not make any difference whether we optimize the switching for the S-R or the S-D link. Furthermore, it can be seen from Fig. 3 that the relay position is generally very crucial for the outage performance, where the optimal position depends on the applied switching scheme.

VI. Conclusion

We have proposed and analyzed three different approaches for performing switch-and-stay transmit diversity at the source node of a three-node cooperative diversity system with regenerative relaying, aiming at minimizing the end-to-end outage probability. The various approaches differ in which entities determine whether or not the source should switch to another element and also the amount of required feedback information. For all schemes, we have derived generic analytical expressions for the corresponding outage probability for arbitrary fading distributions, which were explicitly given in closed-form for the important case of Rayleigh fading on all hops. Besides, we have derived simpler asymptotic expressions for the high SNR regime, based on which we could easily determine the achievable diversity orders. Finally, numerical results were shown to be in perfect agreement with simulated values, thus verifying the accuracy of our theoretical analysis, and they illustrated the significant performance gains that can be achieved compared to the case without any antenna switching.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[2] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74 – 80, Oct. 2004.

[3] Y. Fan and J. Thompson, "MIMO configurations for relay channels: theory and practice," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1774 – 1786, May 2007.

[4] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29 – 43, Jan. 2005.

[5] M. Blanco and K. Zdunek, "Performance and optimization of switched diversity systems for the detection of signals with Rayleigh fading," *IEEE Trans. Commun.*, vol. 27, no. 12, pp. 1887 – 1895, Dec. 1979.

[6] A. A. Abu-Dayya and N. C. Beaulieu, "Analysis of switched diversity systems on generalized-fading channels," *IEEE Trans. Commun.*, vol. 42, no. 11, pp. 2959 – 2966, Nov. 1994.

[7] Y.-C. Ko, M.-S. Alouini, and M. K. Simon, "Analysis and optimization of switched diversity systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 5, pp. 1813 – 1831, Sep. 2000.

[8] C. Tellambura, A. Annamalai, and V. K. Bhargava, "Unified analysis of switched diversity systems in independent and correlated fading channels," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1955 – 1965, Nov. 2001.

[9] G. L. Stüber, *Principles of Mobile Communications*, 2nd ed. Norwell, MA: Kluwer, 2001.

[10] H.-C. Yang and M.-S. Alouini, "Performance analysis of multibranch switched diversity systems," *IEEE Trans. Commun.*, vol. 51, no. 5, pp. 782 – 294, May 2003.

[11] A. Müller and J. Speidel, "Outage-optimal transmit antenna selection for cooperative decode-and-forward systems," in *Proc. IEEE Veh. Technol. Conf.*, Apr. 2009.

[12] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 813 – 815, Dec. 2006.

[13] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700 – 702, Aug. 2005.

[14] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834 – 836, Dec. 2006.