Hybrid Diversity Maximization Precoding for the Multiuser MIMO Downlink

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***Abstract*—In this paper, we present a high-performance hybrid linear precoding scheme for the downlink of multiuser multiple-input multiple-output (MU-MIMO) systems based on the combination of an iterative modified regularized block diagonalization (IMRBD) precoding and minimum mean square error (MMSE) precoding. The proposed hybrid diversity maximization (HDM) scheme aims to maximize the diversity gain by means of this dual-stage precoding strategy while maintaining relatively low complexity. The simulation results show that HDM precoding can provide comparable or even better performance than other iterative precoding schemes, including some of the more complex ones.**

***Index Terms*—Diversity, linear precoding, minimum mean square error (MMSE), multiple-input multiple-output (MIMO), multiuser MIMO (MU-MIMO).**

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) constitutes an integral part of the fourth generation (4G) mobile technologies and beyond due to its great potential for increasing the system capacity of cellular networks. The downlink transmission problem in MU-MIMO systems involves mitigating the multiuser interference (MUI) using some linear or nonlinear precoding scheme at the base station (BS) and optimizing the downlink transmit power allocation for each user subject to an average total power constraint.

Simple linear techniques like channel inversion and regularized channel inversion [1]–[4] are applicable if each user equipment (UE) utilizes a single receive antenna though the performance is generally much lower than that of nonlinear techniques based on dirty paper coding [4], [5]. However, linear MU-MIMO downlink techniques that allow the use of multiple receive antennas at the UE are of particular interest since they can provide higher diversity gain using single-stream transmission or alternatively, multi-stream transmission can be employed to obtain spatial multiplexing gain for the users. Block diagonalization (BD) [6], successive minimum mean square error (SMMSE) [7] and regularized block diagonalization (RBD) [8] are examples of low-complexity precoding techniques for multi-antenna UEs which provide closed-form expressions for the precoding matrices. However, this advantage comes at the cost of lower performance.

Several linear transmission schemes based on iterative processing at the BS have also been proposed in literature (e.g., [8]–[15]). Such schemes are capable of achieving higher performance gains at the expense of significantly increased complexity. Total-MMSE (T-MMSE) [12], the direct optimization scheme of [14], and modified TMMSE (MT-MMSE) [15] are joint transmit-receive optimization techniques based on minimization of the sum of the mean square errors (MSEs) for all simultaneous users. MT-MMSE uses a modified total MMSE criterion resulting in better performance. In [11] and [13], the uplink/downlink duality is exploited to obtain a convex objective function which converges to the exact MMSE solution. Iterative RBD (IRBD) [8] is another interesting iterative scheme which allows the unused row subspace of a user's channel to be utilized for other users' transmissions by iteratively performing RBD. Even though IRBD is generally outperformed by T-MMSE, MT-MMSE and the duality-based schemes, it is much simpler to implement and still provides good performance.

In this paper, we present a new hybrid iterative MU-MIMO downlink transmission scheme referred to as hybrid diversity maximization (HDM). HDM combines a simple iterative modified RBD (IMRBD) scheme and the minimum sum-MSE criterion of [12] and [14] to minimize the average bit-error rate (BER) while maintaining reasonably low system complexity. We analyze the proposed HDM scheme by using single-stream transmission for each multi-antenna UE in order to maximize the diversity gain.

The paper is organized as follows: In Section II, we describe the system model for MU-MIMO downlink transmission. Section III presents a detailed description of the proposed downlink transmission scheme. The simulation results, comparing the performance of the proposed scheme with other techniques, are presented in Section IV. Section V finally concludes the paper.

II. SYSTEM MODEL

The generalized block diagram of a MU-MIMO downlink transmission system is shown in Fig. 1. We consider a K-user MU-MIMO system with N_T transmit antennas at the BS and N_{R_k} receive antennas at UE k, where $k = 1, 2, \ldots, K$. The total number of receive antennas is denoted by

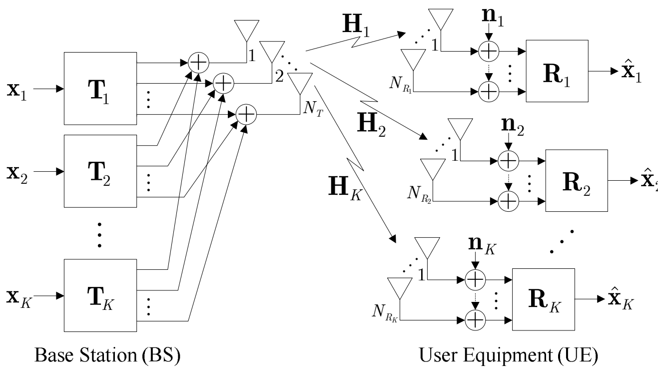

Fig. 1. General block diagram of a multiuser MIMO downlink transmission system.

$N_R = \sum_{k=1}^{K} N_{R_k}$. $\mathbf{x}_k \in \mathbb{C}^{L_k \times 1}$ represents the transmit symbol vector for user k where L_k is the number of simultaneous data streams transmitted to user k. We assume the elements of $\mathbf{x}_k$ to be zero-mean uncorrelated symbols with unit variance i.e., $\mathrm{E}\left[\mathbf{x}_k \mathbf{x}_k^H\right] = \mathbf{I}_{L_k}$ where $\mathrm{E}[\cdot]$ denotes the expectation and $(\cdot)^H$ represents the Hermitian transpose. $\mathbf{H} = \left[\mathbf{H}_1^T \quad \cdots \quad \mathbf{H}_K^T\right]^T \in \mathbb{C}^{N_R \times N_T}$ is the combined multiuser channel matrix consisting of the MIMO channel matrices $\mathbf{H}_k \in \mathbb{C}^{N_{R_k} \times N_T}$ of all the users. We consider a Rayleigh flat-fading channel model where the elements of each $\mathbf{H}_k$ are independent identically distributed (i.i.d.) zero-mean complex Gaussian random variables with unit variance. This is a valid assumption in case of multicarrier transmission like orthogonal frequency-division multiplexing (OFDM). We also assume complete channel state information (CSI) of all UEs to be available at the BS while each UE only has knowledge of its own channel. The transmit precoding matrix for user k and the corresponding receive matrix at the kth UE are represented by $\mathbf{T}_k \in \mathbb{C}^{N_T \times L_k}$ and $\mathbf{R}_k \in \mathbb{C}^{L_k \times N_{R_k}}$ respectively. The estimated symbol vector after receive processing at the kth UE is then given by

$$\hat{\mathbf{x}}_k = \mathbf{R}_k \left(\mathbf{H}_k \sum_{j=1}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k\right) \in \mathbb{C}^{L_k \times 1} \tag{1}$$

where $\mathbf{n}_k \in \mathbb{C}^{N_{R_k} \times 1}$ is the corresponding noise vector consisting of zero-mean additive white Gaussian noise samples with variance σ_n^2. The combined output symbol vector for all users can be written as

$$\hat{\mathbf{x}} = \mathbf{R}\left(\mathbf{H}\mathbf{T}\mathbf{x} + \mathbf{n}\right) \in \mathbb{C}^{L \times 1} \tag{2}$$

where $\mathbf{R} = \mathrm{blockdiag}\left(\left[\mathbf{R}_1 \quad \cdots \quad \mathbf{R}_K\right]\right) \in \mathbb{C}^{L \times N_R}$ and $\mathbf{T} = \left[\mathbf{T}_1 \quad \cdots \quad \mathbf{T}_K\right] \in \mathbb{C}^{N_T \times L}$ represent the combined multiuser receive (block-diagonal) and transmit matrices respectively, $\mathbf{x} = \left[\mathbf{x}_1^T \quad \cdots \quad \mathbf{x}_K^T\right]^T \in \mathbb{C}^{L \times 1}$ and $\mathbf{n} = \left[\mathbf{n}_1^T \quad \cdots \quad \mathbf{n}_K^T\right]^T \in \mathbb{C}^{N_R \times 1}$ are the concatenated transmit symbol and noise vectors respectively, and $L = \sum_{k=1}^{K} L_k$ is the total number of transmitted data streams.

III. Hybrid Diversity Maximization (HDM) Transmission Scheme

A. Iterative Modified RBD (IMRBD) Precoding

IMRBD precoding is a modification to RBD and IRBD [8]. It is capable of providing a slight performance improvement over IRBD for equal number of iterations. Consequently, performance comparable to IRBD can be achieved with fewer iterations. Like IRBD, IMRBD utilizes the unused row subspace of a user's channel matrix (corresponding to the unused singular values) for the transmissions of other users. This requires $L_k < \mathrm{rank}(\mathbf{H}_k)$ for at least one of the users in order to achieve any sort of performance gain. However, in our paper the analysis is restricted to single-stream transmission only i.e., $L_k = 1, \forall k$.

IMRBD precoding constitutes the first stage of HDM. To start with, we calculate the precoding matrices of the users sequentially from user 1 to user K. However, no particular ordering of the users is necessary. During the first iteration, we define the matrix $\tilde{\mathbf{H}}_k$ as

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{U}_1\big|_{(L_1)}^H \mathbf{H}_1 \\ \vdots \\ \mathbf{U}_{k-1}\big|_{(L_{k-1})}^H \mathbf{H}_{k-1} \\ \mathbf{H}_{k+1} \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \in \mathbb{C}^{\left(N_R - N_{R_k} - \sum_{j=1}^{k-1}\left(N_{R_j} - L_j\right)\right) \times N_T} \tag{3}$$

which is a reduced channel matrix with the kth user's channel eliminated along with the unused row subspaces of the preceding users' channel matrices. Each matrix $\mathbf{U}_j\big|_{(L_j)}$ in (3) represents the first L_j columns of the unitary matrix $\mathbf{U}_j$ which contains the left singular vectors of the jth user's equivalent channel. The preliminary precoding matrix for user k is then given by

$$\breve{\mathbf{T}}_k = \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k + \alpha \mathbf{I}_{N_T}\right)^{-1} \in \mathbb{C}^{N_T \times N_T} \tag{4}$$

where $\alpha = N_R \sigma_n^2 / P_T$ and P_T is the average total transmit power. Next, we perform the singular value decomposition (SVD) of the kth user's equivalent channel

$$\mathbf{H}_k \breve{\mathbf{T}}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \tag{5}$$

resulting in the unitary matrices $\mathbf{U}_k$ and $\mathbf{V}_k$ containing the left and the right singular vectors respectively, and the diagonal matrix $\mathbf{\Sigma}_k$ containing the singular values of the equivalent channel. The final precoding matrix of user k is then given by

$$\mathbf{T}'_k = \breve{\mathbf{T}}_k \mathbf{V}_k\big|_{(L_k)} \in \mathbb{C}^{N_T \times L_k} \tag{6}$$

where only the first L_k columns of the product $\breve{\mathbf{T}}_k \mathbf{V}_k$ are retained.

The precoding matrices of all the K users are calculated in a similar fashion. However, for all subsequent iterations, the matrix $\tilde{\mathbf{H}}_k$ is redefined as

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{U}_1\big|_{(L_1)}^H \mathbf{H}_1 \\ \vdots \\ \mathbf{U}_{k-1}\big|_{(L_{k-1})}^H \mathbf{H}_{k-1} \\ \mathbf{U}_{k+1}\big|_{(L_{k+1})}^H \mathbf{H}_{k+1} \\ \vdots \\ \mathbf{U}_K\big|_{(L_K)}^H \mathbf{H}_K \end{bmatrix} \in \mathbb{C}^{\left(N_R - N_{R_k} - \sum_{j=1, j\neq k}^{K}(N_{R_j} - L_j)\right) \times N_T} \tag{7}$$

where each $\mathbf{U}_j\big|_{(L_j)}$ represents the most recent version of the matrix, either computed in the current or the previous iteration. After the ith iteration, we obtain a set of transmit precoding matrices $\{\mathbf{T}'_{k,(i)}\}$ and receive matrices $\{\mathbf{R}_{k,(i)}\} = \{\mathbf{U}_{k,(i)}\big|_{(L_k)}^H\}$ for $i = 1, \ldots, N_I - 1$ where N_I is the total number of iterations used for HDM precoding. The number of iterations to be used depends on the desired performance–complexity tradeoff, and an appropriate stopping criterion can be specified accordingly.

B. *Minimum Sum-MSE Precoding*

After a certain number of iterations, IMRBD does not provide any further performance improvement. Therefore, the second and final stage of HDM employs the minimum sum-MSE criterion of [12] and [14] to obtain the final set of precoding matrices resulting in enhanced performance. This transmit precoding method constitutes the final (i.e., N_Ith) HDM iteration and utilizes the set of receive matrices obtained in the last IMRBD iteration. The optimization problem can be written as

$$\begin{aligned} &\min_{\{\mathbf{T}_k\}} \sum_{k=1}^{K} \mathcal{E}_k \\ &\text{s.t.} \quad \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{T}_k^H \mathbf{T}_k\right) = P_T \end{aligned} \tag{8}$$

where $\mathcal{E}_k$ is the kth user's mean square error and $\mathrm{tr}(\cdot)$ represents the matrix trace operation. With the receive matrices fixed, the objective function in (8) becomes convex over the transmit matrices thus guaranteeing convergence to at least some local minimum. $\mathcal{E}_k$ is given by

$$\begin{aligned} \mathcal{E}_k &= \mathrm{E}\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k\right\|^2 \\ &= \mathrm{tr}\Bigg(\mathbf{R}_k \mathbf{H}_k \left(\sum_{j=1}^{K} \mathbf{T}_j \mathbf{T}_j^H\right) \mathbf{H}_k^H \mathbf{R}_k^H + \sigma_n^2 \mathbf{R}_k \mathbf{R}_k^H \\ &\qquad - \mathbf{T}_k^H \mathbf{H}_k^H \mathbf{R}_k^H - \mathbf{R}_k \mathbf{H}_k \mathbf{T}_k + \mathbf{I}_{L_k}\Bigg) \end{aligned} \tag{9}$$

where $\|\cdot\|$ denotes the Euclidean vector norm. The Lagrange dual objective function for the problem can then be constructed as given in [12]. Solving the Lagrange dual problem by taking the partial derivate with respect to the transmit precoding matrices $\{\mathbf{T}_k\}$ and equating to zero, we get the new precoding matrices as

$$\mathbf{T}'_{k,(N_I)} = \left(\mathbf{A} + \upsilon \mathbf{I}_{N_T}\right)^{-1} \mathbf{H}_k^H \mathbf{R}_{k,(N_I-1)}^H \tag{10}$$

where

$$\mathbf{A} = \sum_{j=1}^{K} \mathbf{H}_j^H \mathbf{R}_{j,(N_I-1)}^H \mathbf{R}_{j,(N_I-1)} \mathbf{H}_j \tag{11}$$

and υ is the Lagrange multiplier which is simply given by $\upsilon = \alpha / K = N_R \sigma_n^2 / K P_T$. This greatly simplifies the generation of the precoding matrices as compared to T-MMSE which requires a numerical search to obtain the Lagrange multiplier. The precoding matrices are finally normalized and scaled so that the power constraint in (8) is fulfilled. The final HDM precoding matrices are thus given by

$$\mathbf{T}_k = \sqrt{\frac{P_T}{\sum_{j=1}^{K} \left\|\mathbf{T}'_{j,(N_I)}\right\|_F^2}}\, \mathbf{T}'_{k,(N_I)} \tag{12}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

The proposed HDM scheme does not necessarily require the receive matrices $\{\mathbf{R}_k\}$ to be transmitted from the BS to the UEs. Instead, each UE can estimate its receive matrix locally which is simply a linear MMSE (LMMSE) receiver given by

$$\hat{\mathbf{R}}_k = \left(\left(\mathbf{H}_k \mathbf{T}_k\right)^H \mathbf{H}_k \mathbf{T}_k + \alpha \mathbf{I}_{L_k}\right)^{-1} \left(\mathbf{H}_k \mathbf{T}_k\right)^H. \tag{13}$$

The product $\mathbf{H}_k \mathbf{T}_k$ in (13) represents the kth user's equivalent channel.

HDM provides maximum diversity gain when single-stream transmission is employed per user. This high diversity gain is a consequence of the two-stage optimization process that constitutes HDM. In the first stage, IMRBD precoding sequentially optimizes the utilization of the unused row subspaces of the users' channel matrices by repeatedly applying the modified RBD precoding. Further performance improvement is then accomplished in the second stage by minimizing the sum-MSE of all users for a given set of receive matrices.

IV. Simulation Results

We have used a quasi-static Rayleigh flat-fading channel model and quaternary phase-shift keying (QPSK) modulation for the simulations. A sufficiently large number of channel realizations are used for each simulated data point in the performance curves. Herein, we use the notation $N_T \times \{N_{R_1}(L_1), N_{R_2}(L_2), \ldots, N_{R_K}(L_K)\}$ to represent a K-user MU-MIMO system consisting of N_T BS antennas and N_{R_k} receive antennas at the kth UE supporting L_k data streams, for $k = 1, \ldots, K$. The performance comparison of various transmission schemes is provided in terms of the combined average BER of all the users versus the signal-to-noise ratio (SNR) defined as $P_T / N_R \sigma_n^2$.

Fig. 2 shows the uncoded BER performance of the proposed HDM scheme in comparison with other techniques for $4 \times \{2(1), 2(1)\}$ MU-MIMO configuration. Per-user SMMSE (PU-SMMSE) proposed in [16] is a modified low-complexity version of SMMSE with similar performance. MMSE power loading (MMSE PL) [8] is used to enhance the performance of PU-SMMSE but it still lags HDM by a huge margin. IRBD with improved diversity (impD) PL [8] and $N_I = 5$ iterations performs quite well. However, further increasing the number of iterations does not result in any significant performance improvement within the given SNR range and the performance gap between IRBD and HDM widens as SNR increases. At higher SNR values, HDM with $N_I = 5$ iterations performs slightly better than T-MMSE with $\varepsilon = 10^{-4}$ which is far more complex to implement and requires more iterations than HDM. Here ε represents the threshold for the stopping criterion used for T-MMSE in [12]. Using a smaller value for ε might improve the performance of T-MMSE by allowing more iterations, with the obvious consequence of increasing the complexity even further.

A major contributing factor to the complexity of the T-MMSE scheme is the procedure needed to obtain the Lagrange multiplier. The Lagrange multiplier υ for T-MMSE is calculated by numerically solving the equation

$$P_T = \sum_{j=1}^{N_T} \frac{\lambda_j}{(\lambda_j + \upsilon)^2} \tag{14}$$

and selecting the value of υ which gives the minimum sum-MSE, $\sum_{k=1}^{K} \mathcal{E}_k$ [12]. The λ_j in (14) represent the singular values of the matrix $\mathbf{A}$ in (11). In the given analysis employing single-stream transmission per user, only the L largest singular values in (14) have been considered while discarding the rest. However, the complexity still remains relatively high.

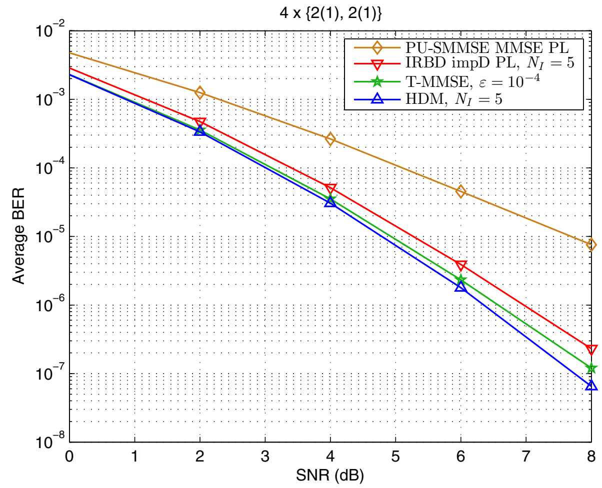

Fig. 2. Performance comparison of the HDM scheme with PU-SMMSE, IRBD and T-MMSE for $4 \times \{2(1), 2(1)\}$ MU-MIMO configuration.

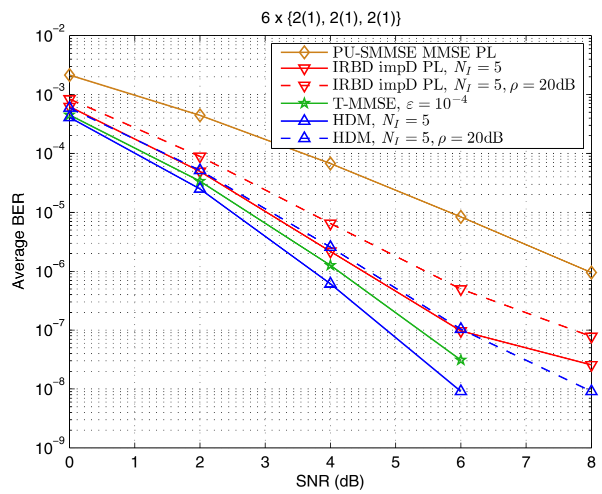

Fig. 3. Performance comparison of the HDM scheme with PU-SMMSE, IRBD and T-MMSE for $6 \times \{2(1), 2(1), 2(1)\}$ MU-MIMO configuration.

Fig. 3 shows the uncoded BER performance comparison for $6 \times \{2(1), 2(1), 2(1)\}$ MU-MIMO configuration. HDM clearly outperforms the other techniques showing its ability to exploit the system diversity more effectively. It provides much higher diversity gain resulting in significant performance improvement over IRBD which has similar complexity. It even outperforms the significantly more complex T-MMSE scheme. The performance of HDM and IRBD with imperfect CSI at the BS is also shown in the figure. The relative strength of the channel estimate available at the BS is represented by $\rho = \|\mathbf{H}\|^2 / \|\mathbf{E}_H\|^2$ where $\mathbf{E}_H \in \mathbb{C}^{N_R \times N_T}$ is the channel estimation error matrix whose elements are i.i.d. zero-mean complex Gaussian random variables and $\|\cdot\|$ denotes the matrix norm. The performance of HDM with $\rho = 20\text{dB}$ is almost identical to that of IRBD with perfect CSI which demonstrates its robustness towards channel estimation errors. In fact at around 8dB, IRBD experiences an error floor and lags behind HDM ($\rho = 20\text{dB}$) as a result.

V. Conclusion

In this paper, we have proposed a hybrid MU-MIMO downlink transmission scheme which effectively exploits the inherent diversity gain of the MIMO broadcast channel (MIMO-BC) to significantly improve system performance, thus providing a higher sum-rate. As seen in the previous section, maximum diversity gain for a certain antenna configuration can be achieved by using single-stream transmission per user. This scheme provides a means of implementing high-performance MU-MIMO downlink transmission systems without any drastic increase in complexity. It can even outperform more complex techniques like T-MMSE despite the lower complexity. The proposed scheme has also shown robustness against channel estimation errors and is capable of maintaining good performance even when perfect CSI is not available at the BS.

References

[1] T. Haustein, C. V. Helmolt, E. Jorswieck, V. Jungnickel, and V. Pohl, "Performance of MIMO systems with channel inversion," in *Proc. IEEE VTC 2002-Spring*, 2002, vol. 1, pp. 35–39.

[2] T. Haustein, A. Forck, H. Gäbler, C. V. Helmolt, V. Jungnickel, and U. Krüger, "Implementation of adaptive channel inversion in a real-time MIMO system," in *Proc. IEEE Int. Symp. Personal, Indoor Mobile Radio Communications (PIMRC'04)*, Barcelona, Spain, Sep. 2004, pp. 2524–2528.

[3] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part I: Channel inversion and regularization," *IEEE Trans. Communications*, vol. 53, no. 1, pp. 195–202, Jan. 2005.

[4] Q. H. Spencer, C. B. Peel, A. L. Swindlehurst, and M. Haardt, "An introduction to the multi-user MIMO downlink," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 60–67, Oct. 2004.

[5] C. B. Peel, B. M. Hochwald, and A. Lee Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part II: Perturbation," *IEEE Trans. Commun.*, vol. 53, no. 3, pp. 537–544, Mar. 2005.

[6] Q. H. Spencer and M. Haardt, "Capacity and downlink transmission algorithms for a multi-user MIMO channel," in *Proc. 36th Asilomar Conf. Signals, Systems, and Computers*, Nov. 2002, pp. 1384–1388.

[7] V. Stankovic and M. Haardt, "Multi-user MIMO downlink precoding for users with multiple antennas," in *Proc. 12th Wireless World Research Forum (WWRF)*, Toronto, ON, Canada, Nov. 2004.

[8] V. Stankovic and M. Haardt, "Generalized design of multiuser MIMO precoding matrices," *IEEE Trans. Wireless Communications*, vol. 7, no. 3, pp. 953 – 961, Mar. 2008.

[9] Z. Pan, K. K. Wong, and T. S. Ng, "Generalized multiuser orthogonal space-division multiplexing," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1969–1973, Nov. 2004.

[10] A. J. Tenenbaum and R. S. Adve, "Joint multiuser transmit–receive optimization using linear processing," in *Proc. IEEE Int. Conf. Commun.(ICC'04)*, Paris, France, Jun. 2004, pp. 588–592.

[11] M. Schubert, S. Shi, E. A. Jorswieck, and H. Boche, "Downlink sum-MSE transceiver optimization for linear multi-user MIMO systems," in *Proc. 39th Asilomar Conf. Signals, Systems, and Computers*, Pacific Grove, CA, USA, Oct. 2005.

[12] J. Zhang, Y. Wu, S. Zhou, and J. Wang, "Joint linear transmitter and receiver design for the downlink of multiuser MIMO systems," *IEEE Commun. Lett.*, vol. 9, pp. 991–993, Nov. 2005.

[13] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver design for multi-user MIMO systems," in *Proc. ITG/IEEE Workshop on Smart Antennas (WSA 2006)*, Ulm, Germany, Mar. 2006.

[14] B. Bandemer, M. Haardt, and S. Visuri, "Linear MMSE multi-user MIMO downlink precoding for users with multiple antennas," in *Proc. IEEE 17th Int. Symp. Personal, Indoor and Mobile Radio Comm. (PIMRC'06)*, 11–14 Sep. 2006, pp. 1–5.

[15] J. Joung and Y. H. Lee, "Regularized channel diagonalization for multiuser MIMO downlink using a modified MMSE criterion," *IEEE Trans. Signal Processing*, vol. 55, no. 4, pp. 1573–1579, Apr. 2007.

[16] M. Lee and S. K. Oh, "A per-user successive MMSE precoding technique in multiuser MIMO systems," in *Proc. IEEE VTC 2007-Spring*, 22–25 Apr. 2007, pp. 2374–2378.