On Asymptotic BER Performance of the Optimal Spatial-Temporal Power Adaptation Scheme with Imperfect CSI

Quan Kuang
Institute of Telecommunications
University of Stuttgart
70569 Stuttgart, Germany
Email: quan.kuang@inue.uni-stuttgart.de

Shu-Hung Leung
Department of Electronic Engineering
City University of Hong Kong
Kowloon, Hong Kong SAR
Email: eeeugshl@cityu.edu.hk

***Abstract*—This paper studies the asymptotic bit-error-rate (BER) performance of the joint spatio-temporal (S-T) power adaptation which has recently been proposed for beamforming space-time-coded multiantenna systems in the presence of imperfect channel state information at the transmitter (CSIT). We derive a closed-form asymptotic solution of the power adaptation, which is instrumental in characterizing the asymptotic BER performance. We show that with imperfect CSIT, the optimal S-T scheme gives exponential diversity gain in the low and medium SNR regions when the quality of CSIT is relatively good, but cannot increase the diversity order of the space-time-coded MIMO link at very high SNR. Compared to the previous design by treating imperfect CSIT as perfect to adjust the temporal power, the optimal S-T is more robust in retaining the exponential decrease in BER over a larger range of SNR. The results sharpen our understanding of the behavior of the optimal S-T strategy.**

I. Introduction

The time-varying fading features of wireless channels have posed an enormous challenge for the system design. In an additive white Gaussian noise (AWGN) channel, the bit-error-rate (BER) decreases exponentially with the signal-to-noise ratio (SNR). However, the average BER in a Rayleigh-fading channel decreases only inversely with SNR (i.e. SNR^{-1}) [1], [2], at least at high SNR. This indicates a severe degradation in BER performance over Rayleigh fading channels. This degradation can be partially mitigated by installing multiple antennas at the transmitter and/or receiver. One popular signaling scheme to obtain the benefits of multiple antennas is the space-time block codes (STBCs) [3], [4], where the transmit diversity can be achieved without the knowledge of channel state information (CSI) at the transmitter (CSIT). By using STBC, the average BER decays with SNR^{-MN}, where M and N are the antenna numbers at the transmitter and receiver respectively, and MN is called the diversity order. If (partial/imperfect) CSIT is available, the BER performance of STBC systems can be further improved via precoding [5]–[10]. The STBCs exploit the space diversity of multiple-input and multiple-output (MIMO) channels whereas the precoding exploits the eigen-property of the CSIT. This STBC and precoding combination is robust to channel fading and provides an efficient way to exploit the available CSIT [11].

For many common forms of imperfect/partial CSIT, such as the channel mean [8], [10], channel correlation [9], compound channel model [12], the optimal precoder consists of adaptive power allocation and multiple eigen-beamforming, where the outputs of the STBC are power-loaded and then transmitted along eigen directions of the autocorrelation matrix of the spatial channel estimate. Among these transmitters, various techniques and criteria have been used to derive the power adaptation strategies. However, most of the existing power adaptation schemes perform power allocation only in the spatial domain among eigenbeams. It has been shown that the diversity order remains MN for precoded STBC systems with spatial-only power allocation [8].

Recently, power allocation both in space and time has been proposed in [13] and [14] for precoded STBC systems. In [13], it shows that with *perfect* CSI, the BER decease exponentially with SNR (i.e. infinite diversity order), as in an AWGN channel. However, when the CSIT is imperfect, the diversity order reduces to MN at high SNR. Since the power adaptation approach adopted in [13] treated the imperfect CSIT as perfect to control the temporal power, it is interesting to investigate whether the optimal design in [14] by explicitly considering the imperfection of CSIT can improve the diversity at large SNR. However, the asymptotic analysis is missing in [14].

This paper studies the asymptotic performance of the optimal spatio-temporal (S-T) power adaptation scheme proposed by [14]. We derive an asymptotic solution of the power adaptation. Compare to the optimal design in [14] which requires numerical search, the closed-form asymptotic solution is simpler to implement. More important, the asymptotic solution enables us to study the diversity gain analytically. We show that with imperfect CSIT, the optimal S-T scheme gives exponential diversity gain in the low and medium SNR regions when the quality of CSIT is relatively good, but cannot increase the diversity order of the space-time coded MIMO link at very high SNR. Although the diversity order cannot be improved at very high SNR, the optimal S-T is more robust than the previous design in [13] in retaining the exponential

ISBN 978-3-8007-3482-5

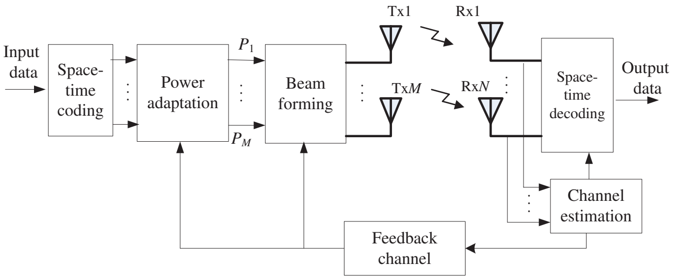

Fig. 1. System diagram

decrease in the average BER over a larger SNR region.

The remainder of this paper is organized as follows. In Section II, we outline our system model and the joint spatio-temporal power adaptation scheme of [14]. In section III, the asymptotic solution is derived. Afterwards, the diversity analysis is provided in Section IV, followed by the numerical results in Section V. Finally, we conclude the paper in Section VI.

II. System model

We consider a wireless multi-antenna communication system with M transmit antennas and N receive antennas operating over a flat and quasi-static Rayleigh fading channel as depicted in Fig.1. The space-time encoder, which is represented by an $M \times T$ orthogonal STBC (OSTBC) transmission codeword matrix $\mathbf{D}$ [4], is used to encode K data symbols into an M-dimensional vector sequence of T time slots with code rate $r = K/T$. The OSTBC vectors are sent along the M eigen-directions of the autocorrelation matrix of the spatial channel estimate at the transmitter with power allocation in space and time.

The channel is represented by an $N \times M$ matrix $\mathbf{H} = \{h_{nm}\}$, where h_{nm} denotes the channel gain from the *m*th transmit antenna to the *n*th receive antenna. It is assumed that h_{nm} remains constant over an OSTBC frame and varies from frame to frame, and $\{h_{nm}\}$ are modeled as independent identically distributed (i.i.d.) complex Gaussian random variables (r.v.s) with zero-mean and variance 0.5 per dimension. At the transmitter, only an imperfect channel estimate $\hat{\mathbf{H}}$ is available for the current frame, modeled as $\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E}$ [15], [16], where $\mathbf{E}$ is the channel error matrix independent of $\mathbf{H}$. The elements of $\mathbf{E}$ are assumed to be i.i.d. complex Gaussian r.v.s with zero mean and variance σ_e^2.

Let $\mathbf{h} = \text{vec}(\mathbf{H})$, $\hat{\mathbf{h}} = \text{vec}(\hat{\mathbf{H}})$, and $\mathbf{e} = \text{vec}(\mathbf{E})$ be the column vectors constructed by stacking the columns of $\mathbf{H}$, $\hat{\mathbf{H}}$, and $\mathbf{E}$ respectively. Based on the Bayesian Linear Model and Theorem 10.3 in [17], the mean and covariance matrices of $\mathbf{h}$ given $\hat{\mathbf{h}}$ are given as

$$E[\mathbf{h}|\hat{\mathbf{h}}] = \mathbf{C_h}(\mathbf{C_h} + \mathbf{C_e})^{-1}\hat{\mathbf{h}} = (1+\sigma_e^2)^{-1}\hat{\mathbf{h}}, \quad (1)$$

$$\mathbf{C}_{\mathbf{h}|\hat{\mathbf{h}}} = \mathbf{C_h} - \mathbf{C_h}(\mathbf{C_h} + \mathbf{C_e})^{-1}\mathbf{C_h} = \sigma_e^2(1+\sigma_e^2)^{-1}\mathbf{I}_{NM} \quad (2)$$

where $E[\cdot]$ denotes the expectation, $\mathbf{C_e} = \sigma_e^2\mathbf{I}_{NM}$ and $\mathbf{C_h} = \mathbf{I}_{NM}$ are the covariance matrices of $\mathbf{e}$ and $\mathbf{h}$ respectively, $\mathbf{I}_{NM}$ is the $NM \times NM$ identity matrix. Hence, conditioned on $\hat{\mathbf{H}}$, the elements $\{h_{nm}\}$ of $\mathbf{H}$ become complex Gaussian r.v.s with mean $(1+\sigma_e^2)^{-1}\hat{h}_{nm}$ and variance $\sigma_e^2(1+\sigma_e^2)^{-1}$.

The received signals of the system can be expressed as

$$\mathbf{Y} = \sqrt{S}\mathbf{H}\hat{\mathbf{U}}\mathbf{P}\mathbf{D} + \mathbf{Z} = \sqrt{S}\bar{\mathbf{H}}\mathbf{P}\mathbf{D} + \mathbf{Z} \quad (3)$$

where $\bar{\mathbf{H}} \triangleq \mathbf{H}\hat{\mathbf{U}}$, $\hat{\mathbf{U}} = \{\hat{u}_{ij}, i,j = 1,\dots,M\}$ is an $M \times M$ unitary matrix containing the M-eigenvectors of $\hat{\mathbf{H}}^H\hat{\mathbf{H}}$ corresponding to the eigenvalues $\{\hat{\zeta}_m\}$ sorted in decreasing order (the superscript H stands for conjugate transpose), $\mathbf{D}$ is the OSTBC codeword matrix with normalized energy as $E[\text{tr}(\mathbf{D}\mathbf{D}^H)]/T = 1$, $\mathbf{Z}$ is an $N \times T$ received noise matrix with i.i.d. entries modeled as complex Gaussian r.v.s with zero mean and variance σ_n^2, S is the total transmit power radiated from the M transmit antennas, $\mathbf{Y}$ is the $N \times T$ received signal matrix, and $\mathbf{P} = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_M})$ denotes a diagonal power allocation matrix which satisfies

$$\sum_{m=1}^{M} P_m = 1 \quad (4)$$

$$P_m \geq 0, \quad m = 1,\dots,M. \quad (5)$$

It is assumed that the receiver perfectly knows the CSI. After space-time decoding, the instantaneous received SNR per symbol at the receiver is expressed as [14]

$$\rho = \frac{S}{r\sigma_n^2}\|\bar{\mathbf{H}}\mathbf{P}\|_F^2 = \frac{S}{r\sigma_n^2}\sum_{m=1}^{M} P_m\beta_m, \quad (6)$$

where $\|\cdot\|_F$ denotes the Frobenius norm, β_m is the *m*th eigen-channel power gain defined as

$$\beta_m = \sum_{n=1}^{N} |\bar{h}_{nm}|^2 = \sum_{n=1}^{N} |\sum_{i=1}^{M} h_{ni}\hat{u}_{im}|^2. \quad (7)$$

A. Optimal Spatio-Temporal Power Adaptation Scheme of [14]

In the scheme of [14], the total power S is subject to the long term (time) average constraint and can be varied from one OSTBC block to another according to the updated CSIT, which is referred to as temporal power allocation. $\{P_m\}$'s are the spatial power allocation parameters. The optimal algorithm is derived from the minimization of a tight BER approximation as follows.

We consider quadrature amplitude modulation (QAM) of size Q and Gray mapping. The following unified BER approximation for all QAM constellations has been used in [14]:

$$\text{BER}_\rho \approx 0.2\exp(-g\rho) \quad (8)$$

where ρ is the received SNR and

$$g = \begin{cases} \frac{1.5}{Q-1} & \text{for square QAM} \\ \frac{6}{5Q-4} & \text{for rectangular QAM.} \end{cases} \quad (9)$$

This approximation is also tight for BPSK by regarding it as a special case of rectangular QAM with $Q = 2$.

With (6), the BER approximation of (8) can be written as

$$\text{BER}_\rho \approx 0.2\exp\left(-\gamma\sum_{m=1}^{M} P_m\beta_m\right) \quad (10)$$

ISBN 978-3-8007-3482-5

where

$$\gamma \triangleq \frac{gS}{r\sigma_n^2}. \tag{11}$$

Since $\{\beta_m\}$ in (10) are not available at the transmitter, the transmitter can rely on the following conditional average BER given $\hat{\mathbf{H}}$ to adapt the power [14]:

$$\begin{aligned} P_{b|\hat{\mathbf{H}}} &= E_{\boldsymbol{\beta}}[\mathrm{BER}_\rho|\hat{\mathbf{H}}] \\ &= 0.2\prod_{m=1}^{M}\frac{1}{(1+\gamma\sigma^2P_m)^N}\exp\left(-\frac{\gamma\tilde{\beta}_mP_m}{1+\gamma\sigma^2P_m}\right) \end{aligned} \tag{12}$$

where

$$\sigma^2 = \frac{\sigma_e^2}{1+\sigma_e^2} \tag{13}$$

$$\tilde{\beta}_m = \frac{1}{(1+\sigma_e^2)^2}\sum_{n=1}^{N}|\sum_{i=1}^{M}\hat{h}_{ni}\hat{u}_{im}|^2 = \frac{\hat{\zeta}_m}{(1+\sigma_e^2)^2} \tag{14}$$

and $\tilde{\beta}_1 \geq \tilde{\beta}_2 \geq \ldots \geq \tilde{\beta}_M$ because the eigenvalues satisfy $\hat{\zeta}_1 \geq \hat{\zeta}_2 \geq \ldots \geq \hat{\zeta}_M$.

The optimization problem of minimizing the average BER by joint spatio-temporal power adaptation can be formulated as

$$\underset{\gamma(\tilde{\boldsymbol{\beta}}),\{P_m(\tilde{\boldsymbol{\beta}})\}}{\text{minimize}} \quad E_{\tilde{\boldsymbol{\beta}}}[P_{b|\hat{\mathbf{H}}}(\gamma(\tilde{\boldsymbol{\beta}}),P_m(\tilde{\boldsymbol{\beta}}),\tilde{\boldsymbol{\beta}})] \tag{15a}$$

$$\text{s.t.} \quad E_{\tilde{\boldsymbol{\beta}}}[\gamma(\tilde{\boldsymbol{\beta}})] = \frac{g\bar{S}}{r\sigma_n^2} \triangleq \bar{\gamma}, \gamma(\tilde{\boldsymbol{\beta}}) \geq 0 \tag{15b}$$

$$\sum_{m=1}^{M}P_m(\tilde{\boldsymbol{\beta}}) = 1,\ P_m(\tilde{\boldsymbol{\beta}}) \geq 0\ \forall m \tag{15c}$$

where $P_{b|\hat{\mathbf{H}}}$ is given in (12), $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_1,\ldots,\tilde{\beta}_M)$ denotes the available CSIT defined as (14), and $\bar{S}$ is the power budget.

This problem can be solved optimally by an inner-outer formulation introduced in [14]. The solution is

$$\gamma^\star(\tilde{\boldsymbol{\beta}}) = \frac{1}{\sigma^2}\left(\sum_{m=1}^{M_0}\mathcal{I}_m(\lambda^\star) - M_0\right) \tag{16}$$

$$P_m^\star(\tilde{\boldsymbol{\beta}}) = \max\left\{0, \frac{1}{\sigma^2\gamma^\star(\tilde{\boldsymbol{\beta}})}\left(\mathcal{I}_m(\lambda^\star)-1\right)\right\} \tag{17}$$

where

$$\mathcal{I}_m(\lambda) = \frac{\sqrt{N^2\sigma^4+4\tilde{\beta}_m\lambda}+N\sigma^2}{2\lambda} \tag{18}$$

and $\lambda^\star$ is uniquely given by the following equation

$$0.2\prod_{m=1}^{M_0}\frac{1}{\mathcal{I}_m(\lambda)^N}\exp\left(\frac{\tilde{\beta}_m}{\mathcal{I}_m(\lambda)\sigma^2}-\frac{\tilde{\beta}_m}{\sigma^2}\right)\lambda = \xi \tag{19}$$

where M_0 is the number of eigenbeams allocated with nonzero power, ξ is the Lagrange multiplier whose value is determined such that the average power constraint in (15b) is satisfied. Note that ξ is a constant determined by the statistical distribution of $\tilde{\boldsymbol{\beta}}$, whereas λ is a function of the realization of $\tilde{\boldsymbol{\beta}}$ as implicitly given by (19). Please see [14] for the method to determine the value of M_0 and ξ.

III. Asymptotic solution

The optimal solution given in (16) to (19) requires numerical search. In this section, we derive the closed-form asymptotic solution to simplify the calculation. Besides, the asymptotic solution enables us to study the asymptotic diversity gain analytically in Section IV.

At high SNR, the optimal spatial power allocation strategy is to distribute the power equally among all the eigenbeams [6], [10]. Considering $P_1^\star = P_2^\star = \cdots = P_M^\star = \frac{1}{M}$, we neglect $\tilde{\beta}_m\lambda$ in the square root in (18) since λ is generally small for high SNR case. Then (18) becomes

$$\mathcal{I}_m(\lambda) \approx \frac{N\sigma^2}{\lambda} = \frac{1}{\tilde{\lambda}}, \quad m = 1,\ldots,M \tag{20}$$

where $\tilde{\lambda} \triangleq \lambda/(N\sigma^2)$.

After we substitute (20) to (19) and consider $M_0 = M$, (19) is simplified into

$$\tilde{\lambda}^{MN+1}e^{b\tilde{\lambda}} = \frac{5\xi}{N\sigma^2}e^b \tag{21}$$

where

$$b \triangleq \frac{\sum_{m=1}^{M}\tilde{\beta}_m}{\sigma^2}. \tag{22}$$

The solution to (21) can be expressed in terms of W-function [18] as

$$\tilde{\lambda}^\star = \frac{MN+1}{b}W\left(\frac{b}{MN+1}\left(\frac{5\xi e^b}{N\sigma^2}\right)^{\frac{1}{MN+1}}\right) \tag{23}$$

where $W(\cdot)$ denotes the principal branch of the Lambert W function, whose value can be accurately calculated [18]. The Lambert function $W(x)$ is a monotonically increasing function of x for $x \geq 0$. Thus the solution of $\tilde{\lambda}^\star$ is unique. With $\tilde{\lambda}^\star$, the temporal power is obtained by substituting (20) into (16), resulting in

$$\gamma^\star(\tilde{\boldsymbol{\beta}}) = \frac{M}{\sigma^2}\left(\frac{1}{\tilde{\lambda}^\star}-1\right). \tag{24}$$

In order to obtain positive value of $\gamma^\star$ from (24), the value of the Lagrange multiplier ξ should satisfy

$$0 < \xi < 0.2N\sigma^2. \tag{25}$$

Interestingly, once (25) is satisfied, $\tilde{\lambda}^\star$ is always less than 1 for all channel realizations as shown in (26), ensuring that $\gamma^\star$ obtained from (24) is always positive

$$\tilde{\lambda}^\star < \frac{MN+1}{b}W\left(\frac{b}{MN+1}e^{\frac{b}{MN+1}}\right) = 1. \tag{26}$$

Equations (23) and (24) provide the closed-form formulae to calculate the temporal power parameter $\gamma^\star$ for high SNR. The value of ξ in (23) can be determined offline from the average power constraint. We propose to adopt Newton's method as follows.

First we express the average power constraint in (15b) using Gauss-Laguerre numerical integration formula [19] as

$$\bar{\gamma} = \int\gamma(b)f_b(b)db \approx \sum_{i=1}^{N_p}w_i\frac{1}{(L-1)!}z_i^{(L-1)}\gamma(b_i) \tag{27}$$

ISBN 978-3-8007-3482-5

where $f_b(b)$ is the pdf of b defined in (22), $L = M \times N$, $\{w_i\}$ denote the weights associated with zeros $\{z_i\}$ of the one-dimensional N_pth order Laguerre polynomial [19], $\gamma(b_i)$ is obtained from (24) in which the $\tilde{\lambda}^\star$ is calculated by substituting $b_i = z_i/[(1+\sigma_e^2)\sigma^2]$ into the parameter b of (23). The detail of the derivation of (27) can be found in Appendix.

From the constraint (27), we let

$$\Phi(\xi) = \bar{\gamma} - \sum_{i=1}^{N_p} w_i \frac{1}{(L-1)!} z_i^{(L-1)} \gamma(b_i) \tag{28}$$

The value of ξ is the root of the monotonic $\Phi(\xi)$, which can be calculated by Newton's method as

$$\xi_{n+1} = \xi_n - \Phi(\xi_n)/\Phi'(\xi_n) \tag{29}$$

where ξ_n is the value of ξ at the nth iteration of the algorithm. The first order derivative of $\Phi(\xi)$ with respect to ξ is

$$\Phi'(\xi) = \frac{M}{\sigma^2} \sum_{i=1}^{N_p} \frac{w_i}{(L-1)!} \frac{z_i^{(L-1)}}{\tilde{\lambda}^2(b_i)} \tilde{\lambda}'(b_i) \tag{30}$$

where

$$\tilde{\lambda}'(b_i) = \frac{W(y_i)}{[1+W(y_i)]} \frac{1}{\xi b_i} \tag{31}$$

with

$$y_i = \frac{b_i}{MN+1} \left(\frac{5\xi e^{b_i}}{N\sigma^2} \right)^{\frac{1}{MN+1}} \tag{32}$$

The initial value of ξ for Newton's method can be derived from the condition of (25). Since ξ should be within the range of $(0,\ 0.2N\sigma^2)$, we set the initial value $\xi_0 = 0.1N\sigma^2$.

Now we complete the derivation of the asymptotic solution. As will be shown in section V, its performance agrees with the optimal solution of [14] perfectly in high SNR regions.

IV. Diversity Gain

In this section, we analyze the diversity gain of the optimal spatio-temporal power adaptation by using the asymptotic solution we derived previously.

For very large SNR, it is noted that the Lagrange multiplier ξ in (23) of the asymptotic solution is very small. Thus, the argument of the Lambert function in (23) becomes small. The Taylor's series expansion of the principle branch of the Lambert function around zero is [18]

$$W(x) = x - x^2 + \frac{3}{2}x^3 - \dots.$$

We can use the following function $g(x)$ to approximate the Lambert function, which is a bounded function whose Taylor's expansion has the first two terms identical to the Lambert function:

$$g(x) = \frac{x}{1+x}.$$

Using $g(x)$, the $\tilde{\lambda}^\star$ in (23) is approximated by

$$\tilde{\lambda} \approx \frac{\left(\frac{5\xi e^b}{N\sigma^2}\right)^{\frac{1}{MN+1}}}{1 + \frac{b}{MN+1}\left(\frac{5\xi e^b}{N\sigma^2}\right)^{\frac{1}{MN+1}}} \approx \frac{\left(\frac{5\xi e^b}{N\sigma^2}\right)^{\frac{1}{MN+1}}}{1 + \epsilon\left(\frac{5\xi e^b}{N\sigma^2}\right)^{\frac{1}{MN+1}}} \tag{33}$$

where $\epsilon \geq 1$ is a constant to make $\tilde{\lambda}$ less than 1, whose value can be determined experimentally. As a result, according to (24), γ is guaranteed to be positvie and expressed as

$$\gamma \approx \frac{M}{\sigma^2} \left[\left(\frac{5\xi e^b}{N\sigma^2} \right)^{-\frac{1}{MN+1}} + \epsilon - 1 \right]. \tag{34}$$

Substituting the γ in (34) into the average power constraint gives

$$\begin{aligned} \bar{\gamma} &= \int_0^\infty \frac{M}{\sigma^2} \left[\left(\frac{5\xi e^b}{N\sigma^2} \right)^{-\frac{1}{MN+1}} + \epsilon - 1 \right] f_b(b) db \\ &= \frac{M}{\sigma^2} \left[\left(\frac{5\xi}{N\sigma^2} \right)^{-\frac{1}{MN+1}} \left(\frac{1}{\kappa_1} \right)^{MN} + \epsilon - 1 \right] \end{aligned} \tag{35}$$

where $f_b(b)$ is the pdf of b defined in (22), which has been derived in Appendix, $\kappa_1 = 1 + \frac{1}{(1+\sigma_e^2)\sigma^2(MN+1)}$.

From (35), we can obtain the Lagrange multiplier ξ as

$$\left(\frac{5\xi}{N\sigma^2} \right)^{\frac{1}{MN+1}} = \frac{M}{\bar{\gamma}\sigma^2 + M(1-\epsilon)} \left(\frac{1}{\kappa_1} \right)^{MN}. \tag{36}$$

Note that in order to have positive ξ, ϵ must satisfy $1 \leq \epsilon < 1 + \frac{\bar{\gamma}\sigma^2}{M}$. Equation (36) verifies that ξ becomes small as $\bar{\gamma}$ increases.

Substituting (36) into (33) and (34) gives

$$\tilde{\lambda} \approx \frac{M\kappa_1^{-MN}}{[\bar{\gamma}\sigma^2 + M(1-\epsilon)]e^{-\frac{b}{MN+1}} + \epsilon M \kappa_1^{-MN}} \tag{37}$$

$$\gamma \approx \frac{\bar{\gamma}\sigma^2 + M(1-\epsilon)}{\sigma^2} \kappa_1^{MN} e^{-\frac{b}{(MN+1)}} + \frac{M}{\sigma^2}(\epsilon - 1) \tag{38}$$

Equations (37) and (38) can be regarded as simplified modifications of the original asymptotic solution (23) and (24).

Now we apply the above results to analyze the asymptotic behavior of the average BER. For large SNR, the spatial power allocation P_m approximately equals $1/M$. Hence, the conditional BER given $\hat{\mathbf{H}}$ is expressed as

$$\begin{aligned} P_{b|\hat{\mathbf{H}}} &\approx 0.2 \left(\frac{M}{M + \gamma\sigma^2} \right)^{MN} e^{-\frac{\gamma\sigma^2 b}{(M+\gamma\sigma^2)}} \\ &= 0.2 \tilde{\lambda}^{MN} e^{b\tilde{\lambda} - b} \end{aligned} \tag{39}$$

Based on (21), the conditional BER given $\hat{\mathbf{H}}$ in (39) is simplified as

$$P_{b|\hat{\mathbf{H}}} = \frac{\xi}{\tilde{\lambda} N \sigma^2}. \tag{40}$$

Based on (40) together with (37), the average BER for large SNR is

$$\begin{aligned} P_b &= \int_0^\infty P_{b|\hat{\mathbf{H}}} f_b(b) db \\ &\approx \frac{\xi}{N\sigma^2} \int_0^\infty \frac{1}{\tilde{\lambda}} f_b(b) db \\ &= \frac{\xi}{N\sigma^2} \left[\frac{\bar{\gamma}\sigma^2 + M(1-\epsilon)}{M} + \epsilon \right]. \end{aligned} \tag{41}$$

ISBN 978-3-8007-3482-5

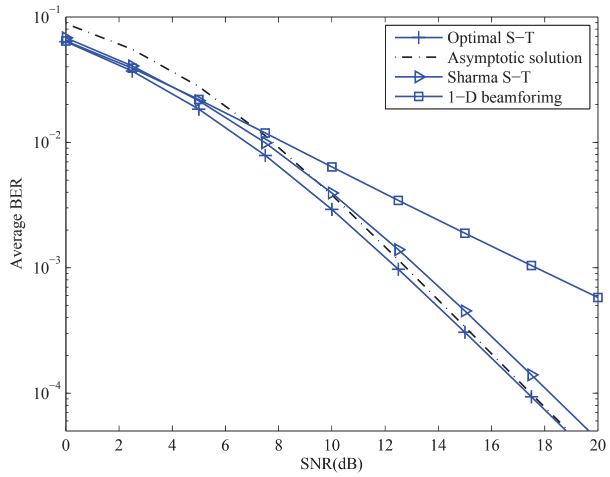

Fig. 2. Asymptotic performance of the optimal S-T power adaptation , $M = 2, N = 1, \sigma_e^2 = 0.4$, and BPSK.

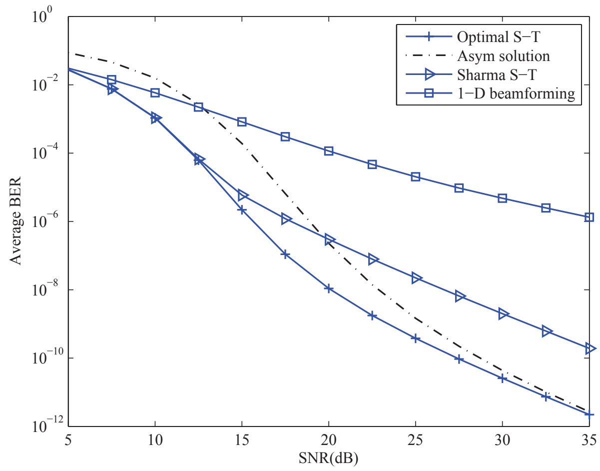

Fig. 3. Asymptotic performance of the optimal S-T power adaptation , $M = 2, N = 1, \sigma_e^2 = 0.01$, and QPSK.

Substituting ξ in (36) into (41) obtains

$$P_b = 0.2\kappa_1^{-MN(MN+1)} \left(\frac{M}{\bar{\gamma}\sigma^2 + M(1-\epsilon)} \right)^{MN} + 0.2\epsilon \left(\frac{M\kappa_1^{-MN}}{\bar{\gamma}\sigma^2 + M(1-\epsilon)} \right)^{MN+1} . \quad (42)$$

For large SNR, $\bar{\gamma}$ is large. Thus, (42) can be approximated as

$$P_b \approx 0.2 \left(\frac{M}{\sigma^2 \kappa_1^{MN+1}} \right)^{MN} \bar{\gamma}^{-MN}. \quad (43)$$

which shows that the diversity gain is MN.

V. Numerical results

In this section, we provide numerical results to validate the asymptotic solution and the diversity result we derived. In the evaluation, G_2 code (Alamouti code) with $r = 1$, and H_3 with $r = 3/4$ are used for illustration. The specific code matrices can be found in [4]. We denote a system with M transmit and N receive antennas as $M \times N$ system. In the following figures, SNR is defined as $\bar{S}/\sigma_n^2$. We refer to the existing spatio-temporal scheme in [13] as Sharma S-T, and the optimal solution in [14] as Optimal S-T. For comparison, we also give the results of using conventional beamforming without STBC where the signal with constant power is transmitted only along the eigen direction with the largest eigenvalue (named as 1-D beamforming).

In our performance evaluation, we use the following formula to determine the average BER:

$$P_b = E_{\hat{\mathbf{H}}}[P_{b|\hat{\mathbf{H}}}] \quad (44)$$

where $P_{b|\hat{\mathbf{H}}}$ has been given in (12), and the expectation is calculated by Gauss-Laguerre numerical integration.

Figs.2 and 3 show the performance comparison. The system being used for evaluation is a 2×1 system with BPSK and QPSK, respectively. We consider moderate estimation error ($\sigma_e^2 = 0.4$) in Fig.2 and small estimation ($\sigma_e^2 = 0.01$) in Fig.3. It is verified in the figures that the asymptotic solution is valid, resulting in the same performance as the optimal S-T solution asymptotically in high SNR regions. It is also observed from Fig.3 that for small estimation error variance the optimal S-T has exponential diversity in the low and medium SNR regions. This result makes the optimal S-T outperform Sharma S-T scheme significantly for small estimation error variance. However, the diversity gain asymptotically reduces to MN(=2) at high SNR. This asymptotic diversity gain will be further illustrated in Fig.4. Constant-power 1-D beamforming lacks the feasibility of adapting the power either in space or in time, leading to the worst performance.

The comparison of the asymptotic BER for very large SNR in (43) with the BER performance is shown in Fig.4. The 2×1 and 3×1 systems are considered. The modulation is QPSK and the estimation error σ_e^2 is set equal to 0.05 and 0.01. As shown in the figure, the closed-form asymptotic BER of (43) is in good agreement with the BER performance curves at high SNR. As derived in Section IV, the temporal power adaptation cannot improve the diversity gain at high SNR when the CSIT is imperfect. The reason is that the BER performance is mainly limited by the additive noise at low SNR and at medium SNR, but largely affected by the errors in the CSIT at high SNR. Through adapting the temporal power to combat fading effects, we can achieve exponential decrease in BER yielding similar performance behavior as in AWGN channel. At very high SNR, however, the diversity gain we can obtain is MN, just like the space-time coding methods with no knowledge of CSIT. This result shows that the imperfection of CSIT at very high SNR causes the feedback knowledge of CSIT is useless. Although the diversity gain cannot be improved at high SNR by the power adaptation, it is clearly shown from Fig.2 and Fig.3 that the optimal S-T scheme outperforms the existing Sharma S-T approach since it is more robust than the existing scheme in retaining the exponential decrease in BER over a

ISBN 978-3-8007-3482-5

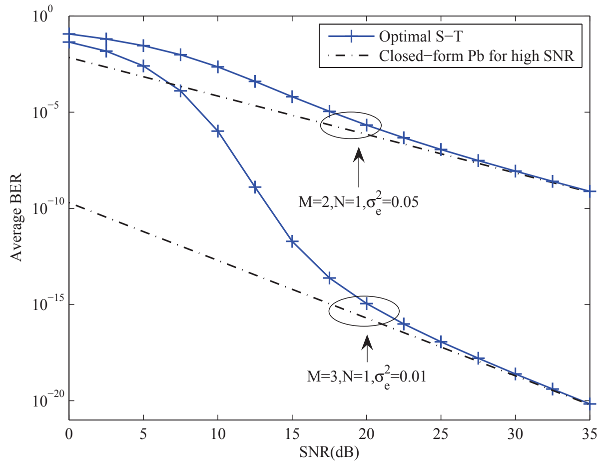

Fig. 4. Diversity analysis for Optimal S-T power adaptation under different system configurations.

larger SNR region for different σ_e^2.

VI. Conclusion

In this paper, we studied the asymptotic performance of the optimal spatio-temporal power adaptation scheme. The average BER is characterized by using an unified tight approximation for QAM constellations, including BPSK as a special case. A closed-form asymptotic solution of the power adaptation has been derived, based on which the asymptotic diversity gain has been analyzed. The results show that by adapting the temporal power to combat fading effect, we can achieve exponential decrease in average BER at low and medium SNR, yielding the performance behavior similar as in AWGN channel. At very high SNR, however, the diversity gain we can obtain is the same as that of space-time coding without the knowledge of CSIT, since the errors in the CSIT becomes a dominant factor making the CSIT less and less useful. Although the diversity order cannot be improved at very high SNR, the optimal design by explicitly considering the CSIT imperfection achieves the exponential decrease in BER over a larger range of SNR under different CSIT uncertainties, in comparison with the previous design. These results provide us better understanding of the behavior of the optimal S-T design with imperfect CSIT.

Appendix

Derivation of the numerical integration of (27)

According to (22), (14) and our channel estimation model, we have

$$b = \frac{\sum_{i=1}^{M} \hat{\zeta}_i}{(1+\sigma_e^2)^2\sigma^2} = \frac{\|\mathbf{H}\|_F^2}{(1+\sigma_e^2)\sigma^2} = \frac{t}{(1+\sigma_e^2)\sigma^2}$$

where we define $t \triangleq \|\mathbf{H}\|_F^2$, which is central chi-square distributed with the degree of freedom of $2L$, and $L = M \times N$. The pdf of t is given as

$$f_t(t) = \frac{1}{(L-1)!} t^{L-1} e^{-t}.$$

According to the Gauss-Laguerre formula [19], the expectation of γ is

$$\begin{aligned} E_{\tilde{\boldsymbol{\beta}}}[\gamma(\tilde{\boldsymbol{\beta}})] &= \int \gamma(\tilde{\boldsymbol{\beta}}) f_{\tilde{\boldsymbol{\beta}}}(\tilde{\boldsymbol{\beta}}) d\tilde{\boldsymbol{\beta}} \\ &= \int_0^\infty \gamma(b) f_b(b) db \\ &= \int_0^\infty \gamma\left(\frac{t}{(1+\sigma_e^2)\sigma^2}\right) f_t(t) dt \\ &\approx \sum_{i=1}^{N_p} w_i \gamma\left(\frac{z_i}{(1+\sigma_e^2)\sigma^2}\right) \frac{1}{(L-1)!} z_i^{L-1} \end{aligned}$$

which gives (27).

References

[1] D.Tse and P.Vishwanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2005.

[2] A.Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2005.

[3] E. G. Larrsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2003.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: performance results," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 3, pp. 451–460, March 1999.

[5] A. Pascual-Iserte, D. P. Palomar, A. I. Perez-Neira, and M. A. Lagunas, "A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 346–360, Jan. 2006.

[6] G. Jongren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Trans. Inf. Theory*, vol. 48, no. 3, pp. 611–627, March 2002.

[7] M. Vu and A. Paulraj, "Optimal linear precoders for MIMO wireless correlated channels with nonzero mean in space-time coded systems," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2318–2332, June 2006.

[8] J. W. Huang, E. K. S. Au, and V. K. N. Lau, "Precoder design for space-time coded MIMO systems with imperfect channel state information," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 1977–1981, June 2008.

[9] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1673–1690, July 2003.

[10] ——, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2599–2613, Oct. 2002.

[11] M. Vu and A. Paulraj, "MIMO wireless linear precoding," *IEEE Signal. Proc. Mag.*, vol. 24, no. 5, pp. 86–105, 2007.

[12] J. Wang and D. P. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3086–3100, 2009.

[13] V. Sharma, K. Premkumar, and R. N. Swamy, "Exponential diversity achieving spatio-temporal power allocation scheme for fading channels," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 188–208, Jan. 2008.

[14] Q. Kuang, S.-H. Leung, and X. Yu, "Optimal joint spatial and temporal power adaptation for space-time-coded systems with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 3335–3339, 2012.

[15] E. G. Larsson, "Diversity and channel estimation errors," *IEEE Trans. Commun.*, vol. 52, no. 2, pp. 205–208, Feb. 2004.

[16] Z. Shi and H. Leib, "Transmit antenna selected V-BLAST systems with power allocation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2293–2304, July 2008.

[17] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theroy*. Englewood Cliffs: NJ: Prentice-Hall, 1993.

[18] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, "On the Lambert W function," *Advances in Computational Mathematics*, vol. 5, pp. 329–359, 1996.

[19] P. J. Davis and P. Rabinowitz, *Methods of Numerical Integration*. New York: Academic Press, 1975.

ISBN 978-3-8007-3482-5